# Fundamentals of Hydraulic Turbine Design \*

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## INTRODUCTION

Water turbines produce approximately one-fourth of the world's electric power. The St. Anthony Falls, just outside my office window, serves as a constant reminder of the role that hydropower has played, not only in the development of Minneapolis (Kane, 1966), but in the development of our civilization as we know it today. It is expected that hydropower will continue to have a place in our energy picture and will have an increasingly important impact on the growth of the less developed parts of our world.

Since we are currently in transition from a relatively stagnant period of design and development to an anticipated renaissance of small turbine development, this paper is structured in such a way that the basics of turbine performance as well as a summary of turbine characteristics which affect the operation of a hydropower site are provided. Cook-book procedures for turbine selection are avoided. Instead, a firm grounding in the basic principles of turbine hydrodynamics is given, which should provide the reader with the necessary tools to not only select a turbine from the current offerings but also to have the ability to evaluate new turbine types which may appear in the future.

The water turbine has a rich and varied history (Rouse and Ince, 1963; Smith, 1980) and has been developed as a result of a natural evolutionary process from the water wheel. Originally used for direct drive of machinery, the use of the water turbine for the generation of electricity is a comparatively recent activity. Much of its development occurred in France, which unlike England, did not have the cheap and plentiful sources of coal which sparked the industrial revolution in the eighteenth century. Nineteenth century France found itself with its most abundant energy resource being water. To this day *houille blanche* (literally white coal) is the French term for water power.

In 1826 the Société d'Encouragement pour l' Industrie Nationale offered a prize of 6000 FF to anyone who "would succeed in applying on a large scale, in a satisfactory manner, in mills and factories, the hydraulic turbines or wheels with curved blades of Bélidor" (Smith, 1980). Bélidor was an eighteenth century hydraulic and military engineer who, in the period 1737-1753, authored a monumental four volume work, "Architecture Hydraulique," a descriptive compilation of hydraulic engineering information of every sort. The water wheels described by Bélidor departed from convention by having a vertical axis of rotation and being enclosed in a long cylindrical chamber approximately one meter in diameter. Large quantities of water were supplied from a tapered sluice at a tangent to the chamber. The water entered with considerable rotational velocity. This pre-swirl combined with the weight of water above the wheel was the driving force. The original tub wheel had an efficiency of only 15 per cent to 20 per cent.

Water turbine development proceeded on several fronts during the period 1750 to 1850.

\*Presented at a conference on Small Hydropower for Asian Rural Development, Bangkok, Thailand, June 8-11,1981.

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The classical horizontal axis water wheel was improved by such engineers as John Smeaton of England (1724-92), who incidentally used in this endeavor the first avowed model experiments and also played an important role in windmill development, and the French engineer J. V. Poncelet (1788-1867). This resulted in water wheels having efficiencies in the range of 60 per cent to 70 per cent. At the same time, reaction turbines (somewhat akin to the modern lawn sprinkler) were being considered by several workers. The great Swiss mathematician, Leonhard Euler (1707-83), investigated the theory of operation of these devices. A practical application of concept was introduced in France in 1807 by Mannoury de Ectot (1777-1822). His machines were, in effect, radial outward-flow machines. The theoretical analyses of Burdin (1790-1893), a French professor of mining engineering who introduced the word "turbine" in engineering terminology, contributed much to our understanding of the principles of turbine operation and underscored the principal requirements for high efficiency. A student of Burdin, Benoit Fourneyron (1802-1867), was responsible for putting his teacher's theory to practical use. His work led to the development of high speed, outward-flow turbines with efficiencies of the order of 80 per cent. The early work of Fourneyron resulted in several practical applications and the winning of the coveted 6000 franc prize in 1833. After nearly a century of development, Bélidor's tub wheel had been officially improved.

Fourneyron spent the remaining years of his life developing some 100 turbines in France and Europe. Some turbines even found their way to the U.S., the first in about 1843. The Fourneyron centrifugal turbines were designed for a wide range of conditions, with heads as high as 114 meters and speeds as high as 2300 rpm. Very low-head turbines were also designed and built.

As successful as the Fourneyron turbines were, they lacked flexibility and were only efficient over a narrow range of operating conditions. This problem was addressed by Hoyd and Boyden (1804-79). Their work evolved into the concept of an inward flow motor due to James B. Francis (1815-92). The modern Francis turbine is the result of this line of development. At the same time, European engineers addressed the idea of axial flow machines, which today are represented by "propeller" turbines of both fixed pitch and the Kaplan type.

Just as the vertical axis tub wheels of Bélidor evolved into modern reaction turbines of the Francis and Kaplan type, development of the classical, horizontal-axis water wheel reached its peak with the introduction of the impulse turbine. The seeds of development were sown by Poncelet in 1826 with his description of the criteria for an efficient water wheel. These ideas were cultivated by a group of California engineers in the late 19th century, one of whom was Lester A. Pelton (1829-1908), whose name is given to the Pelton Wheel, which consists of a jet or jets of water impinging on an array of specially shaped buckets closely spaced around the periphery of a wheel. Thus, it can be said that the relatively high speed reaction turbines trace their roots to the vertical axis tub wheels of Bélidor, whereas the Pelton wheel can be considered as a direct development of the more familiar horizontal axis water wheel. Turbine configurations as we know them today are generally in the form as originally developed. For example, the overwhelming majority of Pelton wheels have horizontal axes. Vertical axis Pelton wheels are a relatively recent development. In over 250 years of development many ideas were tried, some were rejected and others were retained and incorporated in the design of the hydraulic turbine as we know it today. This development has resulted in highly efficient devices, with efficiencies as high as 95 per cent in the larger sizes. In terms of design concept, these fall into roughly three categories, reaction turbines of the Francis and propeller design and impulse wheels of the Pelton type. The rest of this paper is devoted to a review of the principles of operation, the classification and selection of turbines for given operating conditions, and a review of performance characteristics and operational limitations. Most of the development efforts to date have been placed on large turbines, with small turbine technology consisting chiefly of scaling down larger turbines. The validity of this concept is reviewed, and areas where improvements can be made are addressed.

# PRINCIPLES OF OPERATION

# 2.1 Euler's Equation

The torque on the runner of a turbine can be found through conservation of radial momentum. In other words, the torque on a runner is the difference between the rate of angular momentum entering the runner and that exiting. Referring to Fig. 1, this can be written as:

$$T = \rho Q \left( r_1 V_1 \cos \alpha_1 - r_2 V_2 \cos \alpha_2 \right)$$
 (1)

wherein  $\rho$  is the fluid density and Q is the volumetric rate of flow.



Fig. 1. Definition sketch for radial flow turbine runner. Adapted from Daugherty and Francini (1977).

Since the power produced is proportional to the product of mass flow rate and head, we can write the following:

$$T\omega = \rho g Q H_{\mu} \tag{2}$$

$$\omega r_{1} = u_{1} \tag{3}$$

$$\omega r_2 = u_2 \tag{4}$$

Thus

$$H_{u} = (u_{1}V_{1} \cos \alpha_{1} - u_{2}V_{2} \cos \alpha_{2})/g$$
(5)

where  $H_u$  is the head utilized by the runner in the production of power. We must be careful to keep in mind that  $V_1$ ,  $V_2$  are absolute quantities, whereas  $u_1$  and  $u_2$  are the peripheral speeds at entrance and exit, respectively. In a fixed frame of reference the absolute velocity V is related to the vector sum of the relative velocity v and the velocity of a body moving with velocity u.

In vector notation:

$$\mathbf{V} = \mathbf{u} + \mathbf{v} \tag{6}$$

We will define the angles of  $\alpha$  and  $\beta$  as respectively the angles made by the absolute and relative velocities of a fluid with the linear velocity **u** of some body. This is illustrated in the diagram below.



Fig. 2. Typical velocity triangles.

It is obvious from inspection of the figures that

$$V \sin \alpha = \nu \sin \beta \tag{7}$$

$$V \cos \alpha = u + v \cos \beta \tag{8}$$

The energy equation written between two points is given by

$$\begin{pmatrix} P & V^{2} \\ \frac{1}{\gamma} + z_{1} + \frac{1}{2g} \\ = H_{L} + (u_{1}V_{1} \cos \alpha_{1} - u_{2}V_{2} \cos \alpha_{2})/g \end{pmatrix}$$
(9)

wherein  $H_L$  represents frictional losses and the last term on the right hand side of the equation represents the head absorbed by the turbine. It follows from Eqs. (7) and (8) that

$$V^2 = v^2 + u^2 + 2vu \cos \beta$$
 (10)

$$uV\cos\alpha = u(u + v\cos\beta) \tag{11}$$

Combining Eqs. (9), (10), and (11) results in the so-called energy equation in a rotating frame of reference.

$$\left(\frac{P_{1}}{\gamma}+z_{1}+\frac{V_{1}^{2}-u_{1}^{2}}{2g}\right)-\left(\frac{P_{2}}{\gamma}+z^{2}+\frac{V_{2}^{2}-u_{1}^{2}}{2g}\right)=H_{L}$$
 (12)

Note that if there is no flow,  $v_1 = v_2 = 0$  and the equation reduces to that for a vortex. If there is no rotation, the equation reduces to the familiar form of the energy equation.

### 2.2 Turbine Efficiency and Losses

### Definitions

The hydraulic efficiency of a turbine is defined by

$$\eta_h = H_u/H$$

where  $H_u$  is the head utilized by the runner and H is the net head on the turbine, defined as the difference between the total head at the entrance to the turbine proper (entrance to the spiral casing and the total head at the tailrace). The definition of net head is illustrated in Fig. 3. The hydraulic efficiency expresses the effectiveness of the transfer to the runner of the available power in the fluid that flows through it. Also illustrated in Fig. 3 is the definition of gross head; the difference between headwater and tailwater elevations.



