Design of Cost Optimal Solar Flat-Plate Collectors

B.T.Nijaguna
Mechanical Engineering Department
Karnataka Regional Engineering College
Surathkal, Karnataka, India

ABSTRACT

The overall performance of a fin and tube type solar flat-plate collector depends mainly on a) meteorological parameters, b) operating parameters, and c) design parameters. The total system cost using solar flat-plate collectors is mainly the initial cost of the collector.

Over-emphasis on thermal efficiency alone in the design of the tube and sheet type of solar flat-plate collectors would prove to be counter productive unless equal importance is given in its design for cost effectiveness. The design aim should be to judiciously select the fin and tube materials and then determine the optimal fin thickness and width that would result in maximum energy collection for unit price of the system.

In this study, a design methodology based on cost optimization is presented and the cost effective energy equation is obtained. The results obtained show that the cost optimal fin width and thickness are not affected by either the meteorological conditions or the operating conditions and as such the obtained geometries are valid for any operating and meteorological conditions. Design nomographs presented serve as quick and convenient design tools for cost optimal flat-plate collector design.

INTRODUCTION

Solar flat-plate collectors (MacGregor, 1979) can broadly be classified depending on the extent of wetted surface area relative to the absorbing surface area as: 1) Full tube and fin with low wetted area and low water capacity, 2) Full water sandwich where the wetted area and water capacity are both high; and 3) Semi-sandwich type intermediate between type 1 and 2. Brief comparative details of these are given in Table 1. On the basis of overall requirements of low cost, weight, thermal capacity together with good durability, it appears that the tube and fin type is the best choice for domestic water heating systems. The system cost for such application is mainly the initial cost of the collector and this depends on the materials used to construct the collector. As raw materials become scarcer and more expensive they should be used as effectively as possible. Considerable effort has been directed towards improving the thermal collection efficiencies of these units using selective coating, multiple glazing, etc. But over-emphasis on energy efficiency alone without considering the corresponding costs involved could prove counter productive. The aim of the designer should be to obtain best cost effectiveness.

The collector parameters which significantly affect the collection efficiency of flat-plate collectors have been studied by Hahne (1985) and they include radiation characteristics of absorber plate, number of glass covers, insulation values for collector, tube and fin materials and their geometric dimensions. Little economic flexibility is possible at present on the absorber selective coatings that

Table 1 Comparison of different types of flat-plate collectors (FPC).

Type	Advantages	Disadvantages
I (a) Pipe and fin, all copper	Good corrosion resistance, low thermal capacity possible.	Expensive
I (b) Pipe and fin, composite, e.g. copper pipe and aluminum fin.	Fairly cheap. Good internal corrosion resistance. Low thermal capacity possible. Flexibility in choice of materials.	Possibility of external bi-metallic corrosion unless suitably protected.
II Full water sandwich, plastic.	Cheap and light	Limited to low tempeatures. Liable to U.V. damage. High thermal expansion. High thermal capacity.
III (a) Semi water sandwitch, steel (e.g. pressed steel radiators).	Fairly cheap. Readily available.	Long term corrosion problems. Suitable for closed systems only. Heavy. High thermal capacity.
III (b) Semi water sandwich, aluminum (e.g. roll bond type).	Fairly cheap. Light weight.	Very susceptible to internal corrosion specially in mixed metal circuits.

determine the radiation characteristics. Increasing the number of glazings beyond one is marginally beneficial, as is increased insulation thickness beyond a certain value.

Assuming that the materials for the tube and fin plate have been selected based on fabrication and corrosion considerations, the main scope for reducing costs lies in the selection of the optimum combination of tube spacing and fin thickness. Material costs can be reduced by increasing the spacing between tubes and making plate fins less thick. However, this will lead to a reduction in fin thermal efficiency and overall system performance. This means that at one extreme the no fin (tubes butting each other) collector would have the highest thermal collection efficiency - but would be the costliest, while at the other extreme, with very large tube spacing the thermal collection efficiency will be lower - but would be the cheapest, the other parameters being the same. Hence, the aim should be to determine the combination of tube spacing and plate thickness which will minimize the cost input for the desired useful energy collection. In other words, the collector should be designed to obtain maximum thermal effectiveness at minimum cost (or maximum solar energy collection per unit area of the collector at minimum possible cost). A review of literature shows that fins made of different materials, but of the same width will have the same fin efficiency if the thickness-thermal conductivity product 'tk' remains constant (Duffie and Beckman, 1974). If 'tk' is to remain constant for different fin materials, then the thickness must be inversely proportional to the thermal conductivity of fin material. Since, the other two dimensions of the fins remain constant, the volume of the fins will be proportional to their respective thicknesses. Hence for the same fin efficiency, the volume of the fins (and therefore cost of the fins) must be inversely proportional to the thermal conductivities of the respective fin materials. Fin cost alone becomes considered in such an analysis, leaving aside the risers cost.

