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A Probabilistic Analysis of Transmission Right ValuationUnder Market Uncertainty

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ABSTRACT

The paper presents a simulation-based approach for Financial Transmission Right (FTR) and Flowgate Right (FGR) valuation. We calculate the probabilistic distributions of FTR and FGR values under market uncertainties resulting from fluctuating system loads, non-constant supply/ demand bids, and other system contingencies such as unplanned transmission circuit etc. A theoretical framework consisting of a multilateral-transaction model, a nonconforming electric load model, and transmission right valuation models under different market structures incorporating market aforementioned uncertainties is proposed. A Monte Carlo Simulation procedure is employed to obtain the distributions of FTR and FGR values. Numerical implementation of the proposed approach is illustrated on a sample test system.

1. INTRODUCTION

The restructuring process of the electric power industry has attracted a significant amount of attention in the generation sector since its start. Due to the mandatory orders for large utility companies to divest their generation assets, various models for generating assets valuation have been developed, and substantial experience has been gained. However, the equally important issues on how well a market mechanism works in the transmission sector have not been adequately addressed. While it is generally accepted that a highly reliable transmission system is a necessity for the power market operation, transmission valuation issues and reliability have received less attention.

By physically connecting geographically dispersed regions and transporting energy between them, a transmission network is critical for enabling electric power system functions and materializing the benefit of competitiveness and the economy of scale in the generation sector [1]. In a competitive market environment, transmission sector assumes a new role of supporting market trading. To ensure necessary maintenance and expansion in transmission facilities for maintaining system reliability and security, all transmission investments should be correctly valued and adequately compensated. The central problem involved is the proper definition of property rights on transmission and the proper market signals for valuing the transmission rights.

The spot price scheme for electric power systems, as initially proposed in [2], with the option to use financial transmission contracts to hedge against transmission risks as illustrated by [3], provides a platform for establishing correct market signals for resource allocation and capacity investment. In an electric power pool, electricity price fluctuates tremendously over time and across locations as the supply and demand conditions vary. Another influential factor for electricity price volatility is the transmission capacity constraint, which becomes more and more apparent in the power markets. The extraordinary price volatility creates strong demands for trading instruments with underlyings related to transmission risks among risk-averse buyers and sellers of electricity. If liquidly and competitively traded, prices of these instruments would provide economic signals for fair valuation of transmission facilities and investment guidance in capacity addition. A properly designed transmission-pricing scheme combined with well-defined transmission rights ready for trading can contribute to risk mitigation and yield transmission investment incentives.

Transmission capacity of an electric power network is commonly defined in two ways, one is the point-to-point transfer capability, and the other is circuit capacity. FTR, as introduced in [3], is based on receipt point to delivery point transfer capacity. It yields its holder a financial benefit or liability of the locational marginal price difference between two specified nodes without offering its holder any priority in using the underlying physical transmission facilities. It is currently implemented in Pennsylvania-New Jersey-Maryland (PJM) market place. It is also known as the Transmission Congestion Contract (TCC) in the New York system and the Financial Congestion Right (FCR) in the New England system. The implementation of FTRs requires centralized system dispatch and price determination. On the other hand, FGR, as described in [1], is defined in terms of a portion of the total capacity in a certain direction over a particular transmission flowgate. A flowgate can be any line or transformer, or a set of lines and transformers with a certain capacity limit [4]. A FGR provides its holder a financial benefit over the underlying congested flowgate. The inter-zonal Firm Transmission Rights introduced into California system is one application of FGR design. The settlements of FGRs are based on actual marginal values of capacity on the underlying flowgates at the time of congestion, i.e., shadow prices on the constrained transmission facilities. Auctions and bilateral transactions can be used to allocate transmission rights to market participators who value them the most. Ideally, short-term transmission rights with good liquidity would allocate transmission capacity efficiently and capture the scarcity rent, and long-term transmission rights would guide efficient investment in transmission. Paper [5] shows that under certain rules the transmission rights provide correct incentives for transmission investment.

Two approaches are widely employed in studying the price behavior of transmission rights: a fundamental approach that relies on simulation of system operations, and a technical approach that attempts to model the stochastic behavior of market prices from historical data and fundamental analysis. Although plenty of pricing models work well for traditional commodities, they all fall short in capturing the complex price behaviors of electricity such as price spikes, negative price, and stochastic volatility. Various models utilizing reduced-form stochastic processes, fuzzy regression along with neural network, and Fourier Hartley transform based techniques have been explored in modeling electricity prices. However, little success on the application to a bulk power system has been reported. As an alternative, we consider a fundamental approach for modeling electricity prices and apply it to transmission rights valuation.