Fig. 3. Definition sketch for net head.

## 2.3 Similarity Considerations

#### Similitude Theory

The grouping of parameters brought about by dimensional or inspectional analysis permits the writing of any physical relationship in terms of fewer dimensionless quantities ( $\pi$  numbers) representing ratios of significant forces for the problem. This provides a method to extrapolate model test data to prototype situations by equating corresponding dimensionless numbers. As applied to hydraulic machinery, similarity considerations provide, furthermore, an answer to the following important question: Given test data on the performance characteristics of a certain type of machine under certain operating conditions, what can be said about the performance characteristics of the same machine, or of a geometrically similar machine, under different operating conditions? Similarity considerations provide in addition a means of cataloguing machine types and thus aid in the selection of the type suitable for a particular set of conditions.

The problem of similarity of flow conditions can be summarized as follows: Under what conditions will geometrically similar flow patterns with proportional velocities and accelerations occur around or within geometrically similar bodies? Obviously the forces acting on correspond-

ing fluid masses must be proportionally related, as are the kinematic quantities, so as to insure that the fluid will follow geometrically similar paths. An answer to this question can be obtained by examining the fundamental laws of motion and identifying the relevant forces. While these laws cannot yet be used to predict theoretically the flow conditions in a machine with unknown performance characteristics, the information they provide on forces and boundary conditions enables the determination of an answer to the similitude problem.

Similarity of the velocity diagrams at the entrance to the runner is a necessary requirement. Referring to Fig. 1, assuming equal angles  $\alpha_1$  in model and prototype, the ratio  $V_1/u_1$  must be held constant. If  $V_n$  denotes the radial component of the velocity (normal to the flow passages), we have

$$Q_e = f_b \pi \ \frac{B}{D} \ V_n D^2 \tag{14}$$

where  $f_b$  represents the fraction of free space in the inlet passages of the runner  $(f_b - 0.95)$  and B is the width of the passages. With

$$V_n = V_1 \sin \alpha_1 = V_{R_1} \sin \beta_1 \tag{15}$$

Eq. (14) becomes

$$Q_e = (f_b \pi \frac{B}{D} \sin \alpha_1) V_1 D^2$$

Since

$$u = \Omega r = \frac{\pi n}{60} D \tag{16}$$

we have

$$\frac{V_1}{u_1} = \frac{1}{(f_b \pi \frac{B}{D} \sin \alpha_1)}; \quad \frac{Q_e}{u_1 D_1^2} = \frac{1}{(f_b \pi \frac{B}{D} \sin \alpha_1)} \quad \frac{60}{\pi} \quad \frac{Q_e}{n D_1^3}$$
(17)

Constancy of the ratio  $Q_e/nD^3$  or  $Q_e/\Omega D^3$  is then a necessary condition for similarity.

If we now make the assumption that viscous forces are small relative to inertia forces and thus can be neglected in first approximation, and that furthermore the fluid does not change its physical properties as it passes through the machine (which excludes compressibility effects and cavitation, to be dealt with later), the only other forces that appear in the fundamental equations of motion are the pressure forces. Their ratio to inertia forces is proportional to  $\Delta p/\rho V^2$  or, if the head  $H_u$  utilized by the runner is introduced, to  $g H_u/V^2$ . Under the assumptions made, the condition

$$Q_{\rho} / \Omega D^3 = \text{constant}$$
 (18)

is sufficient for similarity. The condition

$$gH_u/V^2$$
 = constant or  $gH_u/\Omega^2 D^2$  = constant (19)

follows from the basic laws and permits calculation of the head  $H_u$  for similar operating conditions. The equality of the ratio  $gH_u/V^2$  for model and prototype also follows from inspection of Euler's equation (5) under the assumption of negligible viscous effects.

Velocity coefficients  $\phi$ ,  $C_1$ , and  $C_2$  are customarily introduced as

$$u_1 = \phi \sqrt{2 g H}, V_1 = C_1 \sqrt{2 g H}, V_2 = C_2 \sqrt{2 g H}$$
 (20)

In terms of these coefficients, the hydraulic efficiency  $\eta_h$  defined by Eq. (13) can be written with the use of Eq. (5) as

$$\eta_{h} = \frac{u_{1}V_{1}\cos\alpha_{1} - u_{2}V_{2}\cos\alpha_{2}}{gH} = 2\phi \left(C_{1}\cos\alpha_{1} - \frac{D}{D_{1}}C_{2}\cos\alpha_{2}\right)$$
(21)

If the viscous losses embodied in  $\eta_h$  can be assumed to occur under hydro-dynamically rough conditions, in the sense that the losses are independent of Reynolds number, Re, and depend only on geometric ratios and relative roughness (Re must be high enough for the losses to be purely turbulence-controlled), then  $\eta_h$  must be the same in model and prototype, provided that the relative roughnesses are the same and the geometry is faithfully reproduced. Under these conditions Eq. (19) becomes

$$gH/\Omega^2 D^2 = \text{constant}$$
(22)

Analogous considerations can be made about the volumetric efficiency  $\eta_{v}$ . Here Reynolds number effects may be more significant due to the smallness of the leakage-flow passages. But if one can assume Re independence, and under strict geometric similarity (including surface roughness and running clearances),  $\eta_{v}$  must be the same in model and prototype and Eq. (19) becomes

$$Q/\Omega D^3 = \text{constant}$$
 (23)

Equations (22) and (23) permit calculation of the net head H and total flow rate Q for similar operating conditions.

#### Scale Effects

The foregoing assumptions are reasonably accurate for turbines of fairly large dimensions operating under non-cavitating conditions. In particular, relative roughnesses and running clearance ratios are the same if similarity considerations are applied to the same machine. For large differences in the size of two geometrically similar machines, such as between model and proto-type, roughnesses and clearances cannot be geometrically scaled due to fabrication limitations. Certain formulas have been developed to correlate model and prototype data, all of them containing a strong dose of empiricism. The Moody step-up equation (Moody and Zowski, 1969)

$$(1 - \eta_1) / (1 - \eta_2) = (D_2 / D_1)^n$$
(24)

has been found to give satisfactory results for turbine flows. The basic assumption in Moody's derivation is Reynolds number independence (hydrodynamically rough flow) and the same degree of surface finish in model and prototype. The derivation is based on assuming losses of the form given by the Durey-Weisbach equation with  $f = A (k/D)^n$ , where A is constant. Since k is assumed to be the same for model and prototype, it disappears from the final result. The empirical exponent n is based on turbine test results:  $n \sim 1/5$  according to Moody, but it may become appreciably smaller if the formula is used with models with very smooth walls and close running clearances. It should be emphasized that Moody's formula has been developed solely with regard to the effect of relative roughness. No consideration is given to changes in the relative size of the running clearances, which obviously affect  $\eta_v$ , nor are mechanical losses explicitly taken into account (although these factors certainly affect the empirical n values). The formula is nevertheless used to correlate overall efficiencies.

The power, P, developed by the turbine is given by

$$P = \eta \gamma Q H = \eta_m \eta_n \eta_h \gamma Q H \tag{25}$$

Thus, even if  $\eta_h$  and  $\eta_v$  are the same in model and prototype,  $\eta_m$  would be different generally due to differences in disk friction losses and in the losses in bearings and stuffing boxes. Obviously, if the mechanical losses are small, changes in  $\eta_m$  would also be small, and could be subsumed in the empirically determined values of the coefficients and/or exponents in the available stepup formulas. Camerer's formula (Camerer, 1924; Nechleba, 1957), as does Moody's Eq. (24), takes into account solely surface roughness effects. A formula of Ackeret's (Muhlemann, 1948), a second formula of Moody (Nechleba, 1957; Mühlemann, 1948), and a formula due to Hutton (1954) all incorporate Reynolds number effects and the net head ratio thus appears explicitly in them, in addition to the size ratio of model and prototype. All of them are based on derivations that ignore volumetric losses and mechanical losses, although the selection of the empirical coefficients is based on actual turbine test data.

It may be noted here that Euler's (5), or the basic relationships of Eqs. (9) and (12) as applied to reaction turbines, imply simplifications which require in actual calculations the use of experimentally determined coefficients. Thus, for example, fluid particles in different streamlines generally have different velocities and, furthermore, their radial distances to the axis of rotation at entry to or exit from the runner are also different, because the entrance or exit edges of the vanes are not always parallel to the turbine axis. Despite these problems, the theory is useful in many ways: it shows the nature of the performance, and it shows how changes in design should be made to alter the characteristics of a machine as obtained from experimental testing. It also sheds light on the nature of the similarity laws and possible scale effects.

Similar considerations can be made regarding the theory of impulse (Pelton) wheels (to be dealt with later) which is based on a simplified version of Eq. (2). Essentially, the same results are obtained regarding similarity relationships. For Pelton wheels, however, the efficiency is nearly independent of size and Eq. (24) or similar equations do not apply. As the size of a Pelton wheel increases, there is a deterioration in the smoothness of the jet before it strikes the buckets, which nullifies any benefits from reduced friction losses. Furthermore, there are no leakage losses to make a difference.

The similarity results in these sections can be used, with due regard to the approximations involved, to predict the performance of a machine under operating conditions different from

those of available experimental data, and the performance of geometrically similar machines if performance characteristics are available for one of them. Some examples of application are given later.

