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The Editors

Renewable Energy Review Journal : Vol. 2, No. 1, July 1980 1

The Application of Solar Cells

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ABSTRACT

This article presents three major terrestrial applications of the photovoltaic conversion of solar energy: water-pumping and irrigation, radio-relays and navigational beacon-lights, rural electrification. For these three fields, the state-of-the-art of the applied techniques is given. As concerns the rural electrification, a method of predicting the payback time of photovoltaic systems in a dynamic economical environment is developed for the example of a village in Africa.

1. INTRODUCTION

The photovoltaic effect was observed for the first time in the first half of the eighteenth century, but remained of purely academic interest until the beginning of the various space experiments. Terrestrial applications have only been investigated since 1954. The modern solar cell is an electronic device, fabricated from semi-conductors, that converts a fraction of the energy radiated from the sun directly into electrical energy.

In any photovoltaic system, the solar cell is the basic component. The photovoltaic generator–a certain number of solar cells put together–is the only energy supply for the electric apparatus that has to be run, so that the characteristics of both the generator and the receiver have to be adapted to one another according to the type of application and the geographical location.

It is obvious that the photovoltaic supply must be installed as close to the place where it is to be used as possible. Generally, this place is fairly remote from other sources of electricity. From the insolation data and the amount of energy required throughout the year, one can have a rough idea of the size of the generator, expressed in units of peak-power. The structure of the system is determined by the type of application: depending on whether it is to work during the daytime only, or in the night-time too, it might be necessary to use storage batteries; and depending on the type of current needed (DC or AC), charge regulators, DC/DC or DC/AC converters might be required.

At the beginning of the 1980's, the main terrestrial applications concern:

- water-pumping and irrigation,
- radio-relays and navigational beacon-lights,
- rural electrification.

The peak-power of installed generators is limited to 100 kW. These three domains of application differ in the structure of their power supply. That is why they will be discussed separately.

2. WATER-PUMPING AND IRRIGATION

Basically, the problem is to run an electric motor. Except for the irrigation station of 50 kW in Nebraska (United States), the required power is not higher than 10 kW.

Fig. 1 represents a typical structure of a pumping station.

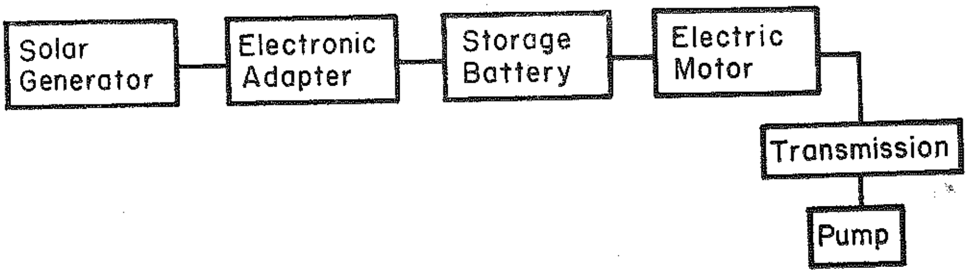

Fig. 1 Schematic structure of a pumping-station.

All the elements of this chain have to be adapted to each other with respect to their characteristics.

The solar generator is made of a number of photovoltaic panels connected in order to get the required voltage (12V, 24V, 36V, 48V, etc..) at their maximum power point under nominal conditions of insolation (generally 1 kW/m^2 at 25°C). The manufacturer provides the corresponding current-voltage (I-V) characteristic. The panels are usually tilted to the South (in the Northern hemisphere) with an angle equal to the latitude. They can also be mounted on a heliostat, which is a panel that tracks the sun throughout the day, but this requires an additional electro-mechanical set to provide the rotation; however, the tracking system can yield a 30% gain in the converted energy.

The electronic adapter is the interface between the generator and the battery. It can merely be a charge limiter that keeps the charging conditions safe for both batteries and photovoltaic panels. For higher power (>1 kW), it can also be a DC/DC converter that tracks the maximum power point of the I-V characteristic of the generator. Its transfer efficiency in both cases is very close to 1.

The storage battery is usually made of a number of lead-acid elements (2V each) having a low self-discharge rate. The capacity is calculated with respect to the energy requirement; it provides generally a few hours' autonomy. Its efficiency is at best 80%. When used without any DC/DC converter, its voltage determines the working point of the photovoltaic panels.

The electric motor can either be an asynchronous AC motor or a DC motor. An AC motor needs no maintenance and may be used immersed in the water to be pumped; however it requires storage batteries and a DC/AC converter. The DC motor cannot be immersed and therefore needs a transmission shaft; nevertheless it may sometimes be directly coupled to the photovoltaic panels, provided that the working points fit together.

The transmission (for DC motors only) is mechanical (shaft) for a centrifugal pump or hydraulic for a volumetric pump.

The pump, whether it is centrifugal or volumetric, determines the structure of the station. A volumetric pump must be used with storage batteries; a centrifugal pump may not. For large power requirements, a centrifugal pump can be driven by an AC motor.