In this paper, we present a framework for modeling electricity prices and valuing congestion revenue rights based on simulation of system operations and market transactions. Specifically, we investigate the price behaviors of FTRs and FGRs under fluctuating system load, non-constant supply/demand bids, and unplanned transmission circuit outages. Probabilistic characteristics of FTR/FGR values are obtained via Monte Carlo simulation. This paper is organized as follows. First, a multilateral transactions model and a nonconforming load model are proposed. We then present an optimal power flow (OPF) formulation to model system dispatch under each different market structures incorporating uncertainties. Solutions of the OPF problems yield the values of transmission rights under consideration. Numerical results for a 6-bus sample system are presented. Finally, we conclude and offer some discussions. Descriptions of the Quadratized Power Flow (QPF) and extension of the costate method for QPF are provided in appendices A and B, respectively.

2. MODEL FORMULATION

2.1 A. Multilateral-Transactions Model

A bilateral transaction consists of one pair of source and sink, while a multilateral transaction consists of multiple sources and sinks. Obviously, a bilateral transaction is a special case of a multilateral transaction. Among various types of the multilateral transactions such as bus-to-area transaction, area-to-area transaction, bus-to-system transaction and area-to-system transaction [6], the area-to-system transaction model is the most general one. Mathematically, an area-to-system multilateral transaction model is defined as:

$$\{(s,p),(d,\rho),(\gamma,P^m)\} = \{(s_i,p_{ks_i}),(d_j,\rho_{kd_j}),(\gamma_k,P_k^m), i=1,2,\cdots,I, j=1,2,\cdots,J, k=1,2,\cdots K\} \quad (1)$$

where, d = a vector of buses where loads are connected,
P^m = total real power of the multilateral transaction m,
p = participation factors of generators to the multilateral transactions,
s = a vector of buses where generators are connected,
γ = transmission loss factors of the multilateral transactions,
ρ = participation factors of loads to the multilateral transactions, and
I, J, and K = total number of suppliers, demands, and multilateral transactions, respectively.

The participation factors of the model satisfy the following equations:

$$\sum_{i=1}^{I} p_{ki} = 1, \qquad \sum_{j=1}^{J} \rho_{kj} = 1$$

By setting I or J to one, or both equal to one, the model could represent any specific type of transactions as listed above.

2.2 A Nonconforming Electric Load Model

For a typical conforming electric load model, load at a specific bus is a certain percentage of the total system load, which implies high correlation between loads. Such model fails to represent the actual behavior of electric loads. Therefore, the following nonconforming electric load model is proposed:

$$PL = pl_0 \cdot PS_0 + pl_1 \cdot PS_1 \cdot v_1 + \sum_{n=2}^{N} pl_n \cdot PS_n \cdot v_n \quad (2)$$

where, pl_0 = a vector of bus valley loads in percentage of system valley load,
pl_1 = a vector of bus peak-valley load in percentage of PS_1,
pl_n = a vector of bus loads in percent of $PS_n, n=1,2,\cdots N$,
PL = a vector of random bus loads,
PS_0 = valley value of system load,
PS_1 = peak-valley value of system load,

PS_n = system load associated with random variable *n*, and
v_n = a set of random variables representing various factors causing the randomness of load,

By setting $v_n = 0 \quad (n = 2, \cdots N)$, the above model becomes the traditional conforming load model.

2.3 A Transmission Right Valuation Models

FTR is a right to receive compensation for the locational marginal price difference between two specific points. FGR is a right to receive compensation based on the congestion of a particular limiting flowgate. When a system operating condition occurs, which results in reaching or violating a transmission capability limit [4], FTR and FGR take non-zero values. The tightening of transmission congestion usually increases FTR and FGR values.

A pool based market dispatch is very close to the traditional power system dispatch, with generation cost functions replaced by market bid functions. Multilateral transactions can be modeled by power injections and withdraws at multiple pairs of source and sink buses. Given a specific system configuration, the system operator usually determines the optimal dispatch by solving an OPF, with a predefined objective and subject to given constraints. The objective is either maximizing social welfare, or minimizing system operating cost, system loss, or customer expense et al. Recent papers [7-9] discussed the selection of various objectives and the corresponding effects on the suppliers and consumers. In this paper, we define the objective to be social welfare maximization. We propose a comprehensive transmission right valuation model based on an OPF formulation that combines pool-based transactions, multilateral transactions, and market uncertainties such as fluctuating system loads, and unexpected transmission circuit outages.