#### Specific Speed

Similar flow conditions are ensured by the constancy of the ratio  $Q/\Omega D^3$ , which implies constancy of the ratio  $gH/\Omega^2 D^2$ . In other words,

$$gH/\Omega^2 D^2 = f(Q/\Omega D^3)$$
<sup>(26)</sup>

This relationship can also be written in terms of a third dimensionless number which does not involve the representative dimension D of the machine, and which can replace either of the two arguments in Eq. (26). Such a number can be obtained by appropriate multiplication of powers of the dimensionless numbers in Eq. (26):

$$N_{s_Q} = (Q/\Omega D^3)^{1/2} (\Omega^2 D^2/gH)^{3/4} = \Omega Q^{1/2}/(gH)^{3/4}$$
(27)

This dimensionless number is called the specific speed. For hydraulic turbines, however, the definition of the specific speed is based on the power P delivered by the turbine as a variable, instead of the flow rate Q. The corresponding dimensionless number for P is  $P/\rho\Omega^3 D^5$ , which is a function of  $qH/\Omega^2 D^2$ . Eliminating D between these two numbers one gets

$$N_{s} = \Omega \left( P/\rho \right)^{1/2} / \left( gH \right)^{5/4}$$
(28)

The two specific speeds are related by

$$N_s = \sqrt{\eta} N_{s_0} \tag{29}$$

which is obtained making use of Eq. (25). If we choose  $N_s$  as the independent variable in these relationships, then all other dimensionless combinations can be expressed as functions of  $N_s$ . These include also the dimensionless torque,  $T/\rho\Omega^2 D^5$ , and the efficiencies, under the assumption of negligible scale of effects.

The specific speed describes a specific combination of operating conditions that ensures similar flows in geometrically similar machines. It has thus attached to it a specific value of the efficiency  $\eta$  (assumed approximately constant for similar flow conditions regardless of size). It is then customary to label each series of geometrically-similar turbines by the value of  $N_s$ which gives maximum  $\eta$  for the series. Unless otherwise stated, this is the  $N_s$  value referred to when the terminology specific speed is used. The value of  $N_s$  thus defined permits the classification of turbines according to efficiency. Each geometric design has a range of  $N_s$  values where it can be used with only one value corresponding to peak efficiency. In subsequent sections this idea will be used to classify turbine designs.

The  $N_{g}$  as defined is dimensionless. It is common in practice to drop g and  $\rho$  from the definition and define  $n_{g}$  as

$$n_{s} = n \sqrt{P} / H^{5/4} \tag{30}$$

with n in rpm. In English units, the units of P are horsepower, and the units of H are feet. In metric units, the unit of P is either the metric horse-power or the kilowatt, and the unit of H is the meter. The relationships of these three definitions of  $n_s$  to the dimensionless  $N_s$  are:

$n_{s} = 43.5 N_{s}$	(English units)	
$n_{s} = 193.1 N_{s}$	(Métric units using metric horsepower)	(31)
$n_{s} = 166 N_{s}$	(Metric units using kW for power)	

2.4 Cavitation

#### Introduction

Cavitation can be defined as the formation of the vapor phase in a liquid flow when the hydrodynamic pressure falls below the vapor pressure of the liquid. It is distinguished from boiling, which is due to the vapor pressure being raised above the hydrodynamic pressure by heating. In its initial stages, cavitation is in the form of individual bubbles which are carried out of the minimum pressure region by the flow and collapse in regions of higher pressure. Calculations, as well as sophisticated laboratory experimentation, indicate that collapsing bubbles create very high impulsive pressures. This results in substantial noise (a cavitating turbine sounds like gravel is passing through it). More important, the repetitive application of the shock loading due to bubble collapse at liquid-solid boundaries results in pitting of the material. As the process continues, cracks form between the pits and solid material is spilled out from the surface. The mechanical effects of cavitation are enhanced by the high temperatures created by collapsing bubbles and the presence of oxygen rich gases which come out of solution. The details of the erosion process are complex, but the results are of practical significance. Many components of a turbine are susceptible to extensive damage as illustrated in Fig. 4. In more developed forms of cavitation, large vapor filled cavities remain attached to the boundary. Each cavity or pocket is formed by the liquid flow detaching from the rigid boundary of an immersed body or flow passage. The maximum length of a fixed cavity depends on the pressure field. Termination may occur by reattachment of the liquid stream at a downstream position on the solid surface or the cavity may extend well beyond the body. The latter case is known as supercavitation. Under these circumstances, the pressure distribution on the boundary can be substantially altered. If developed cavitation occurs on the runner or wicket gates of the turbine, the performance is changed. Cyclical growth and collapse of the cavities can also occur, producing vibration. Thus cavitation can degrade performance and produce vibration, as well as reducing the operational lifetime of the machine through erosion.



Fig. 4. Cavitation erosion on a turbine runner.

### The Cavitation Index

The fundamental parameter in the description of cavitation is the cavitation index

$$\sigma = \left(P_{\rho} - P_{\nu}\right) / \varkappa \rho V_{\rho}^{2} \tag{32}$$

The state of cavitation is assumed to be a unique function of  $\sigma$  for geometrically similar bodies. If  $\sigma$  is greater than a critical value, say  $\sigma_c$ , there is no cavitation and the various hydrodynamic parameters are independent of  $\sigma$ . When  $\sigma$  is less than  $\sigma_c$ , various hydrodynamic parameters such as the lift and drag of various components and the power and efficiency of a turbine are functions of  $\sigma$ . Noise, vibration, and erosion also scale with  $\sigma$ . It should be emphasized that the value of  $\sigma$  where there is a measureable change in performance is not the value of  $\sigma$  where cavitation can first be determined visually or acoustically. The critical  $\sigma_c$  can be thought of as a performance boundary such that:  $\sigma > \sigma_c$  no cavitation effects

 $\sigma < \sigma_c$  cavitation effects: performance degradation, noise and vibration

The precise value of  $\sigma_c$  defined by inception is normally only determined in the laboratory. It should not be confused with more pragmatic definitions such as the value of the cavitation index at a measureable change in hydraulic performance expressed by power, capacity or efficiency, or at a measureable change in vibration level.

When the value of  $\sigma$  is less than  $\sigma_c$ , fixed or attached cavities can form on the suction side of a lifting surface. The minimum pressure is the vapor pressure, independent of upstream velocity and pressure; hence

$$C_{p_m} = -\sigma \qquad \sigma < \sigma_c \tag{33}$$

where  $C_{p_m}$  is the minimum pressure coefficient defined by

$$C_{p_m} \equiv (p_m - p_o) / \frac{1}{2} \rho V_o^2$$

Assuming the pressure distribution on the pressure side to be uninfluenced by cavitation on the suction side, it is easily seen that the lift coefficient,  $C_L$ , should be proportional to  $C_{p_m}$ . This is



Fig. 5. Variation in lift coefficient with cavitation number. (Kermeen, 1956)

shown in Fig. 5. At each value of the angle of attack,  $\alpha$ , there is a value of  $\sigma$  above which  $C_L$  is independent of this parameter. At lower values of  $\sigma$ ,  $C_L$  decreases with decreasing  $\sigma$ . Note that as angle of attack increases,  $C_L$  increases and "cavitation stall" occurs at increasingly higher values of  $\sigma$ . In a turbo-machine the picture is qualitatively the same. As previously mentioned, the angle of attack is proportional to flow coefficient at a fixed wicket gate setting. Obviously the flow is more complicated, but the analogy between a hydrofoil and the blade section of a propeller turbine can be seen.

The introductory material on cavitation was based on simple geometric shapes and an easily defined cavitation parameter. The flow in a turbine is obviously more complex and less easily quantified. There still is, however, a definite need to define operating conditions with respect to cavitation. For example, it is sometimes necessary to specify under what conditions the degree of cavitation will be the same for the same machine operating under different heads and speeds, or for two machines of similar design but different heads and speeds, or for two machines of similar design but different size, e.g. a model and a prototype. The accepted parameter for this purpose is the Thoma sigma,  $\sigma_T$ . This is defined as

$$\sigma_T = H_{nd}/H \tag{34}$$

where  $H_{sv}$  is the net positive suction head. Referring to Fig. 3, this is defined as

$$H_{sv} = H_{a} - z_{1} - H_{v} + (V_{e}^{2}/2g) + H_{Q}$$
(35)

where  $H_a$  is the atmospheric pressure head,  $z_1$  is the elevation of the critical location for cavitation above the tailwater elevation,  $V_e$  the average velocity in the tailrace, and  $H_{\hat{k}}$  the head loss in the draft tube. If we neglect the draft tube losses and the exit velocity head, Thoma's sigma is

$$\sigma_T = (H_a - H_v - z)/H$$
(36)

Each type of turbine will cavitate at a given value of  $\sigma_T$ . Clearly cavitation can only be avoided if the installation is such that  $\sigma_T$  is greater than this critical value. The value of  $\sigma_T$  for a given installation is known as the plant sigma. For a given turbine operating under a given head, the only variable is the turbine setting, z. The critical value of Thoma's sigma,  $\sigma_{TC}$ , controls the allowable setting above tailwater:

$$z_{allow} = H_a - H_v - \sigma_{TC} H \tag{37}$$

It must be borne in mind that  $H_a$  varies with elevation. As a rule of thumb,  $H_a$  decreases from the sea level value of 10.3 meters by 1.1 meter for every 1000 m above sea level. Thus a turbine sited at Leadville, Colorado or Quito, Ecuador, for example, would have an allowable turbine setting that is three meters less than that at sea level. In fact,  $z_{allow}$  could easily be negative, implying a required turbine setting below the tailwater elevation.