Fig. 2 summarizes the most efficient ways to realize a pumping system. The overall efficiency ranges between 2% and 4.5%. Its optimum value has to be computed from the analysis of

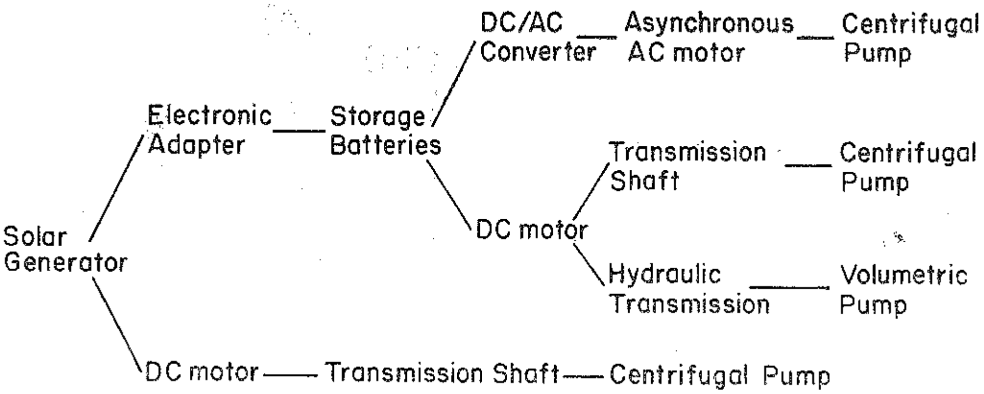

Fig. 2. Different ways to realize a water-pumping system.

the elements of the chain. There are three methods of analysis: the experimental method, which consists of building and comparing different systems, is quite long and costly; the mathematical method, which can be applied only to rather simple systems (with storage batteries for example), aims at modelling each working element and computing the optimum efficiency; the graphical method is suitable for nearly all cases since the manufacturers provide the characteristics of their equipment in the form of curves. This latter method will be applied, as an example, to the structure shown in Fig. 3.

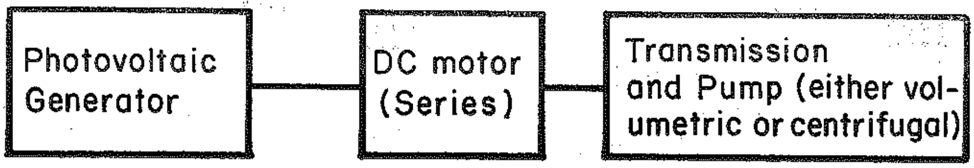

Fig. 3. Example of structure of a pumping-station.

To determine the working point of a motor and a pump, one has to find the intersection of the curves giving the torque as a function of the speed of rotation for both motor and pump. However, this simple procedure can only apply while the motor is connected to a voltage source, storage batteries for example, whereas the solar generator is not a voltage source. Generally, the manufacturers provide the following curves:

- Photovoltaic generator: I-V characteristics at various constant insolation levels,
- DC motor: - Torque-speed of rotation at various constant voltages (Fig. 4 a)
  - Torque-current at various constant voltages (Fig. 4b)
- Pump: Torque-speed of rotation (Fig. 4a)

Combining these curves as in Fig. 5 a, one gets the current-voltage working curve of the electro-mechanical receiver (motor and pump). On the other hand, from the I-V characteristic of the generator, one can draw the locus of the maximum power points with the solar light intensity as a parameter (Fig. 5b). Obviously the best working conditions of the system are achieved when these two curves fit exactly. As a matter of fact, one prefers to compute a posteriori the best grouping of the photovoltaic panels giving such a result.

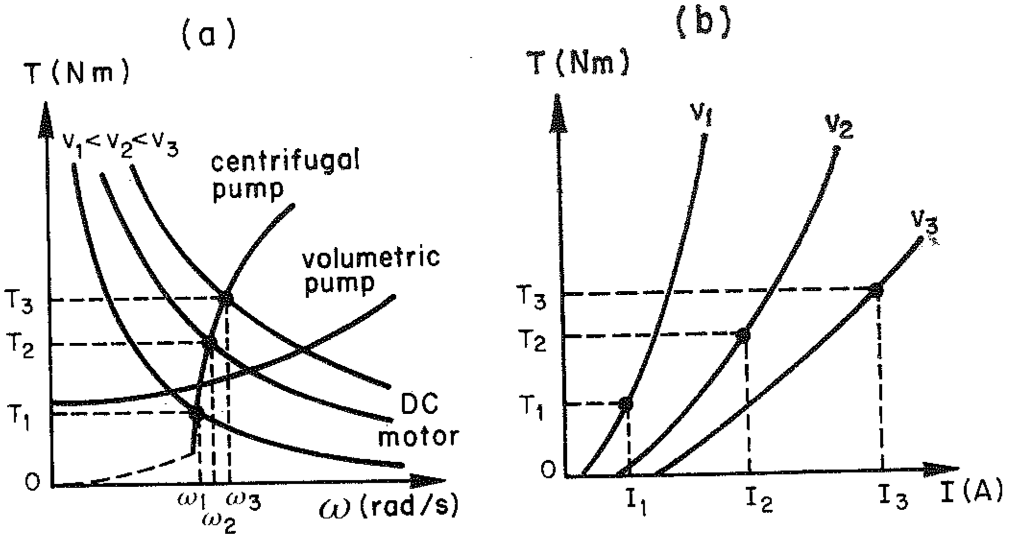

Fig. 4. a—Torque-speed for DC motor at constant voltage for centrifugal and volumetric pumps.
b—Torque-current for DC motor at constant voltage.