2.3.1 A Pool-based System

In a pool-based system, the system operator dispatches the system to maximize the social welfare while observing the physical transmission constraints. The optimization model is defined as follows:

$$\text{Max} \qquad W\left(S^p, D^p\right) \tag{3}$$

Subject to:

$$G\left(x, S^p, D^p\right) = 0, \qquad T(x) \le \overline{T},$$

$$S^p \le S^p_{Max}, \qquad D^p \le D^p_{Max}$$

where, $W\left(S^p, D^p\right)$ = social welfare function of supply and demand quantities,
$S^p = \left[S_i^P\right]$ = vector of supply quantity dispatched,
$D^p = \left[D_j^P\right]$ = vector of demand quantity dispatched,
$T(x) = \left[T_l(x)\right]$ = loading level of circuit $l = 1, 2, \ldots, L$,
$\overline{T}(x) = \left[\overline{T}_l\right]$ = loading capacity of circuit,
x = state variables, and
$G(\cdot)$ = power flow equations.

The objective function of social welfare depends on the supply bid functions $F^S(\cdot)=\lfloor F_i^S(\cdot)\rfloor$ and demand bid functions $F^D(\cdot)=\lfloor F_j^D(\cdot)\rfloor$. Locational marginal prices can be calculated by taking the optimal dispatched quantities into supply and demand bid functions. Therefore, the FTR values can be readily deduced from the price differences, while the FGR values are reflected by the Lagrange multipliers associated with the circuit loading capacity constraints $T(x)\le \overline{T}$.

2.3.2 A Pool-based System with Multilateral Transactions

The multilateral transactions are defined over a source set and a sink set. Let P_k^m be the quantity of real power for multilateral transaction k, p_{ki} and ρ_{kj} be participation factors of supplier i and consumer j for P_k^m, respectively, and γ_k be transmission loss sensitivity for P_k^m. Then $S_i^m = \sum_{k=1}^{K} p_{ki}(1+\gamma_k)P_k^m$ represents total supply at bus i for all the multilateral transactions and $D_j^m = \sum_{k=1}^{K} \rho_{kj}(1+\gamma_k)P_k^m$ represents total demand at bus j for all the multilateral transactions. Therefore, we use vectors $S^m = \lfloor S_i^m \rfloor$ and $D^m = \lfloor D_j^m \rfloor$ to represent injections (supply) and withdraws (demand) associated with the multilateral transactions. Total supply and demand at each node are given by the vectors $S = S^p + S^m$ and $D = D^p + D^m$, respectively. By replacing S^p and D^p in the optimal power flow (3) with S and D, respectively, the model is expressed in terms of the multilateral power transaction variables P_k^m. Solutions of this OPF problem yield the values of transmission rights in a pool-based system with multilateral transactions.

2.3.3 Incorporation of Market Uncertainty

Market uncertainties due to random supply and demand bids, fluctuating system loads, varying fuel price, and unexpected transmission circuit outages are incorporated in the model as follows:

$$\text{Max} \qquad W(S,D) \tag{4}$$

Subject to:

$$G(x,S,D,A^o)=0, \quad T(x)\le \overline{T}^o, \quad S\le S_{Max},$$
$$D\le D_{Max}, \quad S=S^p+S^m, \quad D=D^p+D^m$$

where, $A_i^o = A_i \cdot u_i$ = circuit i parameters under possible circuit outages,
$\overline{T}_l^0 = \overline{T}_l \cdot u_l$ = circuit i loading capacity under possible circuit outage, and
u_l = 0/1 variable where 0 corresponds to circuit outage.

The effect of market uncertainties on network topology is reflected by network parameter variables $\tilde{A}$ and $\tilde{T}$. In the objective function, the supply bid functions become $\tilde{F}^S(\cdot)=\lfloor \tilde{F}_i^S(\cdot)\rfloor$ and the demand bid functions become $\tilde{F}^D(\cdot)=\lfloor \tilde{F}_j^D(\cdot)\rfloor$ by using random variables as the coefficients.