Table 2 Comparison of costs of three fin materials of equal thermal performance.

Material	Thermal conductivity (W/m K)	Density (kg/m³)	Cost (Pounds/kg)	Equivalent thickness (mm)	Cost per unit area (Pounds/m²)	Mass per unit area (kg/m²)
Copper	390	8690	310	0.25	649	2.24
Aluminum	205	2700	175	0.48	227	1.30
Mild steel	50	7850	56	1.95	857	15.30

(June 1977 prices taking 0.25 mm copper as datum)

Table 2 shows a comparison of the cost and weight of copper, aluminum and steel collector plates of equal fin efficiency (MacGregor, 1979). An aluminum plate should be about twice as thick as the equivalent copper plate since its thermal conductivity is about half that of copper. It may be seen that aluminum has a clear lead over copper as regards both price and weight, while steel is marginally worse than copper on cost and very much worse on weight.

MacGregor (1979) considered both collection efficiency and collector cost and gave the tabulated results to find out the optimum dimensions of tube and fin in terms of ratio of total system cost per unit area to fin efficiency for different combinations of fin width to fin thickness.

Kovarik (1978) considered the economic aspects of design of solar collectors consisting of tubes provided with radiation absorbing fins. He formulated a variational problem and obtained a solution that gives the minimum weight or cost per unit heat collection. The solutions are system specific and need individual calculations. His results also show that the optimal fin cross section does not depend on the coefficient of heat transfer, ambient temperature, collecting fluid inlet temperature, solar radiation and cost of cover and insulation but only on the cost of tube and fin materials.

The design methodology suggested here is more general and concise and can be easily and conveniently applied for determining cost optimal tube spacing and fin thickness for any desired tube-fin composite of materials.

COLLECTOR DESIGN PARAMETERS

Most of the parameters that determine the collector performance can be categorised as: 1) meteorological parameters (irradiance, ambient temperature, wind velocity, sky temperature, etc.); 2) operating parameters (inlet fluid temperature and flow rate, slope of collector, etc.) and 3) configuration/design parameters. The following discussion is restricted to the design parameters only, as it will be shown later that the cost optimal design methodology presented here is independent of operating and meteorological parameters.

The collector design parameters which affect the collection efficiency of flat-plate solar collectors to a considerable extent are:

1. Absorptivity and emissivity of the absorber plate.
2. Number of glass covers, their thickness, absorptivity, emissivity, and spacing.
3. Insulation material and thickness.
4. Fin material, its spacing, thickness and thermal properties.
5. Pipe spacing, dimensions.

Absorptivity and emissivity of an absorber plate depend on the type of selective coating used.

Since selective coatings involve considerable initial investment, manufacturers do not and cannot change them frequently. Also most of the selective coatings are protected by patent rights. So one has very limited choice in selecting a selective coating. However by selecting other parameters judiciously one can build a cost effective solar collector. Table 3 shows the effect of number of glass covers on collector heat loss coefficient and collection efficiency on a typical G.I. collector (Sukhatme, 1984).

Table 3 Effect of number of glass covers on the performance of collectors.

Type of absorber surface	Non-selective surface			Selective surface	
No. of glass covers	1	2	3	1	2
U(W/m²K)	6.39	3.37	2.72	3.61	2.51
Collection efficiency (%)	40.6	43.3	41.3	47.0	44.9

From the above data it can be seen that, with non selective absorber surface, collection efficiency increases when the number of glass covers is increased to two. When the number of glass covers is increased from two to three the collection efficiency decreases. The reason for this is that the fraction of incident solar radiation available for the collector absorber decreases due to increased reflective loss and absorption of incident solar radiation by the glass covers. For a selective surface absorber the collection efficiency is maximum when the number of glass covers is just one.