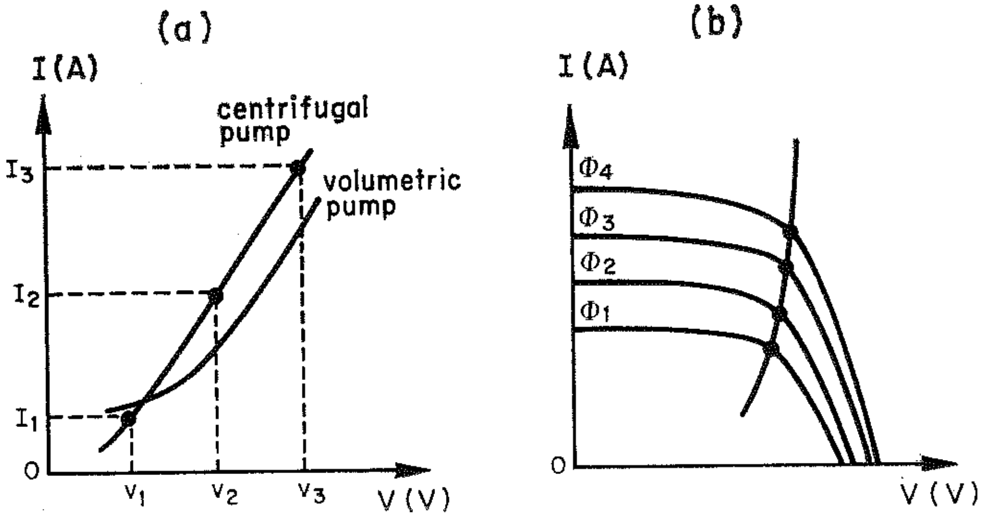

Fig. 5. a—Current-voltage working curve of the electro-mechanical receiver (motor and pump).
b—Current-voltage characteristics of the photovoltaic generator at constant insolation level, with locus of the maximum power points.

The characteristics of both DC motor and pump have to be chosen according to the characteristics of the well, i.e. depth of water and required flow. The output water flow is a function of the speed of rotation, hence of the insolation level (Fig. 6) so that from the insolation data, one knows the average daily amount of pumped water.

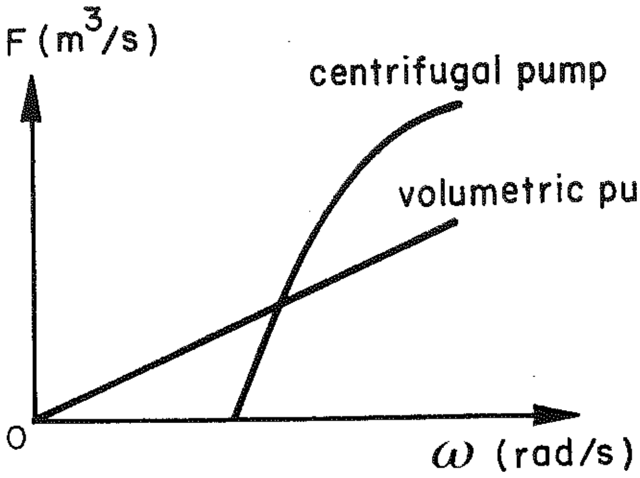

Fig. 6. Water flow as a function of speed of rotation.

Numerous small scale pumping stations (<1kW) have been successfully implemented in Africa. A standard pumping station has the following characteristic:

PV generator peak power	:	600 W
Rated power of the motor	:	350 W
Rated speed of rotation	:	2000 r.p.m.
Manometric height of the well	:	15 m
Maximum daily water flow	:	50 m^3/day
Average daily water flow	:	20 m^3/day
Maximum efficiency	:	3.5%
Total cost of the station	:	15,000 US$

3. RADIO-RELAYS AND NAVIGATIONAL BEACON-LIGHTS

Such equipment requires a permanent, stable and reliable DC voltage source of low power:

- up to 1000 W for telecommunication transmitters,
- up to 20 W for radio relay transmitters,
- up to 10 W for VHF radio-telephone transceivers,
- up to 50 W for radio-beacons, and
- up to 10 W for navigational beacon-lights (air or sea).

Depending on the local type of climate, the required complete autonomy can range between one week and one month, which allows one to determine the capacity of the storage battery. Assume that:

D is the required autonomy (in days of 24 hours) under bad weather conditions,
C is the capacity of the storage battery (in Ah),
V is the voltage of the battery (in V), and
P_u is the average consumed power under the voltage V (in W).