2.4 Efficient Solution of the Transmission Right Valuation Model

Under different system operating conditions, different sets of constraints may become activate in the solutions of the above OPF problems. These binding constraints correspond to congested transmission facilities. To accelerate the computation, we can obtain the OPF solutions via successive linearization at each operating condition and utilization of linear programming techniques. The following formulation of the linearized model is based on quadratized power flow models and costate method.

$$\text{Max} \qquad W(\Delta S, \Delta D) \tag{5}$$

Subject to:

$$\sum_{i=1}^{I} \Delta S_i^p - \sum_{j=1}^{J} \Delta D_j^{pl} + \sum_{k=1}^{K}\sum_{i=1}^{I} p_{ki}(1+\gamma_k)\Delta P_k^m - \sum_{k=1}^{K}\sum_{j=1}^{J} \rho_{kj}(1+\gamma_k)\Delta P_k^m = 0$$

$$\sum_{i=1}^{I} \frac{\partial T_l}{\partial P_i}\Delta S_i^p - \sum_{j=1}^{J} \frac{\partial T_l}{\partial P_j}\Delta D_j^p + \sum_{k=1}^{K} \frac{\partial T_l}{\partial P_i} p_{ki}(1+\gamma_k)\Delta P_k^m - \sum_{k=1}^{K} \frac{\partial T_l}{\partial P_j}\rho_{kj}(1+\gamma_k)\Delta P_k^m \le \overline{T}_l - T_l^0$$

$$\Delta S_i^p + \sum_{k=1}^{K} p_{ki}(1+\gamma_k)\Delta P_k^m \le S_i^{Max} - S_i^0$$

$$\Delta D_j^p + \sum_{k=1}^{K} \rho_{kj}(1+\gamma_k)\Delta P_k^m \le D_j^{Max} - D_j^0$$

where, T_l^0 = loading on circuit l under current operating conditions,
S_i^0 = committed quantity of supply i under current operating conditions,
D_j^0 = load quantity of demand j under current operating conditions, and
Δ = the deviations of a variable from its previous operating condition.

Note that $\frac{\partial T_l}{\partial P_i}$ is the impact on flowgate l with one unit of incremental power injection at bus i. It is based on AC power flow which accounts for reactive power and voltage variation. It is different from the traditional DC power flow based Power Transfer Distribution Factor (PTDF). Therefore $\frac{\partial T_l}{\partial P_i}$ reflects the system operating conditions more accurately. The detailed computational procedures are referred to [6], [10], and [11].

3. MONTE CARLO SIMULATION

Monte Carlo simulation has been extensively utilized in power system probabilistic analysis. A general framework was proposed in Pereira et al 1992 to combine an analytical model with a Monte Carlo simulation model. The Monte Carlo simulation method is successfully applied in many power systems problems such as the adequacy assessment of distributed generation systems, the generation cost computation under operating constraints, the generation system well-being analysis, and system reliability analysis. A Monte Carlo procedure simulates a specific system with a reasonable number of random draws from all possible system states according to their probabilistic distributions. The simulation of a randomly selected system condition is done with power system analysis tools and subsequent computations. Two key issues in the Monte Carlo simulation are: first, number of trials should be large enough to adequately capture all possibilities relative to the application, and second, the analysis methods must be appropriate with respect to the application, namely, they should accurately compute the quantities of interest.

In our proposed Monte Carlo simulation procedure, each trial represents a set of market and system operating conditions. Subsequently, the transmission right valuation models are implemented to obtain the transmission right values under each specific scenario. After performing a sufficient number of trials, we obtain the probabilistic characteristics of the transmission right value and its sensitivities to system conditions.

4. NUMERICAL EXAMPLE

The proposed method is applied to a 6-bus test system taken from [1], as illustrated in Fig. 1. We provide a short description of the system and then show the computational results for FTR and FGR valuation.

4.1 System Description

The test system consists of six buses. Three generators are located at buses 1, 2, and 4, respectively. The three generators submit supply bids. Three loads reside at buses 3, 5, and 6, respectively. The three loads submit demand bids. Bus 6 is selected as the slack bus. The circuit admittances are $0.01\text{-}j\,0.1$ for circuit 1-6 and 2-5, and $0.02\text{-}j\,0.2$ for all others. The numbers in the figure illustrate the circuit capacities in MVA. In addition, a flowgate from N to S is defined with capacity 340 MVA.