The determination of  $\sigma_{TC}$  is usually done by a model test. A schematic of the correlation between performance breakdown and  $\sigma_{T}$  is shown in Fig. 6. This figure is based on information



Fig. 6. Schematic of the correlation between performance breakdown, noise, and erosion with cavitation index

presented by Deeprose et al (1974). Note the similarity in the trend of performance with  $\sigma_T$  and the correlation of lift coefficient at fixed  $\sigma$  with  $\sigma$  as shown in Fig. 5. As has already been emphasized, a measureable drop in efficiency occurs at value of  $\sigma_T$  that is well below the value corresponding to the detection of cavitation inception acoustically. Note also that maximum noise and presumably maximum erosion rate occur at a value of  $\sigma_T$  intermediate between the value at inception and that at performance breakdown. A slight rise in efficiency is often also noted at intermediate values of  $\sigma_T$ .

#### Suction Specific Speed

The critical value of  $\sigma$  is a function of the type of turbine involved, i.e. the specific speed of the machine. A cavitation scaling parameter often used in pump application is the suction specific speed

$$S = \Omega \sqrt{Q} / (gH_{sy})^{3/4}$$
(38)

The suction specific speed is a natural consequence of considering dynamic similarity in the low pressure region of a turbomachine. The dynamic relations are

$$gH_{sv}D_e^4/Q^2 = \text{const}, \quad gH_{sv}/\Omega^2 D_e^2 = \text{const}$$
 (39)

where  $D_e$  is the eye or throat diameter. These relations hold when the kinematic condition for similarity of flow in the low pressure region of the machine is satisfied:

$$Q/\Omega D_e^3 = \text{const}$$
 (40)

Elimination of  $D_e$  in Eq. (39) yields the suction specific speed. Using Eq. (26) for the power developed by a turbine, the relationship between  $\sigma_T$ ,  $N_s$ , and S is given by

$$\sigma_T = (1/\eta^{2/3}) \left( N_s / S \right)^{4/3}$$
(41)

If S can be assumed to be constant, then Eq. (41) produces a relationship between  $\sigma_T$  and  $N_s$ . Allowable values of S do vary, but an acceptable conservative value in non-dimensional units is 3. A comparison between Eq. (41), assuming  $2 \le S < 4$  and actual turbine experience, is shown in Fig. 7. An efficiency of 0.9 is assumed. The allowable S for turbines appears to be higher than an equivalent pump. Note also that the trend of limiting  $\sigma_T$  for turbines has a steeper slope than the constant S lines. This could imply that different specific speed designs are not equally close to the optimum with regard to cavitation or that the factor S cannot be considered a constant. It should be kept in mind that since the flow direction for a pump and a turbine are in opposite directions, only the inception point should be similar. Under developed cavitating conditions, the flow situation could be quite different with cavity closure occuring on the runner in the case of a pump, whereas it would occur downstream of the runner in the case of the equivalent turbine.



Fig. 7. Allowable Thoma's sigma as a function of specific speed. (Adpated from Moody and Zowski, 1969)

Figure 7 is a useful chart for estimating the turbine setting for various types of turbines in conjunction with Eq. (37). This is a useful procedure for preliminary design and comparison between different types of turbines for the same installation. However, the manufacturer's recommendation should be followed in the final design.

# TURBINE TECHNOLOGY

### 3.1 Overall Description of a Hydropower Installation

Semi Spiral

The hydraulic components of a hydropower installation consist of an intake, penstock, guide vanes or distributor, turbine, and draft tube. The intake is designed to withdraw flow from the forebay as efficiently as possible, with no or minimal vorticity. Trash racks are commonly provided to prevent ingestion of debris into the turbine. Intakes usually require some type of shape transition to match the passageway to the turbine and also incorporate a gate or some other means of stopping the flow in case of an emergency or turbine maintenance. Some types of turbines are set in an open flume; others are attached to a closed conduit penstock. In all cases, efforts should be made to provide uniformity of the flow, as this uniformity has an effect on the efficiency of the turbine. For low head installations, the diameter of a closed penstock must be quite large to accomodate the large discharges necessary for a given power output. Its size is a compromise between head loss and cost. The selection of the actual penstock configuration is dependent on the location of the powerhouse with respect to the dam.

For some types of reaction turbines, the water is introduced to the turbine through casings or flumes which vary widely in design. The particular type of casing is dependent on the turbine



INTAKES & CASES

Fig. 8. Typical intake and case dimensions. (Mayo, 1979)

size and head. For small heads and power output, open flumes are commonly employed. Steel spiral casings are used for higher heads, and the casing is designed so that the tangential velocity is essentially constant at consecutive sections around the circumference. This requirement necessitates a changing cross-sectional area of the casing. Some examples of intakes and casings are shown in Fig. 8 where dimensions are given in terms of the runner diameter. As the inflow has an effect on the turbine efficiency, the design of the special casing is carried out by the turbine manufacturer.

Discharge control for some types of reaction turbines is provided by means of adjustable guide vanes or wicket gates around the outer edge of the turbine runner. The vanes are tied together with linkages and their positioning is regulated by a governor. The adjustable vanes are shown schematically in Fig. 1. The flow area can be readily varied from zero to a maximum by rotation of the vanes. In addition, the velocity diagrams at the entrance and exit are a function of the guide vane position and therefore the efficiency of the turbine also changes. Wicket gates can also be used to shut off the flow to the turbine in emergency situations. Various types of valves are installed upstream of the turbine for this purpose for turbines without wicket gates.

One purpose of the draft tube is to reduce the kinetic energy of the water exiting the turbine runner. Within limits a well-designed draft tube will permit installation of the turbine above the tailwater elevation without losing any head. Different designs of a draft tube are common, ranging from a straight conical diffuser to configurations with bends and bifurcations. Some typical shapes and relative dimensions are shown in Fig. 9, where the dimensions again are given in terms of the runner diameter.

The simplest form of draft tube is the straight conical diffuser. Efficiency of energy conversion is dependent on the angle of the diverging walls. Small divergence angles require long diffu-



Fig. 9. Typical draft tube dimensions (Mayo, 1979)

sers to achieve the area necessary to reduce the exit velocity. Long diffusers increase construction costs, and therefore the angle in some cases may be increased up to about 15 degrees from the typical optimum value of about 7 degrees. In addition to the increased loss through a large angle diffuser, flow separation can lead to unstable flow. Flow instability is to be avoided, as it has an adverse effect on turbine performance.

For some types of turbine installations, such as a vertical axis turbine, the flow must be turned through a 90 degree angle after leaving the turbine. This is accomplished by adding an elbow between the turbine and draft tube, which has an influence on the draft tube performance, and requires careful design.

Experimental data on diffusers are available in the literature. However, the flow leaving the turbine runner can have a swirl component of velocity which has an effect on the draft tube efficiency. The magnitude of the swirl is dependent on the type of turbine and operating conditions. Excessive swirl can result in surging in the draft tube, as well as load fluctuations and pressure fluctuations that can cause mechanical vibrations of severe magnitude. However, a small swirl component has been found to be beneficial.

A draft tube design adequate for one type of runner may not be satisfactory for another. Therefore, the draft tube is considered an essential part of the turbine, and its design is carried out by the turbine manufacturer.

An example of some typical losses and their sources are shown in Fig. 10 for a Kaplan turbine. For small discharges, major losses occur in the runner and distributor. This is typical for a turbine operating at off-design conditions, and is associated with the shock losses in the runner. As the discharge increases, the runner and distributor losses decrease to relatively small values. The draft tube losses increase, but the largest increase is in the losses at the draft tube exit. It becomes obvious that efforts should be made to reduce this loss, which can be accomplished by enlarging the draft tube exit area. However, as such enlargement increases the construction cost, compromises must again be made.



Fig. 10. Energy balance for a medium speed Kaplan turbine model as a function of unit discharge for a 1 m diameter and 1 m head. (Kovalev, 1965).

## 3.2 Turbine Classification and Description

There are two basic types of turbines, denoted as impulse and reaction. In an impulse turbine the available head is converted to kinetic energy before entering the runner, the power available being extracted from the flow at atmospheric pressure. In a reaction turbine the runner is completely submerged and both the pressure and the velocity decrease from inlet to outlet. The velocity head at the inlet to the turbine runner is typically less than 50 per cent of the total head available. In either machine the torque is equal to the rate of change of angular momentum through the machine as expressed by the Euler equation.

### Impulse Turbines

Modern impulse units are generally of the Pelton type and are restricted to relatively high head applications (Fig. 11). One or more jets of water impinge on a wheel containing many curved buckets. The jet stream is directed inwardly, sideways, and outwardly thereby producing a force on the bucket which in turn results in a torque on the shaft. All of the available head is converted



Fig. 11. Double-overhung impulse wheel. (Daily, 1950)

to kinetic energy at the nozzle. Any kinetic energy leaving the runner is "lost". It is essential that the buckets are designed in such a manner that exit velocities are a minimum. No draft tube is used since the runner operates under essentially atmospheric pressure and the head represented by the elevation of the unit above tailwater cannot be utilized.\* Since this is a high head device, the loss in available head is relatively unimportant. As will be shown later, the Pelton wheel is a low specific speed device. Specific speed can be increased by the addition of extra nozzles, the specific speed increasing by the square root of the number of nozzles. Specific speed can also be increased by a change in the manner of inflow and outflow. As shown in Fig. 12, a Turgo turbine can handle relatively larger quantities of flow at a given speed and runner diameter by passing the jet obliquely through the runner in a manner similar to a steam turbine. The jet impinges on several buckets continuously, whereas only a single bucket per jet is effective at any instant in a Pelton wheel.

<sup>\*</sup>In principle, a draft tube could be used, which requires the runner to operate in air under reduced pressure. Attempts at operating an impulse turbine with a draft tube have not met with much success.