One has:

$$P_u . D . 24 = V . C,$$
$$C = P_u . D . 24/V. \qquad \ldots\ldots\ldots (1)$$

The peak-power of the photovoltaic generator is calculated from the local yearly insolation data and from the yearly energy requirement. If W_s is the yearly insolation level in kWh/m^2 year, and if P_g is the peak-power to be determined (in W), one has:

$$K . P_g . W_s = P_u . 8760,$$
$$P_g = P_u . 8760/K . W_s \qquad \ldots\ldots\ldots (2)$$

K is a coefficient taking into account:

- the transfer efficiency of the storage battery (around 70% for standard lead/acid),
- the meteorological uncertainties,
- the non-uniformity in the characteristics of the photovoltaic modules, and
- the losses in solar energy due to the fixed orientation of the panels.

Usually, K ranges between 0.45 and 0.60.

Table 1 presents examples of two existing applications.

Table 1. Examples of small-scale photovoltaic applications

25 W HF radio-beacon (air navigation) in TOGO		
Peak-power of the PV generator	:	224 W
Yearly insolation level	:	1650 kWh/m^2
Average consumed power	:	20 W
Storage batteries	:	24 V, 200 Ah
Autonomy	:	10 days
15 W television relay transmitter in FRANCE		
Peak-power of the PV generator	:	176 W
Yearly insolation level	:	1100 kWh/m^2
Average consumed power	:	15 W
Storage batteries	:	24 V, 300 Ah
Autonomy	:	20 days

4. RURAL ELECTRIFICATION

The power requirements for rural electrification depend essentially on the type of application:

- 100 W DC for remote school television sets,
- 1 kW DC for remote refrigeration units for vaccines, flowers, etc......,
- 10 kW AC for hospitals, and
- 100 kW AC for small villages.

The first two applications can be classified with those of the above section requiring small scale DC supplies.

In the case of the last two applications, the cost aspect is much more important, so that the optimization of the size of the generator and the storage batteries has to be carefully investigated with respect to the local insolation data and the user's needs in energy throughout the day. To facilitate the understanding of such an engineering design, the example of a village in Africa will be treated.

1. *Assessment of the problem*

Consider a hypothetical village of 4,000 inhabitants located in a tropical region of Africa. The local insolation data have been collected for three years; their average is reported in Table 2. These data, measured by a calibrated pyranometer on a horizontal surface, have to be correlated with the measured diffuse/direct ratio of the sun illumination, which is fairly high (up to 30%, even in the best season) because of the high tropical humidity; this already allows one to determine the mounting of the photovoltaic panels: fixed orientated towards the equator with a tilt angle equal to the local latitude. The required peak-power has to be divided by a cosine factor (around 0.8, depending on the value of this diffuse/direct ratio) to obtain the nominal peak-power of the panels. Note that a lower diffuse/direct ratio would have allowed one to install the photovoltaic modules on a sun-tracking heliostat and avoid the effect of this factor.

Table 2. Average local daily insolation level throughout the year

Month	J	F	M	A	M	J	J	A	S	O	N	D
W_s (kWh/m^2-day)	5.7	5.6	6.7	5.6	6.3	5.3	4.9	3.5	5.0	6.1	5.7	5.5

The only existing electrical appliance is a pumping station which is supplied by a 35 kW alternator run by a Diesel motor. It provides water for a 300 m^3 water tower. The urgent needs, planned in the development program for the village, are:

- a telecommunication station: about 5 kW ;
- a dispensary equipped with an air-conditioning unit, a refrigerator, fans and neon-lights : about 10 kW;
- a school with 10 classrooms, fans and neon-lights : about 11 kW; and
- an official building (administration, post office, etc. . . .) : about 8 kW.

The private users are to be included only in the next two years' development program. It is assumed that the daily load curve remains the same shape throughout the seasons, with changes only in its absolute value. Fig. 7 displays this daily load curve for a day of May (hot season) when the demand is maximum.

Table 3 presents the seasonal variations of the daily demand, estimated from the example of other urban zones in the same country.

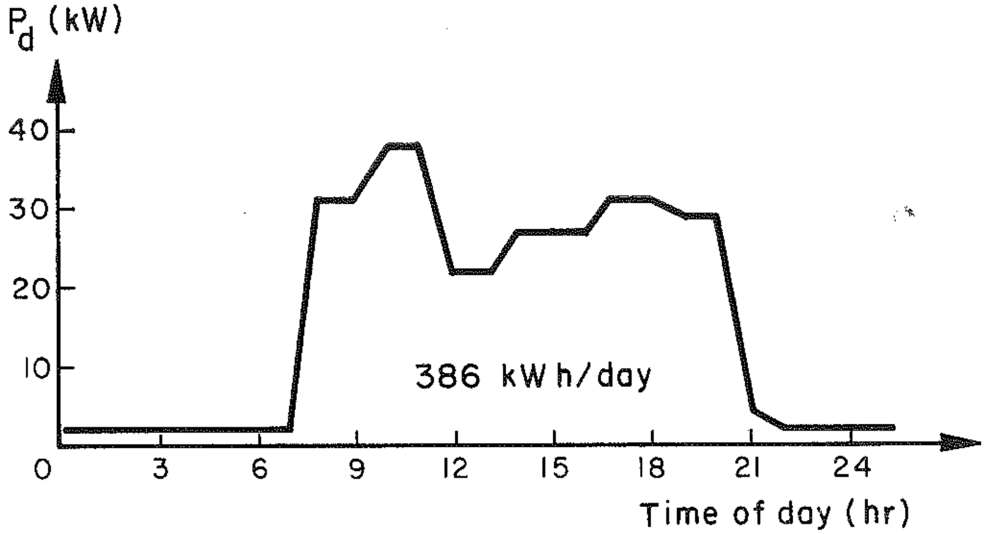

Fig. 7. Average daily load curve in May.