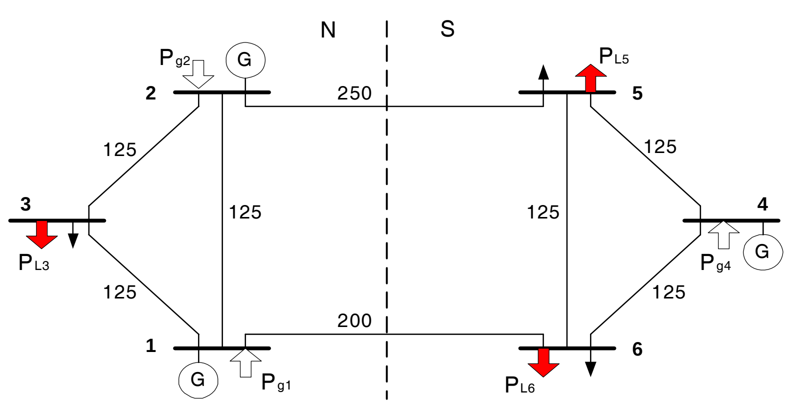

Fig.1 Test 6-Bus System

The bidding functions of suppliers (generators) and consumers (loads) are $P_i = a_i + b_i P_{gi}$ $(i = 1,2,4)$ and $P_j = a_j - b_j P_{Lj}$, $(j = 3,5,6)$. The bid parameters in the base case are:

$$[a_1, a_2, a_3, a_4, a_5, a_6] = [10.0, 15.0, 37.5, 42.5, 75.0, 80.0]$$
$$[b_1, b_2, b_3, b_4, b_5, b_6] = [0.05, 0.05, 0.05, 0.25, 0.10, 0.10]$$

The dispatched quantities in the base case are computed and listed in Table 1.

Table 1 Base Case Supply and Demand Quantities

Generation (MW)			Load (MW)		
Bus 1	Bus 2	Bus 4	Bus 3	Bus 5	Bus 6
330.02	229.98	239.63	220.00	264.90	314.74

We next examine the value of FTR from bus 1 to bus 6 and FGR values over flowgates N-S and 4-6, which we denote as FTR_{16}, FGR_{NS}, and FGR_{46}, respectively. For the base case, their values are listed in Table 2.

Table 2 Base Case FTR/FGR Values

FTR/FGR	FTR_{16}	FGR_{NS}	FGR_{46}
Values ($/MWh)	28.88	26.80	10.36

4.2 Transmission Right Valuation with Fluctuating Load

We vary the coefficients of demand bid functions to reflect the non-conforming system load as proposed in (2). It is assumed that the coefficients vector a in the demand bid functions takes random values as:

$$\begin{bmatrix} a_3 \\ a_5 \\ a_6 \end{bmatrix} = \begin{bmatrix} 37.50 \\ 75.00 \\ 80.00 \end{bmatrix} + \begin{bmatrix} 0.25 \\ 0.25 \\ 0.50 \end{bmatrix} \cdot PS_1 \cdot v_1 + \begin{bmatrix} 0.45 \\ 0.11 \\ 0.26 \end{bmatrix} \cdot PS_2 \cdot v_2$$

where v_1 and v_2 are independent identical uniform $(-1,1)$ random variables, $PS_1 = 10$, and $PS_2 = 10$. By Monte Carlo sampling and applying the proposed transmission right valuation model, we compute distributions of the FTR/FGR values and their statistical characteristics such as mean and variances. Table 3 provides the value means and variances of the selected FTRs and FGRs. Fig. 2 illustrates the probability density function of the FGR_{NS} value distribution. Fig. 3 illustrates the variation of FGR_{NS} value with respect to the two independent random variables and.

Table 3 FGR/FGR Value Means and Variances with Fluctuating System Loads

FTR/FGR	FTR_{16}	FGR_{NS}	FGR_{46}
Mean ($/MWh)	28.73	26.63	10.00
Variance	84.97	46.92	182.37