The cost of a hardened and tempered glass cover normally used in a solar collector is nearly one third of the total cost of the collector. So, providing a second glass cover to a non-selective flat-plate solar collector increases the cost by about thirty per cent while improving the collection efficiency by a mere three per cent. So, providing a second glass cover is certainly not cost effective. Also, at least one glass cover is essential to protect the absorber plate and its selective coating from the aggressive atmosphere as well as to reduce convective heat losses.

Some of the insulation materials that can be used for insulating the back and sides of a flat-plate solar collector are: glass wool, rock wool, polyurethene foam, and thermocole. Costs of glass wool and rock wool are more or less the same and are much lower than that of either polyurethene foam or thermocole. Both have reasonably low thermal conductivity (around .04 W/m K). Hence both rock wool and glass wool are widely used for insulating the solar flat-plate collectors. A back insulation thickness of about 5 cm is normally used. Though increasing the insulation thickness over 5 cm increases the heat collected per unit cost, the increase of the latter is so small that the aesthetic and transportation considerations score over it.

The fin efficiency F of a fin and tube type solar flat-plate collector is given as (Duffie, 1974):

$$F = \frac{\tanh\left(\sqrt{U/kt}\,(w - D)/2\right)}{\left(\sqrt{U/kt}\,(w - D)/2\right)} \tag{1}$$

From Eq. (1) it becomes clear that fins made of different materials but of the same width and attached to riser tubes of diameter D will have the same fin efficiency if the thickness-thermal conductivity product 'tk' remains constant. So, the cost of the fin can be evaluated for different materials such that 'tk' remains constant (So that F and therefore the heat collected will remain the same for all the materials) and the fin material which costs the least can be chosen.

If 'tk' is to remain constant for different fin materials then the thickness must be inversely proportional to the thermal conductivity of the material. Since, the other two dimensions of the fins remain constant, the volumes of the fins will be proportional to their respective thickness. Hence for the same fin efficiency F, the volumes of the fins must be inversely proportional to the thermal conductivities of the respective fin materials. So for the same heat output, costs of fins of different materials will be proportional to

$$\frac{\text{density of the fin material X cost per kg}}{\text{thermal conductivity}}$$

The above factor can be evaluated for different materials which can be used as a fin, and the one which minimizes the above factor can be chosen. However, it may be noted that in the above analysis only the fin cost is considered leaving the riser tube costs.

Tube pitch and fin thickness have considerable influence over both heat collected and the cost of the collector. Hence they must be chosen such that the heat colleted per unit cost is maximized. Logically, the main collector parameters for cost optimization gets short listed to absorber plate fin and tube materials and their geometric interdependence.

COST OPTIMAL DESIGN ANALYSIS

A typical fin and tube arrangement used in flat-plate solar collectors is shown in Fig. 1. Fig. 1 also gives the temperature profile over the fin. The actual temperature profile over the riser tube would be slightly different from the one shown in the figure. The temperature profile shown in Fig. 1 has been used because it greatly simplifies the analysis. This also helps in taking into consideration the heat collected by that part of the fin which is directly over the tube.

The heat balance for the small element t dx dy (dy is the elemental thickness in the direction of fluid flow) shown in Fig. 1 can be written as:

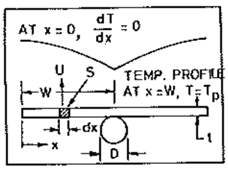

Fig. 1 Fin and tube arrangement and temperature profile over the fin.

Heat conducted in + Solar radiation absorbed = Heat conducted out + Heat lost to the atmosphere by conduction, convection and radiation.

Substituting for the above quantities, we get:

$$-\,kt\,dy\,\frac{dT}{dx}\Big|_x + S\,dx\,dy = -kt\,dy\,\frac{dT}{dx}\Big|_{x+dx} + U(T-T_a)dx\,dy \tag{2}$$

$$\frac{d^2T}{dx^2} = m^2(T - T_a - S/U) \tag{3}$$

where $m = \sqrt{U/kt}$

Taking
$$Z = T - T_a - S/U \tag{4}$$

Eq. (3) reduces to

$$\frac{d^2Z}{dx^2} = m^2Z \tag{5}$$

A general solution for the above equation is

$$Z = A \cosh (mx) + B \sinh (mx) \tag{6}$$

The following boundary conditions are applied to obtain the constants A and B in Eq. (6)

$$\text{At } x = 0, (dT/dx) = 0 \text{ and so } (dZ/dx) = 0 \tag{7}$$

$$\text{At } x = w, T = T_r \text{ and therefore } Z = T_r - T_a - S/U \tag{8}$$

where T_r is the root temperature of the fin.