Table 3. Average daily electrical energy demand throughout the year

Month	J	F	M	A	M	J	J	A	S	O	N	D
W_d (kWh/day)	230	247	285	333	386	319	302	285	305	319	285	247

This demand is expected to increase with respect to an annual growth rate related to the development of the region. This explains why the design of the generator has to be modular.

2. *Structure of the Generator*

The most commonly proposed structure is shown in Fig. 8.

The photovoltaic generator is made of a number of independent modular panels (500-1000 W each), each connected to a DC/DC converter working on the maximum power point tracking principle; these electronic devices have a transfer efficiency better than 92%. All their outputs are connected in parallel to the storage batteries. A high voltage (at least 100 V) is required to run the Pulse-Width-Modulation DC/AC converter with a good efficiency (90%). They provide the mains line with 380 V, 50 Hz on three phases. The state of charge of the storage batteries is continuously controlled: a Diesel-run alternator provides charging through a battery charger and suppliers electric power to the mains during the periods when insolation is low and the load is high (particularly the month of August).

3. *Sizing the power plant*

It is first assumed that the power of the alternator is over the maximum consumed power, i.e. around 40 kW (see Figure 7). Suppose that the alternator is made of five units 10 kW each.

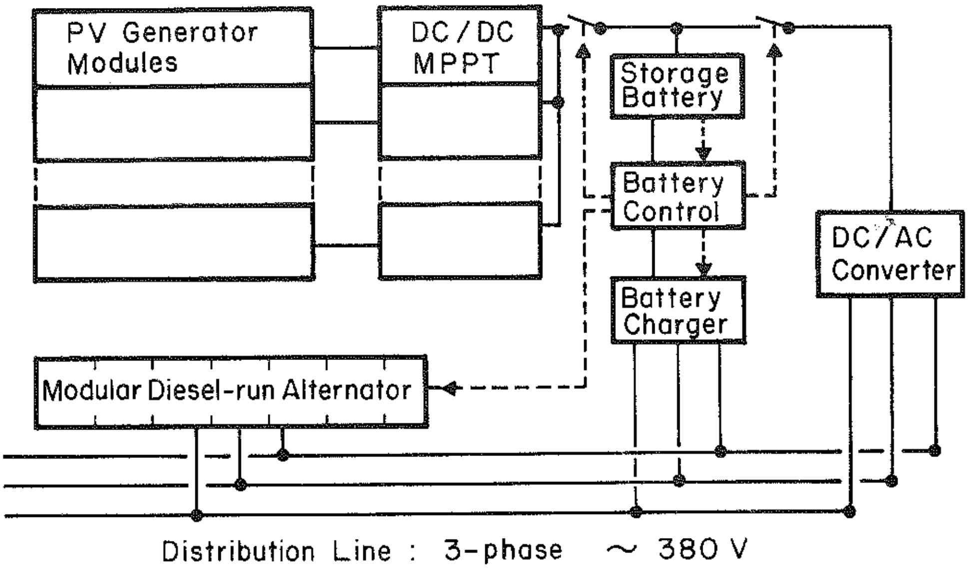

Fig. 8. Structure of a large-scale photovoltaic power plant.

Assume also that the daily insolation W_s in kWh/m^2 is strictly equivalent to a daily number H of hours with a 1kW/m^2 insolation level, so that:

$$\underset{(\text{kWh/m}^2)}{W_s} = \underset{(\text{kW/m}^2)}{1} \times \underset{(\text{hr.})}{H}.$$

Then built Table 4 containing the monthly average values of:

H_m : number of hours equivalent to the monthly insolation level, equal to the product of W_s (see Table 2) and the number of days in the month, and

W_m : electrical energy demand (in kWh/month), equal to the product of W_d (see Table 3) and the number of days in the month.