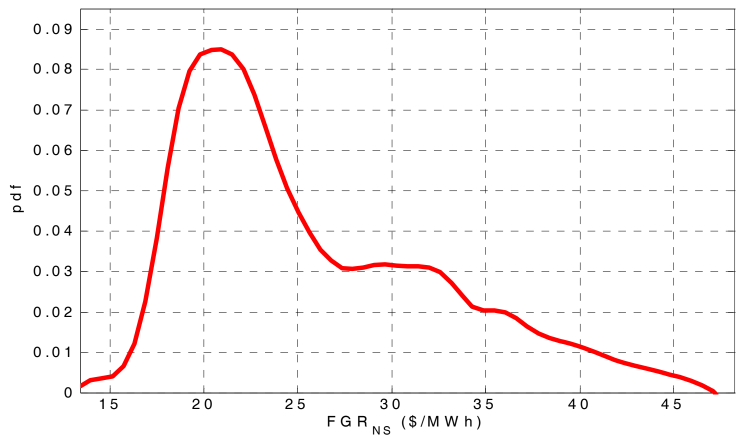

Fig. 3 Probability Density Function of FGR_{NS}

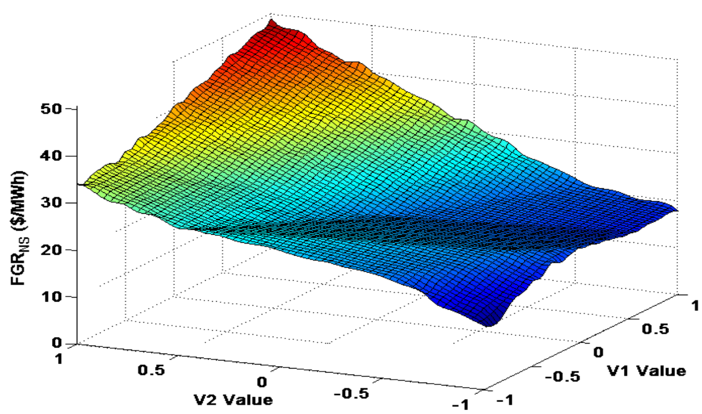

Fig. 4 FGR_{NS} Values w.r.t. v_1 & v_2 Values

From Table 3 and two Figs. of 3 and 4, we conclude that the values of FTR and FGR fluctuate significantly with random loads as compared with constant system loads. The non-conforming system load model provides a more realistic set up for modeling the electric load uncertainty and studying its effect on transmission right values.

4.3 Transmission Right Valuation with Individual Supply/Demand Bids

Market participants usually monitor the markets closely and respond to the market activities in real time. As a result, the supply/demand bids would not be constants. Also, gaming activities is not avoidable. Market participants always look for ways to exert influence on market price possibly by manipulating their bid functions. We study the sensitivity of transmission right values with respect to bidding functions of various market participants in the simple test system. It is important to understand the impacts of system reliability constraints on the values and volatilities of transmission rights subject to random bid functions. We also examine the FTR/FGR values under postulated contingency test, and contrast them with the FTR/FGR values without considering contingencies. Sensitivity results are presented in Figs. 2 through 5. The data are color and shape coded, with the legend given only in Fig. 5. The cases with the '_C' denote the transmission right values for the case of the network with outage of circuit 4-5. The results are self-explanatory.