Substituting the obtained values of constants A and B in Eq. (6) we get the temperature distribution in the fin as:

$$T - T_a - S/U = (T_r-T_a-S/U) \cosh (mx)/\cosh (mw) \tag{9}$$

Heat conducted into the tube by one fin of unit length can be obtained as:

$$q_F = - kt\frac{dT}{dx} \Big|_w \tag{10}$$

Differentiating Eq. (9) with respect to x and substituting in Eq. (10), gives q_F as:

$$q_F = (S-U(T_r-T_a)) \quad (\tanh (mw))/m \tag{11}$$

Dividing q_F by area of one fin one gets heat collected per unit area of the collector, q as:

$$q = [S-U(T_r-Ta)](\tanh (mw))/(mw) \tag{12}$$

Total cost of a solar flat-plate collector can be considered to be composed of three parts as follows:

1. cost of the riser tubes;
2. cost of the absorber plate; and
3. all other costs (both material and manufacturing).

The total cost of the collector associated with one fin of unit length is given by

$$C_F = C_t/2 + (C_f\, w\, t) + (C_o\, w) \quad (13)$$

where C_F = Cost of half a tube + Cost of one fin + Other costs associated with one fin.

Total cost of the collector per unit collector area, C, is obtained by dividing C_F by area of the fin. Thus,

$$C = C_F/w = (C_t/2w) + (C_f\, t) + C_o \quad (14)$$

For a particular set of values of U, K, C_t etc. assumed here for illustration the heat collected per unit area of the collector q, and the cost of the collector per unit collector area C, were found for different fin widths. The ratio q/C which is the heat collected per unit cost was also evaluated. The values of the variable U, k, C_t, etc. used in the calculations and the values obtained are given in Table 4 (Assume $U = 5\ W/m^2K$, $k = 211\ W/mK$, $C_t = 15\ Rs/m$, $C_f = 80{,}000\ Rs/m^3$, $C_o = 400\ Rs/m^2$, $S = 250\ W/m^2$, $T_r = 35\ °C$, $T_a = 25\ °C$, $t = 0.00057\ m$).

Using the calculated data from Table 4, q, C and q/C were plotted against fin width as shown in Fig. 2. From Fig. 2 it is seen that both q and C decrease with increasing fin width. But heat collected per unit cost increases with increasing fin width reaches a maximum and then starts decreasing. The fin width which maximizes 'heat collected per unit cost' is the most desirable fin width. Though the

Table 4 Variation of q, C and q/C with fin width.

w (m)	q (W/m^2)	C (Rs/m^2)	q/C (W/Rs)
.0029	199.98	3031.31	.0660
.0229	198.56	773.11	.2568
.0429	195.95	620.43	.3144
.0629	189.71	564.84	.3359
.0829	182.90	536.07	.3412
.1029	175.94	518.49	.3376
.1299	166.52	506.63	.3287
.1429	157.71	498.08	.3166
.1629	148.89	491.64	.3028

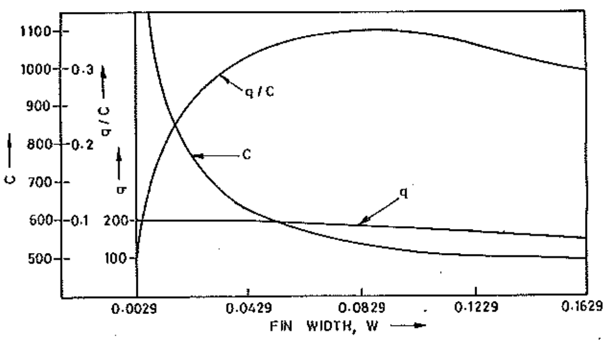

Fig. 2 Trend of C, q/C and q for different fin width (for one t).

fin width which maximizes the heat collected per unit cost is the most desirable fin width for that assumed fin thickness, it is seen that it is still a function of fin thickness. For different fin thicknesses different fin widths will maximize heat collected per unit cost. So both fin width and thickness must be optimized simultaneously.