Fig. 12. Turgo and Pelton wheels contrasted. The jet on the turgo strikes three buckets continuously, whereas on the Pelton it strikes only one. A similar speed increasing effect can be had on the Pelton by adding another jet or two.

The Banki-Mitchell turbine illustrated in Fig. 13 is a variation on this theme. The flow passes through the blade row twice, first at the upper portion of the wheel and again at the lower portion. The flow exits the blade in the opposite direction from the first pass and hence this configuration tends to be self-cleaning since debris impinging on the periphery of the runner at



Fig. 13, Ossberger cross flow turbine.

the top dead center is removed by the flow on the second pass at essentially bottom dead center.

Most Pelton wheels are mounted on a horizontal axis, although newer vertical axis units have been developed. Because of physical constraints on orderly outflow from the unit, the maximum number of nozzles is generally limited to six or less. Whilst the power of a reaction turbine is controlled by the wicket gates, the power of the Pelton wheel is controlled by varying the nozzle discharge by means of an automatically adjusted needle, as illustrated in Fig. 14. Jet deflectors, Fig. 14a, or auxilliary nozzles arranged as in Fig. 14b are provided for emergency unloading of the wheel. Additional power can be obtained by connecting two wheels to a single generator or by using multiple nozzles. Since the needle valve can throttle the flow while maintaining essentially constant jet velocity, the relative velocities at entrance and exit remain unchanged, producing nearly constant efficiency over a wide range of power output. This is a de-



Fig. 14a. Pelton 45° elbow-type needle nozzle with jet deflector. (Daily, 1950)



Fig. 14b. Pelton nozzle with auxiliary reflief nozzle. (Daily, 1950)

sirable feature of Pelton and Turgo wheels. Throttling of the Banki-Mitchell turbine is accomplished differently, as illustrated in Fig. 13. An adjustable guide vane is used which functions in a manner similar to the wicket gates in a reaction turbine. If operating conditions require, the guide vanes can be divided into two separately controlled sections. For most installations, the lengths of the two guide vane sections are in the ratio 1:2, allowing for utilization of 1/3, 2/3, or the entire runner, depending on the flow conditions. This combination provides a relatively flat efficiency curve over the power range 15 per cent to 100 per cent.

#### **Reaction Turbines**

Reaction turbines are classified according to the variation in flow direction through the runner. In radial and mixed flow runners, the flow exits at a radius different (in modern designs the inlet flow is always inward) than the radius at the inlet. If the flow enters the runner with only radial and tangential components, it is a radial flow machine. The flow enters a mixed flow runner with both radial and axial components. Francis turbines are of the radial or mixed flow type, depending on the design specific speed. Two Francis turbines are illustrated in Fig. 15. The radial flow runner (Fig. 15a) is a low specific speed design, whereas the mixed flow runner (Fig. 15b) achieves peak efficiency at considerably higher specific speed.



Fig. 15. Two examples of Francis turbine. (Daily, 1950)

Axial flow propeller turbines are generally either of the fixed blade or Kaplan (adjustable blade) variety. The "classical" propeller turbine, illustrated in Fig. 16, is a vertical axis machine with a scroll case and a radial wicket gate configuration that is very similar to the flow inlet for a Francis turbine. The flow enters radially inward and makes a right angle turn before entering the runner in an axial direction. The Kaplan turbine has both adjustable runner blades as well as adjustable wicket gates. The control system is designed in such a manner that the variation in blade angle is coupled with the wicket gate setting in a manner which achieves best overall efficiency over a wide range of flow rate. The classical design does not take full advantage of the geometric properties of an axial flow runner. The flow enters the scroll case in a horizontal direction, issues radially inward from the guide case where it forms a vortex and discharges into the draft tube in a vertical direction with very little whirl component remaining. The flow must then again be turned through  $90^{\circ}$  to discharge into the tailwater in a horizontal direction. From a



Fig. 16. Smith-Kaplan axial-flow turbine with adjustable-pitch runner blades, N<sub>8</sub> - 3.4 (Daily, 1950)

design point-of-view, this is less than desirable for many reasons. The flow field entering the runner is highly complex and it is difficult to design the proper pitch distribution from hub to tip for minimal shock losses. There are additional losses in the elbow and the tortuitous flow path required from inlet to outlet requires additional civil works.

More modern designs take full advantage of the axial flow runner; these include the tube, bulb, and Straflo types illustrated in Fig. 17. The flow enters and exits the turbine with minor





Fig. 17a. Comparison of structures required for Straflo vs bulb turbine with same output and head.

Fig. 17b. Various tube turbine arrangements.

changes in direction. A wide variation in civil works design is also permissible. The tube type can be fixed propeller, semi-Kaplan, or fully adjustable. An externally mounted generator is driven by a shaft which passes through the flow passage either upstream or downstream of the runner. The bulb turbine was originally designed as a high output, low head unit. In large units, the generator is housed within the bulb and is driven by a variable pitch propeller at the trailing end of the bulb. Smaller units are available in which an externally mounted generator is driven by a right angle drive which is housed within the bulb (Fig. 18). Because of the simplicity of installation, the various modern axial flow machines are of considerable interest for low head applications.



Fig. 18. Right-angle drive bulb turbine.

In addition to the radial flow and mixed flow Francis and axial flow propeller units, there is the Deriaz turbine which is a mixed-flow propeller unit of the Kaplan type. This turbine was originally developed for pumped storage applications, but shows great promise for applications in the medium head range. The turbine consists of a series of controllable pitch blades mounted on a conical hub. The turbine can have either a conventional scroll case and gate apparatus or a more specialized flap system for controlling the inlet flow. This unit provides the same flat efficiency curve over a wide range of power as the standard Kaplan propeller units, but because of the mixed flow design, is applicable to higher head applications. However, at the present time (1981) there do not appear to be any units available of a small size (less than 1 MW capacity).

### 3.3 Performance Characteristics

#### **Comparative Performance of Impulse and Reaction Turbines**

The two basic types of turbines tend to operate at peak efficiency over different ranges of specific speed. This is due to geometric and operational differences. In order to give the reader a perspective of the operational characteristics of each type, a brief discussion of operational principles is presented below. This is followed by a summary of the performance of commercially available equipment in subsequent sections.

#### Impulse Wheels

Typical types of impulse wheels are illustrated in Figs. 11 to 13. For a given pipeline there is a unique jet diameter that will deliver maximum power to a jet. Denoting the jet diameter by  $d_j$  the power is given by

$$P_{j} = \gamma Q V_{j}^{2}/2g = \gamma (\pi/8g) d_{j}^{2} V_{j}^{3}$$
(42)

Let  $\triangle h$  denote the difference between the reservoir surface elevation and the nozzle elevation. Neglecting losses at the entrance to the pipe and in the nozzle, Eq. (12) yields

$$\frac{V_j^2}{2g} + f \frac{L}{d_p} \frac{V_p^2}{2g} = \Delta h$$
 (43)

where  $V_j$  is the jet velocity,  $V_p$  is the velocity in the pipe, and  $d_p$  and L denote, respectively, the pipe diameter and length. As the size of the nozzle opening is increased, the flow rate Q gets larger while the jet velocity  $V_j$  gets smaller, since the losses in the pipeline increase with Q. Using  $V_p = V_j (d_j/d_p)^2$  and Eq. (43), it can be shown that maximum power is obtained when

$$\Delta h = 3f \frac{L}{d_p} \frac{V_p^2}{2g}$$
(44)

$$H = V_j^2 / 2g = (2/3) \Delta h$$
 (45)

and 
$$d_j = (d_p^{5}/2fL)^{1/4}$$
 (46)

Thus, for a given penstock geometry and  $\Delta h$ , the maximum power available to the turbine can be calculated. From Eqs. (25) and (45), it should be noted that the maximum possible plant efficiency is 2/3 for this case. It follows from this argument that the net head to be used for turbine selection lies between  $\Delta h$  and  $2/3 \Delta h$ . For a given Q and  $\Delta h$ , a value of  $d_j$  can be calculated for the two head extremes. The minimum penstock diameter can be determined from Eq. (46) with a fixed f and L using the latter  $d_j$ . However, to increase the plant efficiency, the penstock diameter should be slightly larger than the minimum diameter. Computations of the plant efficiency should be made for several diameters. The final selection of  $d_p$  and  $d_j$  will depend on the available turbines and on an economic analysis of the installation.

Of the head available at the nozzle inlet, a small portion is lost to friction in the nozzle and to friction on the buckets. The rest is available to drive the wheel. The actual utilization of this head depends on the velocity head of the flow leaving the turbine and the setting above tailwater. Optimum conditions corresponding to maximum utilization of the head available dictate that the flow leave at essentially zero velocity. Under ideal conditions, this occurs when the peripheral speed of the wheel is one-half the jet velocity. In practice, optimum power occurs at a speed coefficient,  $\phi = u / \sqrt{2gH}$  somewhat less than 0.5. In fact, it can be shown that best efficiency will occur when  $\phi = \frac{1}{2}C_{v}\cos\alpha_{1}$ , where  $C_{v}$  is the velocity coefficient for the nozzle

$$C_{v} = V_{i} / \sqrt{2gH} \tag{47}$$

and  $\alpha_1$  represents the effective angle between the jet velocity and the peripheral velocity of the runner entrance to the bucket. Since maximum efficiency occurs at fixed speed for fixed H,  $V_j$  must remain constant under varying flow conditions. Thus the flow rate, Q, is regulated with an adjustable nozzle. There is some variation in  $C_{\nu}$  and  $\alpha_1$  with regulation and maximum efficiency occurs at slightly lower values of  $\phi$  under partial power settings. Present nozzle technology is such that the discharge can be regulated over a wide range at high efficiency.