Ley P (in kW) be the peak-power of the photovoltaic generator,

η_1 be the transfer efficiency of the DC/DC converter, assumed to be: $\eta_1 = 0.95$,

η_2 be the transfer efficiency of the storage battery, assumed to be : $\eta_2 = 0.75$,

C_b be the capacity (in kWh) of the storage battery, and

η_3 be the transfer efficiency of the DC/AC converter, assumed to be: $\eta_3 = 0.90$

Table 4. Average monthly values of insolation level and electrical energy demand

Month	J	F	M	A	M	J	J	A	S	O	N	D
H_m (hr/month)	176.7	156.8	207.7	168.0	195.3	159.0	151.9	108.5	150.0	189.1	171.0	170.5
W_m (MWh/month)	7.13	6.92	8.84	9.99	11.97	9.57	9.36	8.84	9.15	9.89	8.55	7.66

The method of determining the value of P is based on the analysis of the yearly balance of energy flow through the battery. This balance has to be integrated through the year with a time constant which depends essentially on the type of climate. In this case, where the weather is fairly uniform, the time constant is as long as one month. For other climatic conditions, it can be one week, and more accurate data concerning insolation and energy demand have to be collected.

For given P and C_b, define the monthly values of:

- the available energy produced:

$$A_m = P.\, H_m.\, \eta_1.\, \eta_2 \text{ in MWh/month, and}$$

- the energy to be supplied to the mains through the DC/AC converter:

$$B_m = Wm/\eta_3 \text{ in MWh/month.}$$

Obviously there cannot be any complete accordance between these two values throughout the year, which means that the production cannot be perfectly adapted to the demand. This leads either to an excess of production or to a lack of production. An excess of production occurs when the capacity of the battery is saturated; it can be avoided by modifying the connections between the photovoltaic modules (i.e. by putting some of them out of work). A lack of production occurs when the demand is greater than the production and the capacity of the storage battery; the Diesel-run alternator makes up the difference and keeps the batteries charged at a safe level (generally 10% of the capacity).

Each month, these quantities are calculated as explained below: Let m be the index of the month ($1 \leq m \leq 12$), and let C_m be the charge level of the storage battery ($0.1\ C_b \leq C_m \leq C_b$). Compare the values of A_m and B_m:

If $A_m > B_m$, the charge level becomes:

$$C_m = C_{m-1} + A_m - B_m,$$

and $C_m > C_b$, implies that the excess of production is:

$$E_m = C_{m-1} + A_m - B_m - C_b,$$

and the charge level is C_b; while $C_m < C_b$ implies that there is no excess or lack of production.

If $A_m < B_m$, the charge level becomes:

$$C_m = C_{m-1} + A_m - B_m,$$

and $C_m > 0.1\ C_b$ implies that there is no excess or lack of production while $C_m < 0.1\ C_b$ implies that the lack of production is:

$$D_m = B_m - A_m - (C_{m-1} - 0.1\ C_b),$$

and the charge level is $0.1\ C_b$.

Thus for given P and C_b, one can compute the monthly values of E_m and D_m, and the yearly values:

$$E = \sum_{m=1}^{12} E_m.$$

$$D = \sum_{m=1}^{12} D_m.$$

The quantity $(E + D)$ has to be minimized.

An approximate value of P is given by:

$$P.\ \eta_1\ .\ \eta_2\ .\ \sum_{m=1}^{12} H_m = (1/\eta_3) \sum_{m=1}^{12} W_m,$$

which yields

$$P \simeq 84 \text{ kW}.$$

Because the climate is favorable and because of the presence of the Diesel-run alternator, one can state that the battery must provide an autonomy of a few days at most, say not more than 800 kWh. For each P and C_b ranging around these values, build the table for computing the yearly removed energy $(E + D)$. Table 5 is given as an example. Then draw the curves: $E + D = f\ (P, C_b)$. Figure 9 shows the minimum value of the removed energy for:

$$P = 82 \text{ kW}.$$

Table 5. Example of calculation of yearly removed energy for P = 85 kW and C_b = 200 kWh (all values in MWh)

Month	J	F	M	A	M	J	J	A	S	O	N	D	Total
A_m	10.70	9.50	12.58	10.17	11.83	9.63	9.20	6.57	9.08	11.45	10.36	10.33	121.40
B_m	7.92	7.69	9.82	11.10	13.30	10.63	10.40	9.82	10.17	10.99	9.50	8.51	119.85
$A_m - B_m$	2.78	1.81	2.76	-0.93	-1.47	-1.00	-1.20	-3.25	-1.09	0.46	0.86	1.82	
C_m	0.2	0.2	0.2	0.02	0.02	0.02	0.02	0.02	0.02	0.2	0.2	0.2	
E_m	2.78	1.81	2.76	0	0	0	0	0	0	0.28	0.86	1.82	10.31
D_m	0	0	0	0.75	1.47	1.00	1.20	3.25	1.09	0	0	0	8.76
												E+D =	19.07