For algebraic ease, two relative cost parameters C_1 and C_2 are defined as:

$$C_1 = C_o/C_f \tag{15}$$

$$C_2 = C_t/C_f \tag{16}$$

Using Eq. (12) and Eq. (14) the heat output per unit cost Q can be written as:

$$Q = q/C = \frac{[S-U(T_r-T_a)]\tanh(w\sqrt{U/kt})}{C_f\sqrt{U/kt}\,(C_2/2 + w(C_1+t))} \tag{17}$$

For maximum heat collection per unit cost both $(\delta Q/\delta w)$ and $(\delta Q/\delta t)$ must be equated to zero.

Partially differentiating the expression for Q with respect to w and equating the result to zero gives:

$$(C_1 + t) = \frac{\sqrt{U/kt}\,(C_2/2 + w(C_1 + t))}{\sinh(w\sqrt{U/kt})\cosh(w\sqrt{U/kt})} \tag{18}$$

Partially differentiating the expression for Q with respect to t and equating the result to zero gives:

$$\frac{\sqrt{U/kt}\,(C_2/2 + w(C_1 + t))}{\sinh(w\sqrt{U/kt})\cosh(w\sqrt{U/kt})} = -2t + C_2/2w + C_1 + t \tag{19}$$

RHS of Eq. (18) and LHS of Eq. (19) are same, and therefore

$$t = C_2/4w \tag{20}$$

Substituting the above expression for t in Eq. (18) and simplifying gives:

$$\frac{\sinh(\sqrt{16Uw^3/C_2k})}{\sqrt{16Uw^3/C_2k}} = \frac{4w\,C_1 + 3\,C_2}{4w\,C_1 + C_2} \tag{21}$$

The above equation relates optimum width of the fin w, to some important parameters of a solar flat-plate collector. If values of the parameters U, k, C_t, and C_o are known, optimum fin width w can be obtained by solving Eq. (21). Once optimum fin width w is found, optimum fin thickness t can be found from Eq. (20).

If the values of U, k, C_t, C_f and C_o are known (the same values assumed for illustrative purposes earlier are used for further elucidation), Eq. (21) can be solved for optimum fin width w, by plotting its LHS and RHS as a function of fin width w. The value of w corresponding to the point of intersection of the two curves plotted will give the optimum fin width. Eq. (21) was solved for the following set of values ($U = 5 W/m^2K$, $k = 211$ W/m K, $C_t = 15$ Rs/m, $C_f = 80,000$ Rs/m^3, $C_o = 400$ Rs/m^2, $C_1 = C_o/C_f = 0.005$ m, $C_2 = C_t/C_f = 0.0001875$ m^2) of the collector parameters using the above method and optimum fin width and thickness have been found. Table 5 and Fig. 3 show calculations and plot of Eq. (21).

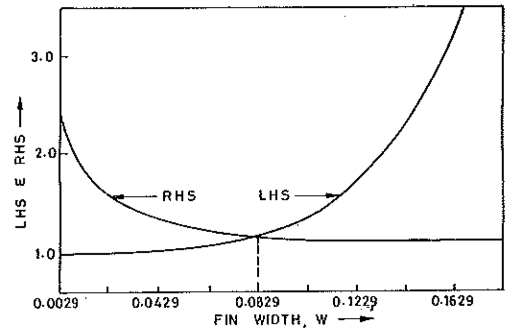

Fig. 3 Variation of LHS and RHS of Eq. (21) with fin width.

Table 5 Values of LHS and RHS of Eq. (21) for different fin widths.

w	LHS	RHS
.0029	1.000	2.527
.0229	1.004	1.581
.0429	1.027	1.359
.0629	1.086	1.259
.0829	1.203	1.204
.1029	1.410	1.167
.1229	1.754	1.142
.1429	2.318	1.123
.1629	3.244	1.109
.1829	4.786	1.098

From Fig. 3 and Table 5 it is seen that LHS and RHS become equal at a fin width of 0.0829 m, which is the cost optimum fin width. Hence optimum fin thickness is found from Eq. (20) as:

Optimum fin thickness = $C_2/4w = 0.00057$ m.