A given head and penstock configuration establishes the optimum jet velocity and diameter. The size of the wheel determines the speed of the machine. For a wheel of diameter D, the speed in radians per second is

$$\Omega = 2u_{\perp}/D = (2\phi/D)\sqrt{2gH}$$
(48)

Using Eq. (25) for the power P and the equation

$$Q = V_j \pi d_j^2 / 4 = C_v \sqrt{2gH} \pi d_j^2 / 4$$
(49)

for the flow rate Q, one obtains for the specific speed of the machine the relationship

$$N_{e} = 2^{1/4} \sqrt{2\pi\eta C_{v}} \phi d_{i}/D$$
 (50)

or approximately

$$N_s = 1.3 d_i / D$$
 (51)

Practical values of  $d_j/D$  for Pelton wheels to ensure good efficiency are in the range 0.04 to 0.1, corresponding to  $N_s$  values in the range 0.05 to 0.13 (10 to 25 in metric units using the metric horsepower). In Turgo turbines the relative wheel diameter can be half that of a Pelton wheel resulting in specific speeds approximately twice that of the conventional design. Higher specific speeds are possible with multiple nozzle designs. The increase is proportional to the

square root of the number of nozzles. Crossflow turbines can operate at even higher specific speed  $(N_s = 0.6)$  because the length of the runner can be much larger than the diameter, which permits large values of flow through a relatively small diameter runner. This is possibly one of the reasons why the crossflow turbine has seen application over such a wide range of head and power (Fig. 24). However, in considering an impulse unit, one must remember that efficiency is based on net head and the net head for an impulse unit is generally less than the net head for a reaction turbine at the same gross head because of the lack of a draft tube.

#### **Reaction Turbines**

The main difference between impulse wheels and reaction turbines is the fact that a pressure drop takes place in the rotating passages of the reaction turbine. This implies that the entire flow passage from the turbine inlet to the discharge at the tailwater must be completely filled. A major factor in the overall design of modern reaction turbines is the draft tube. This was not always the case. In earlier days, when low speed, large diameter Francis turbines were installed under low heads, the lack of a draft tube or very short conical tube resulted in a nominal velocity head loss from the runner. This was not particularly critical for installations which are under developed based on today's standards since water was spilled over the dam much of the year. However, since today it is desirable to reduce the overall equipment and civil construction costs by using high specific speed propeller runners, the draft tube is extremely critical from both a flow stability and an efficiency viewpoint. Since the runner diameter is relatively small, a substantial percentage of the total energy is in the form of kinetic energy leaving the runner. To recover this efficiently, considerable emphasis should be placed on the draft tube design.

The practical specific speed range for reaction turbines is much broader than for impulse wheels. This is due to the wider range of variables which control the basic operation of the turbine. As an illustration of the design and operation of reaction turbines for constant speed, refer again to Fig. 1. The pivoted guide vanes allow for control of the magnitude and direction of  $V_1$ , i.e.  $V_1$  and  $\alpha_1$ . The relationship between blade angle, inlet velocity, and peripheral speed for shock free entry can be obtained from Eqs. (7) and (8) as

$$\cos \beta_1 = \frac{V_1 \cos \alpha_1 - u_1}{V_1 \sin \alpha_1}$$
(52)

Without the ability to vary the blade angle, it is obvious that shock free entry cannot be completely satisfied at partial flow. This is the distinction between the power efficiency of fixed propeller and Francis types at partial loads and the fully adjustable Kaplan design.

Referring to Eq. (21), optimum hydraulic efficiency would occur when  $\alpha_2$  is equal to 90°. However, overall efficiency of the turbine is dependent on the optimum performance of the draft tube which occurs with a little whirl in the flow. Thus, best overall efficiency occurs with  $\alpha$  $\approx 85^{\circ}$  for low specific speed Francis turbines to  $\alpha_2 \approx 75^{\circ}$  for high specific speed turbines. The hydraulic efficiency (Eq. 21) is approximately

$$\eta_{\mu} = 2\phi C_{\mu} \cos \alpha_{\mu} \tag{53}$$

With  $\alpha_1$  in the range of 10° to 25° and  $C_1 \cong 0.6$ , the speed coefficient  $\phi$ , is approximately 0.8 compared with a little less than 0.5 for an impulse turbine. Note also that  $C_1 \cong 0.6$  implies that only 40 per cent of the available head is converted to velocity head at the turbine inlet compared with 100 per cent for the impulse wheel.

The determination of optimum specific speed in a reaction turbine is more complex since there are more variables. For a radial flow machine (refer to Fig. 1), a relatively simple expression can be derived. Combining Eqs. (14), (15), (20), (25), and (48), the expression for specific speed, Eq. (28), is

$$N_{s} = 2^{5/4} (2 \pi \eta f_{b} C_{1} \sin \alpha_{1} B/D)^{1/2} \phi$$
(54)

or approximately

$$N_{s} = 5.5 (C_{1} \sin \alpha_{1} B/D)^{-1/2} \phi$$
 (55)

Using standardized design charts for Francis turbines (Fig. 17),  $N_s$  is normally found to be in the range 0.3 to 2.5 (58 to 480 in metric units using metric horsepower).



Fig. 19. Empirical design constants for reaction turbines. (Daily, 1950)

### Performance Comparison

The physical characteristics of various runner configurations are summarized in Fig. 20. It is obvious that the configuration changes with speed and head. This can be expressed in terms of peak efficiency versus specific speed, as illustrated in Fig. 21. As already discussed, impulse turbines are efficient over a relatively narrow range of specific speed, whereas Francis and propeller turbines have a wider useful range. Variable geometry is an important consideration when a



Fig. 20. Physical characteristics of various turbine runners compared. (Mayo, 1979)



Fig. 21. Efficiency of various types of turbines as a function of specific speed.

turbine is required to operate over a wide range of load. Pelton wheels and Turgo wheels tend to operate efficiently over a wide range of power loading because the needle valve is capable of metering flow at a constant discharge velocity. Thus the relative velocities through the runner remain fixed in magnitude and direction which allows for maximum runner efficiency independent of flow rate. A comparison of efficiency variation with load as a function of the level of sophistication of a tube turbine is illustrated in Fig. 22. Fixed gates and blade settings result in peak efficiency at 100 per cent load and the efficiency drops off rapidly with changes in load. On the



Fig. 22. Turbine efficiency as a function of load. Comparison between an impulse turbine and various configurations of the high speed propeller unit.

other hand, a Kaplan type tube turbine can maintain efficiency over a relatively broad range of conditions. Also illustrated in the same diagram is the variation of efficiency for an impulse wheel. Although its peak efficiency is less than the high speed tube turbine, the impulse unit is able to maintain a relatively high efficiency over a wide range of conditions. Both Francis and Deriaz units are designed to operate at medium specific speed. Like the Kaplan type, the Deriaz unit maintains its high efficiency over a wide range of load, but has not been used in small applications. The decision of whether to select a simple configuration with a relatively "peaky" efficiency curve or to go to the additional expense of installing a more complex machine with a broad efficiency curve will depend on the expected operation of the plant. If the head and flow are relatively constant, then the less expensive choice is justified. On the other hand, many run-of-the-river plants may be more economical with the installation of Kaplan or Deriaz units.

Because various types of turbines tend to operate best over different specific speed ranges, the head and power available at a given site dictate what options are practical. This is illustrated in Fig. 23, where the various types of turbines that would be useful at various combinations of head and desired power output are plotted over a range of head and power from 2 to 400 m and 10 kW to 20,000 kW. The figure is constructed with the following assumptions: speed in the range 600 - 3600 rpm<sup>\*</sup>, direct drive, and specific speed in the range of optimum efficiency for a given design. At constant *n* and *n<sub>e</sub>*, the head is related to the power by

$$H \sim (n/n_s)^{4/5} P^{2/5}$$
 (56)

<sup>\*</sup>The speed range is based on the assumption of 60 Hz current and a maximum of 12 poles in the generator. (Number of poles equals 7200/n.)





Fig. 23. Range of application of various types of turbines.

Thus the upper limit represents maximum rpm (if possible without cavitation) and minimum  $n_s$ . The lower boundary is determined from the lowest rpm and maximum  $n_s$  without cavitation. Cavitation limits are based on a net positive draft head of one atmosphere. For example, the lower curve for the propeller turbine is determined with n of 600 rpm and  $n_s$  of 764 ( $N_s = 4.6$ ). Since the critical cavitation number is a function of  $n_s$  (Fig. 7), and the plant sigma is a function of draft head and net head (Eq. 36), a point is reached where the two sigmas are equal. This occurs at the break in the lower curve. If it is desired to maintain the same speed at higher heads,  $n_s$  must decrease to avoid cavitation. Thus,  $n_s$  decreases along the lower curve to the right of the breakpoint.

#### Survey of Commercially Available Equipment

The recent interest in small-scale hydropower has stimulated the turbine manufacturers to produce turbines suitable for this application. Larger units have been scaled down to match the lower head and power requirements. As cost of equipment has a significant impact on the economic feasibility of a small-scale installation, a major thrust has been made to develop standardized units to reduce cost. Many of these standardized units are supplied complete with the generator and auxiliary equipment. The larger and well established equipment manufacturers are adding such equipment as a line item, and the number of smaller manufacturers is rapidly increasing in response to the anticipated demand. The small companies are in general developing equipment for the lower power outputs in the range of less than 200 kW.