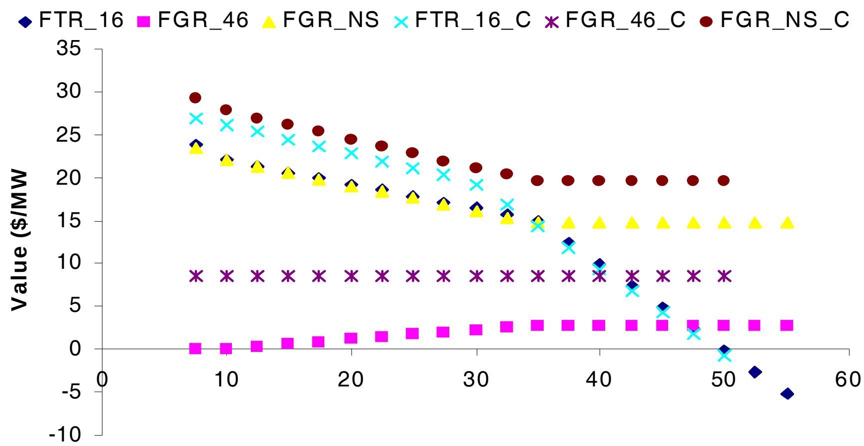

Fig. 5 Transmission Right Values w.r.t. a_1

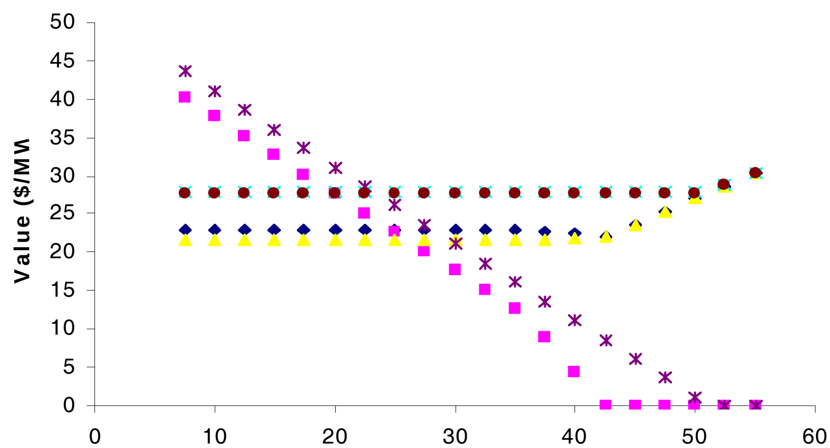

Fig. 6 Transmission Right Values w.r.t. a_4

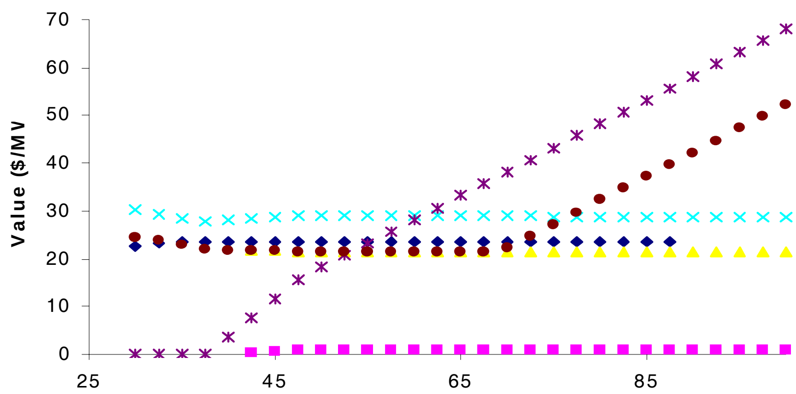

Fig. 7 Transmission Right Values w.r.t. a_3

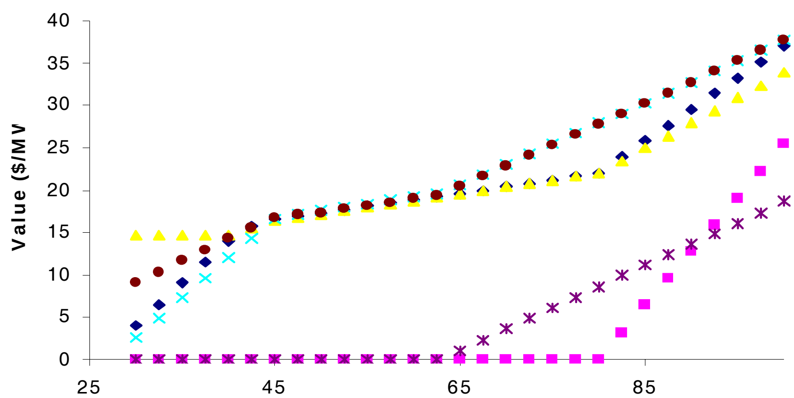

Fig. 8 Transmission Right Values w.r.t. a_6

We observe that the decrease of supplier bids in the high-price zone or the increase of supplier bids in the low-price zone relieves the congestion. It therefore reduces the values of FGR/FGR from low-price zone to high-price zone, and makes the FTR/FGR within the high-price zone more valuable. By the same argument, the increase of demand bids tends to increase values of all transmission rights. The incorporation of the outage of circuit 4-5 makes these transmission rights more valuable. It is important to note that the volatility of the results for the simple example is by and large due to the internal constraints and the uncertainties of the electric power system itself.

4.4 Transmission Right Valuation Circuit Outage

Transmission circuits may exhibit varying outage rates depending on their design, exposure to disturbances, such as lightning, etc. It is therefore important to ask the question how FTR/FGR values are affected by the outage rate of transmission circuits. The answer to this question links transmission reliability to valuation of transmission facilities and as such it is a very important question. The proposed method provides a comprehensive approach for assessing the effects of circuit outage rates to FTR/FGR values. Specifically, the methodology is applied to the example system with varying outage rates for circuit 2-5. The results are presented in Table 4. Note that with the increase of outage rate of circuit 2-5, the values of FTR_16, and FGR_46 increase, while the FGR_NS decreases, because flowgate N-S will become less likely to be congested due to the capacity of circuit 1-6, and bus 6 relies more on supplier 4.