GRAPHICAL SOLUTION AND NOMOGRAPH

A closer look at Eq. (21) reveals that for any set of values of U, k, C_2 and w the remaining variable C_1 could be found easily. So it must be possible to vary the variables U, k, C_2 and w over a wide range and find out the corresponding values of the variable C_1. The data thus obtained could be plotted in a graphical form so that for any known set of values of U, k, C_1, and C_2 the value of the optimum fin width w could be found directly from the graphs. To reduce the number of graphs to be plotted to a minimum number and to generalize, it is necessary to reduce the number of variables to the minimum. So new variables V and B are defined as:

$$V = U/kC_2 \text{ and } B = \text{LHS of Eq. (21)} \tag{22}$$

where

$$B = (\sinh\sqrt{16\,Vw^3})/(\sqrt{16Vw^3}) \tag{23}$$

Now Eq. (21) can be rewritten as:

$$B = \frac{4wC_1 + 3C_2}{4wC_1 + C_2} \text{ or } C_1/C_2 = (3 - B)/[4w(B - 1)] \tag{24}$$

Eq. (21) has now been modified into two simultaneous equations namely Eq. (23) and (24) containing only three variable viz. w, V and C_1/C_2. So it should be possible to represent the relationship among the variables w, V and C_1/C_2 in a graphical form. To get the necessary data a particular value is first given to w and then V is varied over a range and the corresponding values of C_1/C_2 are obtained from Eq. (24). The process is repeated for different values of w.

Using the data so generated and taking V and C_1/C_2 in the x and y axes respectively curves were drawn for different values of w. The curves drawn resembled a hyperbola and the readability of the graph was poor. So it was decided to use log (V) and log (C_1/C_2) instead of V and C_1/C_2. Accordingly two new variables x and y are defined as:

$$x = \log(V) = \log(U/kC_2) \tag{25}$$

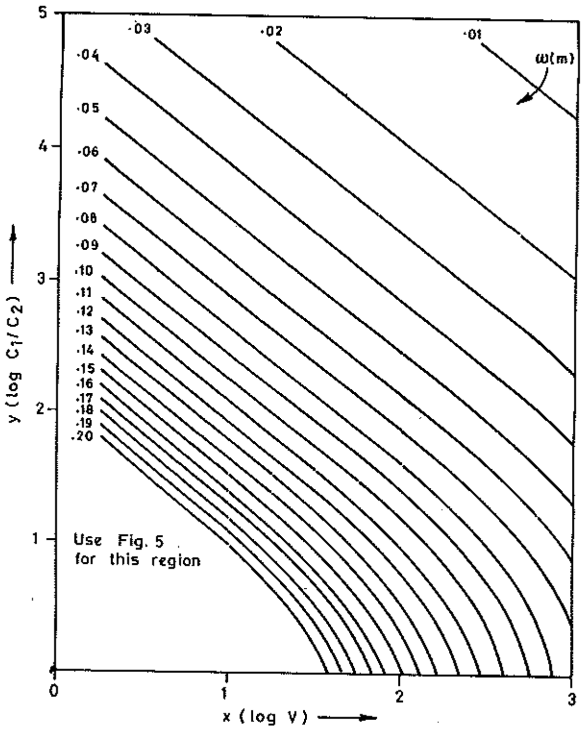

Fig. 4 Design nomograph for determining optimum fin width.

$$y = \log (C_1/C_2) \qquad (26)$$

For selected values of w and x, the corresponding y values were computed. x and w were varied over a wide range and the corresponding values of y were found. Using the data generated nomographs namely Figs. 4 & 5 were drawn connecting x, y and w. There x varies from 0.0 to 3.0 in Fig. 4 while x varies from 0.0 to 1.5 in Fig. 5.

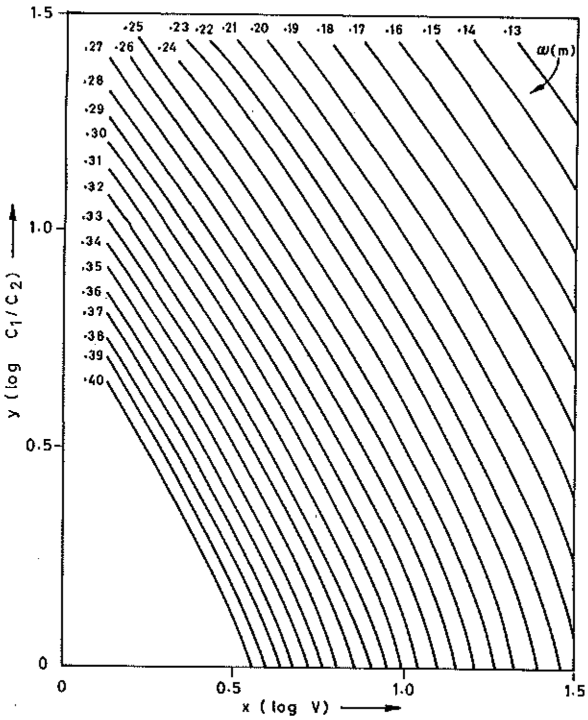

Fig. 5 Design nomograph for determining optimum fin width.