A summary graph based on commercially available equipment as a function of head and



Fig. 24. Summary chart of commercially available turbines.

power output is shown in Fig. 24 to indicate the coverage as it currently exists. This graph should be compared with Fig. 23. This summary should be used only as a guide to available turbines, as different units are rapidly being offered by the various manufacturers. It can readily be seen that several types of turbines are available for a given head and power output. The crossflow turbine covers a wide range of conditions. The propeller turbine classification includes the vertical Kaplan, bulb, and tube turbine units. The Kaplan turbine is commonly used for the higher heads and power output, and the bulb for essentially the same output at lower heads. The tube turbine range includes the lower heads and power outputs. Standardized tube turbines are available in this range, and may be economically attractive for the mini-hydro projects and also in the upgrading or rehabilitation of old hydropower stations.

For the micro-hydropower sites, which are lowhead and have power outputs of less than 100 kW, some standardized propeller turbines are also available. These small units have been developed specifically for this application, and attempts have been made to simplify the machine and thus lower initial equipment costs. The simplification may result in reduction of efficiency of the unit, and this should be considered in the assessment of economic feasibility.

Accurate performance data are usually not available for smaller turbines. In fact, model tests are often not performed for turbines smaller than about 5 - 10 MW. As an example, consider a 500 rpm, 15 MW turbine operating at its design point with an efficiency of 92 per cent under a 100 m head. The same design could be scaled down to 500 kW at 1200 rpm and 50 m head. Since the specific speed is constant, the size, speed, and head will vary according to

$$H_{1}/H_{2} = n_{1}^{2} D_{1}^{2} / n_{2}^{2} D_{2}^{2}$$
(57)

Thus  $D_1/D_2 = 3.39$ . The change in efficiency could be estimated by using the Moody formula (Eq. 24) in reverse (a Moody step-down equation, if you will). This yields  $\eta_2 = 89.9\%$ . Scaling down to even smaller sizes would bring about even more dramatic reductions in efficiency. This is true only for reaction turbines in which leakage and frictional losses are disproportionately higher in the smaller sizes.

#### **Present Trends in Turbine Development**

As previously mentioned, attention is being directed toward the development of standardized turbines to cover a wide range of applications. Some turbine manufacturers are exploring the possibility of using pumps operated as turbines. It is expected that continued efforts will be made in this area. For many remote and relatively inaccessible sites, lightweight turbines of small size would be attractive. The use of plastics, etc. for various elements could perhaps reduce cost through mass production techniques as well as weight for the smaller units. These elements may require more maintanence, but the lower cost of the parts may offset the increased maintenance cost.

### HYDRAULIC STRUCTURES AND OPERATIONAL CONSIDERATIONS

# 4.1 Integration of Turbine with Inlet and Outlet Works

From the discussion in Section 3, it is apparent that several types of turbines may be appropriate for given flow conditions. It is therefore necessary to make a decision as to which particular unit is most economical. In addition to the cost of the turbine and generator, the cost of the associated civil works must be considered, as this cost can represent a large portion of the total cost of a small scale hydropower installation. Some types of turbines require larger civil works than others. Several alternate preliminary layouts should be evaluated, each of which may have different inlet and draft tube requirements. The necessity of this evaluation has become increasingly evident with the recent interest in retrofitting existing sites for power production. The original turbines either may not exist or may not be capable of repair, or it may be desirable to replace the unit with a modern turbine of different capacity. The condition and the extent of the existing civil works may influence the type of turbine to be finally selected. For example, if an open flume Francis turbine was originally installed but is no longer useable, it may prove to be more economical to install a new turbine of the same type or a different design with roughly the same overall dimensions as the original equipment if the rest of the structure is still in good condition. In other cases, it may be most cost effective to abandon the existing structure and select a different type of turbine. It is therefore necessary to analyze each site on an individual basis. In so doing considerable cost savings may be realized.

### 4.2 Cavitation or Turbine Setting

Another factor that must be considered prior to equipment selection is the elevation of the turbine with respect to tailwater elevations. As previously discussed, hydraulic turbines are subject to pitting due to cavitation. For a given head, a smaller, lower cost, high speed runner must be set lower (i.e., closer to tailwater or even below tailwater) than a larger, higher cost, low speed turbine runner. Also, atmospheric pressure or plant elevation above sea level is a factor as are

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tailwater elevation variations and operating requirements. This is a complex subject which can only be accurately resolved by model tests. Every runner design will have different cavitation characteristics, therefore, the anticipated turbine location or setting with respect to tailwater elevations is an important consideration in turbine selection.

Cavitation is not normally a problem with impulse wheels. However, by the very nature of their operation, cavitation is an important factor in reaction turbine installations. The susceptibility for cavitation to occur is a function of the installation and the turbine design. As already discussed, this can be expressed conveniently in terms of Thoma's sigma. The critical value of  $\sigma_T$  is a function of specific speed, as illustrated in Fig. 7. As the specific speed increases, the critical value of  $\sigma_T$  increases dramatically. For minimization of cavitation problems, the plant  $\sigma_T$  must be in excess of the critical  $\sigma_T$  denoted on the chart. This can have important implications for turbine settings and the amount of excavation necessary. The criteria for establishing the turbine setting have already been discussed in Section 2.4. As an example, consider a 500 KW machine operating at 500 rpm under a head of 10 meters. The specific speed is 3.8. This will be an axial flow turbine having a critical  $\sigma_T$  of about 0.9. At sea level the maximum turbine setting would be

$$z_{B_m} = 10 - 0.09 - (0.9 \ge 10) = 1$$
 meter

If the same turbine was installed at Leadville, Cororado, elevation  $\sim 3000$  m, the maximum turbine setting would be

$$z_B = 6.7 - 0.09 - (0.9 \times 10) = -2.3$$
 meters

Considerable excavation would be necessary. Thus, cavitation can be an important consideration.

# 4.3 Speed Regulation

The speed regulation of a turbine is an important and complicated problem. The magnitude of the problem varies with size, type of machine and installation, type of electrical load, and whether or not the plant is tied into an electrical grid. It should also be kept in mind that runaway or no load speed can be higher than the design speed by factors as high as 2.5. This is an important design consideration for all rotating parts, including the generator.

It is beyond the scope of this section to discuss the question of speed regulation in detail. However, some mention of this should be made since much of the technology is derived from large units. The cost of standard governors is thus disproportionately high in the smaller sizes. Regulation of speed is normally accomplished through flow control. Adequate control requires sufficient rotational inertia of the rotating parts. When load is rejected, power is absorbed, accelerating the flywheel and when load is applied, some additional power is available from deacceleration of the flywheel. Response time of the governor must be carefully selected since rapid closing time can lead to excessive pressures in the penstock.

A Francis turbine is controlled by opening and closing the guide vanes which vary the flow of water according to the load. A powerful governor is required to overcome the hydraulic and frictional forces and to maintain the guide vanes in fixed position under steady load. On the other hand, impulse turbines are more easily controlled. This is due to the fact that the jet can be deflected or an auxiliary jet can bypass flow from the power producing jet without changing the flow rate in the penstock. This permits long delay times for adjusting the flow rate to the new power

conditions. The spear or needle valve controlling the flow rate can close quite slowly, say 30 to 60 seconds, thereby minimizing any pressure rise in the penstock.

Several types of governors are available which vary with the work capacity desired and/or the degree of sophistication of control. These vary from pure mechanical to mechanical-hydraulic and electro-hydraulic. Electro-hydraulic units are sophisticated pieces of equipment and would not be suitable for remote regions. The precision of governing necessary will depend on whether the electrical generator is synchronous or asynchronous (induction type). There are advantages to the induction type of generator. It is less complex and therefore cheaper and typically has slightly higher efficienty. Its frequency is controlled by the frequency of the grid it is feeding into, thereby eliminating the need of an expensive conventional governor. It cannot operate independently but can only feed into a network and does so with a lagging power factor which may or may not be a disadvantage, depending on the nature of the load. Long transmission lines, for example, have a high capacitance and in this case the lagging power factor may be an advantage.

Some general features of the overall regulation problem can be demonstrated by examination of the basic equation for a rotating system

$$J d\Omega/dt = T_t - T_t$$
<sup>(58)</sup>

where

J = angular velocity  $T_t = torque of turbine$  $T_t = torque due to load$ 

Three cases may be considered in which  $T_{t}$  is equal to, less than, or greater than  $T_{t}$ 

J =moment of inertia of rotating components

For the first case, the operation is steady. The other two cases imply unsteady operation, since  $d\Omega/dt$  is not constant, and usually a governor is provided so that the turbine output matches the generator load.

Speed regulation is a function of the flywheel effect of the rotating components and the inertia of the water column of the system. The starting up time of the rotating system (Bureau of Reclamation, 1966) is given by

$$T_s = J\Omega^2 / P = Jn_o^2 / 6818 \,\mathrm{HP}_r \tag{59}$$

where

 $n_{p}$  = normal turbine speed, rpm

HP, = rated metric horsepower

The starting up time of the water column is given by

$$T_{p} = \Sigma L V/gh_{r} \tag{60}$$

where

L = length of water column

V = velocity in each component of the water column

J = flywheel effect of generator and turbine, kg m sec<sup>2</sup>

 $h_{\rm r}$  = rated head

For good speed regulation it is desired to keep  $T_g/T_p \leq 8$ . Lower values can also be used, although special precautions are necessary in the control equipment. It can readily be seen that higher ratios of  $T_g/T_p$  can be obtained by increasing J or decreasing  $T_p$ . Increasing J implies a larger generator, which also results in higher costs. The startup time of the water column can be reduced by reducing the length of the flow system, lower velocities, or addition of surge tanks, which essentially reduce the effective length of the conduit. A detailed analysis should be made for each installation, as for a given length, head, and discharge, the flow area must be increased to reduce  $T_p$  which leads to associated higher construction costs.