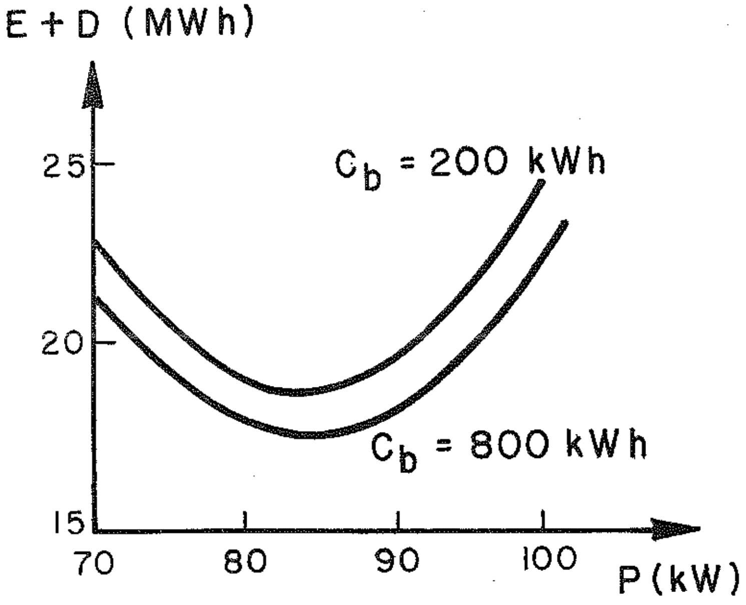

Fig. 9. Yearly removed energy as a function of generator peak-power.

Figure 10shows that *(E + D)* is nearly a linear function of the capacity C_b, with a slope independent of the generator peak-power *P*. Accounting for the good climatic conditions, one can state that an autonomy greater than 2 days is not necessary, so that: C_b = 500 kWh is a satisfactory result.

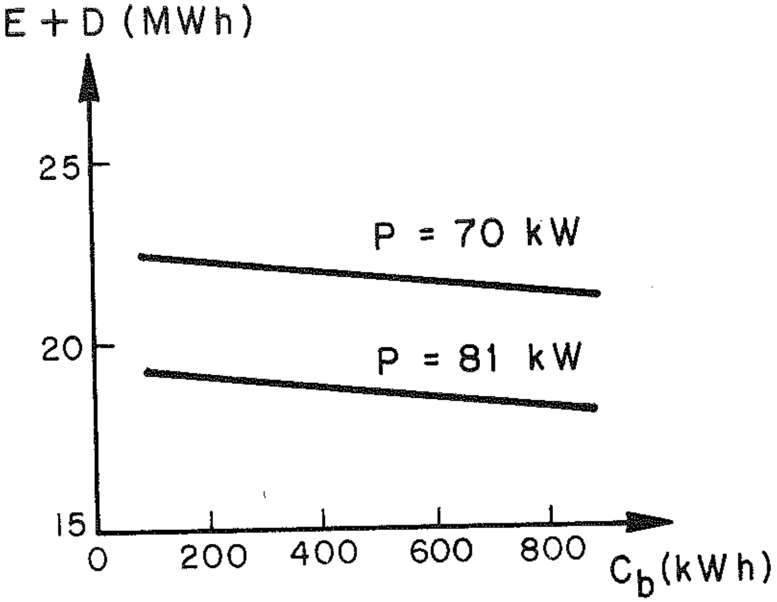

Fig. 10. Yearly removed energy as a function of capacity of the storage battery.

The calculation of the removed energy is given in Table 6 with the rated characteristics of the power plant.

Table 6. Calculation of the removed energy for the rated characteristics for P = 82 kW and C_b = 500 kWh (all values in MWh)

Month	J	F	M	A	M	J	J	A	S	O	N	D	Total
A_m	10.32	9.16	12.13	9.81	11.41	9.29	8.87	6.34	8.76	11.05	9.99	9.96	117.09
B_m	7.92	7.69	9.82	11.10	13.30	10.63	10.40	9.82	10.17	10.99	9.50	8.51	119.85
$A_m - B_m$	2.40	1.47	2.31	-1.29	-1.89	-1.34	-1.53	-3.48	-1.41	0.06	0.49	1.45	
C_m	0.5	0.5	0.5	0.05	0.05	0.05	0.05	0.05	0.05	0.11	0.5	0.5	
E_m	2.40	1.47	2.31	0	0	0	0	0	0	0	0.10	1.45	7.73
D_m	0	0	0	0.84	1.89	1.34	1.53	3.48	1.41	0	0	0	10.49
												E + D =	18.22

The yearly production rate of the Diesel-run alternator is:

$$W_A = D.\ \eta_3,$$

which gives

$$W_A = 16.4 \text{ MWh}.$$

Allowing for the weather uncertainties, it can be estimated at:

$$W_A = 20 \text{ MWh}$$

instead of

$$\eta_3 \cdot \sum_{m=1}^{12} B_m = 110 \text{ MWh.}$$

The average consumption of Diesel-oil by Diesel-run alternators is about 250 kg/MWh. Hence, the installation must be provided with a 5m^3 fuel tank. Each time that the growth of the village requires an extension of the power plant, every element has to be re-adjusted according to the above method.

The maintenance consists of

- one control visit a year for the photovoltaic generator and the electronic equipment,
- two control visits a year for the storage batteries to check the liquid and charge levels, and
- one control visit a year for the Diesel-run alternator eventually to change parts.

5. THE COST ASPECT

1. *Introduction*

As far as the photovoltaic market is concerned, the main producing countries are developed countries (United States, France, Germany, Israel, Japan), while their external trade is mostly orientated towards well-insolated developing countries, funded by local or international promotion programs for renewable sources of energy. However, even in the developing countries, electricity production by photovoltaic systems is still completely marginal compared to the other means of production. As long as it remains marginal (until it reaches about 1% of the total energy production), prices will remain high.