If the values of U, k, C_t, C_f and C_o of a collector are known the two parameters x and y can be evaluated. For these values of x and y the optimum fin width w can be read from Fig. 4 or Fig. 5. The corresponding optimum fin thickness t can then be obtained using Eq. (20). The heat collected per unit input cost can be obtained using the obtained optimum fin dimensions in Eq. (17).

EXAMPLES

Example 1: The use of nomographs for the design cases of aluminum fin-copper tube, copper fin - copper tube and mild steel fin-G.I. tube collectors are given. The assumed design data, the resulting values of x and y co-ordinates and the read values of optimum w from the nomograph are also listed in Table 6. The optimum t obtained together with the heat collected per unit input cost Q is also listed in Table 6 for comparative purposes. It may be noted that heat collected per unit cost Q was obtained from Eq. (17) taking $S = 400$ W/m^2, $T_r = 35$ °C, $T_a = 25$ °C and the obtained optimized w and t values.

It may be seen that aluminum fin - copper tube will have the maximum heat collected per unit cost.

Example 2: The cost related energy Eq. (21) shows that the optimum fin dimensions are independent of meteorological and operating parameters. In this example variations of Q, the heat collected per unit cost (Eq. 17) are calculated from different solar radiation and operating conditions for the case of an aluminum fin-copper tube collector. The design values of U, k, C_t, C_f and C_o are taken as the same in Example 1. Q calculated using Eq. (17) for different selected w and t are listed in the Tables 7 to 11. The corresponding S, T_r and T_a values are also stated. The desirable fin dimensions for each case would be the ones that maximize the heat collected per unit cost. From the tables it could be read that this value of Q corresponds to the fin width of 0.0829 m and fin thickness of 0.00057 m in all cases. These values are the same as those given by the nomograph method. Hence it may be stated that the design methodology suggested in this work is general and the obtained optimum fin dimensions are independent of meteorological and operating parameters.

Table 6 Optimal fin dimensions and the heat collected per unit input cost for three types of collectors.

Values of	Aluminum fin copper tube	Copper fin copper tube	Mild steel fin G.I. tube	Remarks
U,W/m²K	5.0	5.0	5.0	
K,W/m-K	211.0	385.0	47.5	
C_t,Rs/m	15.0	15.0	20.0	Assumed
C_f,Rs/m³	8x10⁴	45x10⁴	6x10⁴	
C_o,Rs/m²	400.0	400.0	400.0	
x	2.1017	2.466	2.499	Calculated
y	1.426	1.301	1.301	Calculated
w,m	0.083	0.071	0.07	From nomograph
t,m	0.00057	0.000156	0.00119	From Eq. (20)
Q,W/Rs	0.5971	0.5042	0.4996	From Eq. (17)

Table 7 Variation of Q with fin width and thickness for S = 250W/m², T_r = 35°C and T_a = 25°C.

t(m)	w(m)				
	.0629	.0729	.0829	.0929	.1029
.00037	.3365	.3381	.3366	.3327	.3272
.00047	.3371	.3402	.3403	.3382	.3343
.00057	.3359	.3400	.3412	.3402	.3376
.00067	.3337	.3385	.3405	.3404	.3386
.00077	.3310	.3362	.3387	.3393	.3382

Table 8 Variation of Q with fin width and thickness for $S = 300\ W/m^2$, $T_r = 35°C$ and $T_a = 25°C$.

t(m)	w(m)				
	.0629	.0729	.0829	.0929	.1029
.00037	.4206	.4226	.4208	.4159	.4090
.00047	.4214	.4253	.4254	.4228	.4179
.00057	.4199	.4250	.4265	.4253	.4220
.00067	.4171	.4231	.4256	.4255	.4223
.00077	.4138	.4203	.4234	.4241	.4228