Fig. 25. Speed rise for full gate load rejected with no water hammer. (Bureau of Reclamation, 1966)

A method for determining the speed rise as a result of load rejection is incorporated in Fig. 25 for several specific speed machines. The abscissa is the ratio of  $T_G/T_S$ , where  $T_G$  is the full closing time of the governor

$$T_{C} = 0.25 + T_{C}$$
 (61)

and  $T_{\mathcal{C}}$  is the rated governor time in seconds, which generally varies from 3 to 5 seconds. With the ratio determined, the percent speed rise  $S_R$  for no water hammer and a given specific speed can be found in Fig. 35. This value should be modified to include the startup time of the pipeline, or

$$S_R = S_R (1 + T_p/T_c)$$
 (62)

It is desired to keep the speed rise for full load rejection to less than 45 per cent, although in some cases higher percentages can be permitted if regulating ability is sacrificed. If the speed rise is excessive, consideration should be given to providing surge tanks. Further discussion is beyond the scope of this section, and the reader is referred to the previously mentioned reference for more detail.

### 4.4 Emergency and Abnormal Conditions

Emergency conditions can arise if the system experiences a sudden drop in load and the guide vanes remain open as a result of failure in the regulating system. The speed will rise rapidly until a maximum is reached, which is called the runaway speed. The runaway speed is dependent on the type of turbine, distributor opening, head, and in the case of a Kaplan turbine, the runner blade angle.

Based on field tests, an equation (Bureau of Reclamation, 1966) has been developed for use in predicting the runaway speed,  $n_r$ , for various types of turbines. This formulation is

$$n_r / n_d = K_n \left( H_{\text{max}} / H_d \right)^{1/2}$$
(63)

where and  $K_n = 0.28 N_s + 1.45$   $n_d = \text{design speed, rpm}$   $N_s = \text{specific speed}$   $H_{\text{max}} = \text{maximum head}$  $H_d = \text{design head}$ 

Thus, the ratio of runaway to normal speed is higher for propeller turbines than for Francis turbines and may attain values up to about 2.6. This factor must be considered in the design of the turbine and generating equipment as the increase in centrifugal force can be substantial.

As previously mentioned, for an adjustable blade runner the runaway speed is a function of the blade angle. If the blade angles are increased from their optimum value, the runaway speed is decreased. At the larger blade angles, the shock losses are higher and equilibrium conditions are reached at lower speeds. However, an increase in blade angle can also result in serious vibration, which can cause damage to the turbine unit.

If the blade angle is decreased, the velocity vectors are changed so that the losses are reduced, and the runaway speed is increased. In fact, the theoretical runaway speed with a closed blade runner approaches infinity. This is not actually realized, however, due to frictional windage losses in the generator.

# **FUTURE NEEDS**

#### 5.1 Economics

Figure 26 contains cost information developed during a preliminary analysis of small, low head hydropower potential in the State of Minnesota. Twenty four existing dams were considered, most which had produced power in the past. Flow duration curves were established from data available from the Geological Survey. Estimates of yearly energy production were based on the flow duration curves and net head, assuming the plant would be operating at full capacity at a flow rate equal to the 25 per cent exceedance level. The total cost of refurbishing was based on guidelines established by the U.S. Bureau of Reclamation and include new turbines, generators and switchyard equipment, miscellaneous power plant equipment, transmission lines over level terrain, and/or new refurbished civil works. As an indication of the relative cost, the total cost of the project is divided by the total kW hour production for one year. Figure 27 provides information on pay-back period as a function of relative cost. Since the proposed developments are publicly owned, a 7 per cent interest rate is assumed. The pay-back period has been computed for energy valued at three different rates and two inflation rates. For example, assuming a 20 year pay-back period with a 10 per cent inflation rate and energy at 3 ¢ per kW hour, relative costs greater than \$0.45 are not feasible. Inspection of Fig. 26 shows that a dramatic increase in relative costs for installations with a capacity less than 200 kW. This is reflective of the fact that



Fig. 26. Relative cost of hydropower plant installment at existing dam sites vs design hydraulic head and design turbine discharge (\$ (1980)/kWh produced annually).





Fig. 27. Payback period vs relative cost of hydropower facility at three initial energy rates. 7 per cent interest assumed.

much of the large turbine technology is simply not viable in the smaller sizes. Improvements must be made in both the costs of the equipment and the civil works. This can include, for example, new technology in power regulation, new techniques in production such as injected molded plastic parts etc. Reduction in civil works could be achieved through the use of pre-cast modular components, reduction in size of draft tube, turbines with more resistance to cavitation allowing higher settings etc.

# 5.2 Non-Conventional Uses of Hydropower

In many parts of the world, electrical grids are non-existent. Power development will depend on the establishment of a base load. Since load diversion is an attractive regulation technique, consideration should be given to the development of localized absorption of the energy produced. This could include, but not be limited to electrolytic manufacture of fertilizer (Anonymous, 1978; Treharne et al) and electrolytic production of hydrogen (Nuttall). If attractive and safe methods for producing and storing hydrogen can be developed, the use of fuel cells become an attractive method of energy storage where large impoundments of water are not feasible or uneconomic. Hagen (1976) shows that methanol can be produced electrolytically in regions having a source of limestone. The cost of production could be as small as \$0.14/1. Consideration of these possibilities in the economic analysis could substantially change the cost benefit ratio at remote sites.

# SUMMARY

It has been shown that the head utilized by a turbine runner to produce power can be derived from a suitable form of Euler's equation of motion. The head utilized, and consequently the power developed, is dependent on the velocity vectors of the inlet and exit flow of the runner. The velocity vectors are determined by the operational conditions and the turbine design. Overall efficiency of the turbine is the product of the hydraulic, volumetric, and mechanical efficiencies. Each of these are dependent on various energy losses in the turbine unit, and the origin of these losses has been briefly discussed.

Similarity considerations permit the formulation of dimensionless numbers. These numbers are useful in the extrapolation of test data taken with a model turbine to full-scale conditions and therefore predict performance. One of the most significant dimensionless numbers is the specific speed, which consists of a combination of operating conditions that ensures similar flows in geometrically similar machines. Each type of machine has a value of specific speed that gives maximum efficiency, and it is therefore convenient to classify the various turbine designs by the specific speed at best efficiency.

Cavitation must be avoided in turbines, as it results in loss of performance and can cause erosion damage to the runner and possibly other parts of the structure. Each particular turbine type has its own cavitation limits which are determined from model tests. High specific speed turbines are more susceptible to cavitation than low specific speed units. The setting of the turbine with respect to the tailwater elevation must be carefully considered to ensure cavitation-free operation, and is based on the manufacturer's recommendations.

Turbines can be classified in two broad groups, impulse and reaction turbines. An impulse turbine is driven by a high velocity jet impinging on buckets around the periphery of the wheel, whereas the reaction turbine requires that the flow passages be completely filled. Reaction turbines can be subdivided further into radial, mixed, or axial flow types. The radial and mixed flow types have fixed runner blades, except for the Deriaz turbine, and the axial flow machine may have either fixed or adjustable blades. In addition to differences in blade geometry, each type of reaction turbine has different requirements for a draft tube. The draft tube is considered part of the turbine, and its energy losses are charged to the turbine performance. Some draft tube configurations may require a large amount of excavation to achieve the desired turbine setting.

The overall efficiency of an impulse turbine is quite constant over a broad range of operating conditions, which is achieved by throttling of the flow at the nozzle. Fixed-blade reaction turbines have a more peaky efficiency curve, whereas the efficiency curve for adjustable blade units is relatively flat. The latter unit is particularly suited for installations subject to a wide variation in flow conditions. However, an economic analysis must be made to justify the higher cost of the fully adjustable turbine.

A wide variety of turbines is becoming commercially available for application to small scale hydropower sites. Standardized units are offered by manufacturers in a range of sizes to reduce equipment costs. With increased demand for lower cost units to make marginal sites feasible, it is expected that further developments in standardization will be made. The manufacturers should be contacted for their recommendations.

Several operational conditions are also significant. Most turbines are designed to operate at a constant rotational speed which is controlled by a governor. Some general guidelines are given concerning speed regulation. With a sudden loss in electrical load, the turbine will reach a run-

away speed that is considerably greater than the normal speed. The turbine and generator must be designed to tolerate the additional centrifugal forces. Provisions should also be made for emergency shutdown of the flow to the turbine, either by wicket gates or appropriate valves.

A review of small turbine technology indicates that most of our knowledge is based on the design and operation of large machines. The laws of similarity indicate that current designs are adequate for the needs of small facilities (defined as 3 m to 300 m head and power in the range 10 kW to 1000 kW). The only exception to this appears to be at ultra low head in the 500 kW to 1000 kW range. However, it does appear that current manufacturing techniques are not amenable to the smaller sizes where relative costs on a per kW basis rise sharply. New regulation technology and the use of plastics etc. could sharply reduce the relative cost in the smaller sizes. Smaller turbines are not usually tested, and reliable information on efficiency is generally not available. Impulse wheels are less sensitive to reductions in efficiency as size decreases. However, Francis and Kaplan units are more susceptible to reductions in efficiency in the smaller sizes where leakage and mechanical losses are relatively larger.

### ACKNOWLEDGMENTS

Preparation of this paper was supported in part by grants from the St. Anthony Falls Hydraulic Laboratory Seed Research Fund and the Minnesota Energy Agency. The author appreciates the help of Dr. Paul Cooper of Ingersoll-Rand Corporation who was kind enough to supply preliminary data on the utility of existing pumps as hydroturbines.

Much of the material contained here is based on an abridged version of Chapter 5 "Hydraulic Turbines" in *Small Hydropower Systems Design*, McGraw-Hill, 1982, by R. E. A. Arndt, C. Farell, and J. Wetzel (edited by J. Fritz).

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