Table 4 FTR/FGR Value Means ($/MWh) with Different Outage Rates of Circuit 2-5

Outage Rates	0.00%	0.25%	0.50%	0.75%	1.00%
FTR_{16}	22.0200	21.9253	21.7998	21.6567	21.5312
FGR_{46}	0.0600	0.1924	0.3680	0.5682	0.7438
FGR_{NS}	22.0300	22.1109	22.2181	22.3404	22.4476

5. CONCLUSIONS

The paper presents a comprehensive computational framework for transmission right valuation. The framework incorporates system uncertainties into different market structures, Numerical results using a sample power system reveal that the FTR/FGR values vary with internal transmission constraints and market uncertainties such as fluctuating loads, market participants' activities, and unplanned circuit outages. For a simple system like the one used in our example, it may be possible to analytically predict the impacts of different market events on the system operations, market prices and values of transmission rights. However, when analyzing a more realistic and complex, it becomes very difficult or even impossible to analytically demonstrate the same impacts without resorting to power system computational tools. We employ the Monte Carlo method to construct a powerful power system analysis tool for tackling the problem of transmission right valuation. By simulating the power system operations and market transactions subject to system and market uncertainties, we obtain realistic estimation of the values of transmission rights. This helps the price discovery in both transmission right auctions and the bilateral transactions in the secondary market. In the long run, accurate market simulations provide correct market signals for valuing congested transmission facilities and inducing capital investments in transmission system expansions.

6. ACKNOWLEDGMENTS

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8. APPENDICES

8.1 Quadratized Power Flow (QPF)

QPF is based on modeling any power system component as a set of linear or quadratic equations. This can be achieved with the introduction of additional state variables. No compromising simplification is necessary. Application of connectivity constraints (Kirkoff's current law) yields the quadratized power flow equations:

$$G_Q(x_Q, u_c) = [x_Q, u_c]^T \cdot [A] \cdot [x_Q, u_c] + B \cdot [x_Q, u_c] + b = 0 \qquad (6)$$

The solution to the quadratic equations is obtained with the Newton-Raphson method:

$$x_Q^{k+1} = x_Q^k - Jac \cdot \left(x_Q^k\right)^{-1} \cdot G_Q\left(x_Q^k\right) \qquad (7)$$

Iteration terminates when the norm of the QPF equations is less than certain tolerance. Sparsity techniques are applicable to get fast solutions for large systems.

8.2 Extension of Co-state Sensitivity Method to QPF

The co-state method is applied to compute the linearized model of any system constraint or function. Suppose some system performance index function (such as total transmission loss, and circuit loading index).

$$F_I = f_I\left(x_Q, u_c\right) \qquad (8)$$

Differentiation of the QPF equations gives:

$$\frac{\partial G_Q\left(x_Q, u_c\right)}{\partial u_c} + \frac{\partial G_Q\left(x_Q, u_c\right)}{\partial x_Q} \cdot \frac{dx_Q}{du_c} = 0 \qquad (9)$$

Therefore, we can get

$$\frac{dx_Q}{du_c} = -\left(\frac{\partial G_Q(x_Q, u_c)}{\partial x_Q}\right)^{-1} \cdot \frac{\partial G_Q(x_Q, u_c)}{\partial u_c} \tag{10}$$

The derivative of the system performance index function with respect to u_c is given by:

$$\frac{dF_I}{du_c} = \frac{\partial f_I(x_Q, u_c)}{\partial u_c} + \frac{\partial f_I(x_Q, u_c)}{\partial x_Q} \cdot \left(\frac{\partial G_Q(x_Q, u_c)}{\partial x_Q}\right)^{-1} \cdot \frac{\partial G_Q(x_Q, u_c)}{\partial x_Q} \tag{11}$$

Let co-state vector $\hat{x}_Q^T = \frac{\partial f_I(x_Q, u_c)}{\partial x_Q} \cdot \left(\frac{\partial G_Q(x_Q, u_c)}{\partial x_Q}\right)^{-1}$. For a function F_I which is not an explicit function of control variable u_c, we have $\frac{dF_I}{du_c} = \hat{x}_Q^T \cdot \frac{\partial G_Q(x_Q, u_c)}{\partial x_Q}$.