Table 9 Variation of Q with fin width and thickness for $S = 300 W/m^2$, $T_r = 40°C$ and $T_a = 25°C$.

t(m)	w/(m)				
	.0629	.0729	.0829	.0929	.1029
.00037	.3785	.3803	.3787	.3743	.3681
.00047	.3793	.3828	.3829	.3805	.3761
.00057	.3779	.3825	.3839	.3828	.3798
.00067	.3754	.3808	.3830	.3830	.3810
.00077	.3724	.3783	.3811	.3817	.3805

Table 10 Vartiation of Q with fin width and thickness for $S = 350\ W/m^2$, $T_r = 45°C$ and $T_a = 25°C$.

t(m)	w(m)				
	.0629	.0729	.0829	.0929	.1029
.00037	.4206	.4226	.4208	.4159	.4040
.00047	.4214	.4253	.4254	.4228	.4179
.00057	.4199	.4250	.4265	.4253	.4220
.00067	.4171	.4231	.4256	.4255	.4233
.00077	.4138	.4203	.4234	.4241	.4228

Table 11 Variation of Q with fin thickness and width for $S = 400\ W/m^2$, $T_r = 45°C$ and $T_a = 25°C$.

t(m)	w(m)				
	.0629	.0729	.0829	.0929	.1029
.00037	.5047	.5071	.5050	.4991	.4908
.00047	.5057	.5104	.5105	.5074	.5015
.00057	.5039	.5100	.5118	.5104	.5064
.00067	.5005	.5077	.5107	.5106	.5080
.00077	.4966	.5044	.5081	.5089	.5074

CONCLUSIONS

A closer examination of cost related energy Eq. (21) reveals that the optimum fin width and therefore optimum fin thickness are not functions of T_r, S and T_a as is the case in Eq. (17). This means that even if the operating conditions (T_r, S or T_a) change, the cost optimal fin width and thickness still remain the same. Cross checking of the resulting w and t values for different T_r, S or T_a values (assuming of course U remains the same) using the nomographs substantiates this conclusion. Hence the design nomographs presented here serve as a design tool for the cost optimal design of flat-plate solar collectors. It could be used for any material combination of fin and tube construction, and to obtain the corresponding cost effective fin thickness and width.

NOMENCLATURE

C	Total cost of the collector per unit collector area (Rs/m^2)
C_F	Total cost of the collector associated with one fin of unit length (Rs)
C_f	Cost of the fin material per unit volume (Rs/m^3)
C_t	Cost of riser tubes per unit length (Rs/m)
C_o	Other costs of the collector per unit collector area (Rs/m^2)
C_1	C_o/C_f (m)
C_2	C_t/C_f (m^2)
D	Outer diameter of riser tubes (m)
F	Fin efficiency
k	Thermal conductivity of fin material (W/m K)
m	$(U/kt)^{1/2}$
q_F	Heat conducted into the tube by one fin of unit length (w)
q	Heat collected per unit area of the collector (W/m^2)
Q	Heat collected per unit cost of the collector (W/Rs)
S	Solar radiation absorbed per unit collector area (W/m^2)
t	thickness of fin (m)
T	Temperature at any point over the absorber (°C)
T_a	Ambient temperature (°C)
T_r	Temperature at the root of the fin (°C)
U	Effective overall heat loss coefficient of collector (W/m^2.K)
V	U/kC_2
w	Width of the fin (m)
x	Log (U/kC_2)
y	Log (C_1/C_2)
Z	$T - T_a - S/U$
B	Sinh $(\sqrt{(16\ Vw^3)})/\sqrt{(16\ Vm^3)}$

REFERENCES

1. Duffie, J.A. and W.A. Beckman (1974), *Solar Thermal Processes*, John Wiley, New York.
2. Hahne, E. (1985), Parameter effects on design and performance of flat-plate solar collectors,

Solar Energy, Vol. 34, No. 6, pp.497-504.

3. Kovarik, M. (1978), Optimal distribution of heat conducting material in the finned pipe solar energy collectors, *Solar Energy*, Vol. 21, pp.477-484.
4. MacGregor, A.W.K. (1979), Economic use of materials in the design of solar water heating collector plates of the pipe and fin type, *Sun*, Vol. 2, pp.945-949.
5. Sukhatme, S.P. (1984), *Solar Energy*, pp.115-116, Tata McGraw Hill Publishing Company, New Delhi.