As a matter of fact, Africa among these developing countries is considered to be the most suitable field for experiments on photovoltaic devices from both technical and economical points of view, until the photovoltaic market is ready to be self-extending. Fortunately, most of these experiments have been—and are still—successful, giving a good opportunity to study the sociological impact of new techniques on the rural development. In the developed countries, the problem is different, for it merely consists in implementing decentralized energy supplies, and the social aspect is not involved in the same manner. For both cases, *as the energy production from renewable resources will remain marginal until about 1990, its economical analysis still cannot be other than marginal: it can only be given as an example allowing one to outline the main features of the future photovoltaic energy production market.* Nevertheless, such an analysis will be carried out on the example of Section 4, and will serve as a basis for more general considerations about the economics of photovoltaic solar energy.

Remember the characteristics of the photovoltaic power station described in Section 4; it is made of:

- Photovoltaic panels, total peak-power: $P = 82$ kW,
- MPPT DC/DC converters,
- Storage batteries, total capacity: $C_b = 500$ kWh,

- Electronic charge controller and battery charger,
- DC/AC converter (3 phase): 3 × 15 kW, and
- Diesel-run alternator: 5 units 10 kW each and a 5 m^3 fuel tank.

Now investigate successively the value of the investment, the expenses and the income, to determine the balance of costs and calculate the payback time. All the prices below are given in 1980 US$.

2. *Investment*

The first year, when the power plant is installed, the investment is the sum of the investments devoted to each of the above-mentioned parts:

- Photovoltaic generator: 82 kW-peak at 15 US$/W-peak,
  $I_1 = 15 \times 82{,}000 = 1.23 \times 10^6$ US$.
- DC/DC converters: 164 units 500 W each at 100 US$ each,
  $I_2 = 1.64 \times 10^4$ US$.
- Storage batteries: 500 kWh at 80 US$/kWh
  $I_3 = 4 \times 10^4$ US$.
- Electronic control and battery charger:
  $I_4 = 5 \times 10^3$ US$.
- DC/AC converter and transformer: 3 × 15 kW at 2 US$/W,
  $I_5 = 9 \times 10^4$ US$.
- Diesel-run alternator: 5 units 10 kW each at 6,000 US$ each,
  $I_6 = 3 \times 10^4$ US$.
- Miscellaneous (electric cables, out-house, etc..), estimated at 2,000 US$/kW on the generator peak-power,
  $I_7 = 1.64 \times 10^5$ US$.
- Packing, transportation (personnel included), installation, estimated at 4,000 US$/kW on the generator peak-power,
  $I_8 = 3.28 \times 10^5$ US$.

The total investment for this particular example is then:

$$I = I_1 + I_2 + I_3 + I_4 + I_5 + I_6 + I_7 + I_8,$$
$$= 1.9 \times 10^6 \text{ US\$}.$$

Consider that over the years the value of this invested capital increases because of economic inflation, and decreases because of the general depreciation of the capital. For such a new technical investment, the inflation rate is greater than the depreciation rate, so that one can attribute to the value of this investment an annual increasing rate r:

- the first year, the value of the investment is : $C_o = 1.9 \times 10^6$ US$,
- the second year, it is: $C_1 = C_o \,.\, (1 + r)$,
- the n^{th} year, it is: $C_{n-1} = C_o \,.\, (1 + r)^{n-1}$, and
- at the end of the n^{th} year, it is: $C_n = C_o \,.\, (1 + r)^n$. . . . . (3)

Usually r can range between 3% and 6%.

Note that for further investigations, the price of the photovoltaic generator (15 US$/W-peak) will be taken as a parameter.

3. *Expenses*

The yearly expenses include taxes, insurance, maintenance and eventually replacement of parts at the end of their lifetime. They are often referred to as an annual fixed charge rate on the up-to-date value of the capital m. But they have to be corrected by the annual inflation rate i:

- at the end of the first year, the yearly expense has been $m.C_o$; but, corrected by the inflation rate i, it is equal, at the end of the n^{th} year, to:

$$E_1 = m.\ C_o\ .\ (1 + i)^{n-1};$$

- at the end of the second year, the yearly expense has been: $m.C_1$; but, corrected by the inflation rate i, it is, at the end of the n^{th} year, equal to:

$$E_2 = m\ .\ C_1\ (1 + i)^{n-2} = m.\ C_o\ .\ (1 + r).\ (1 + i)^{n-2};$$

- at the end of the $(n-1)^{th}$ year, the yearly expense is: $m.\ C_{n-2}$; but, corrected by the inflation rate i, it is, at the end of the n^{th} year, equal to:

$$E_{n-1} = m\ .\ C_{n-2}\ .\ (1 + i) = m\ .\ C_o\ (1 + r)^{n-2}.\ (1 + i)^{n-(n-1)};$$

- at the end of the n^{th} year, it is merely:

$$E_n = m\ .\ C_o\ .\ (1 + r)^{n-1}.$$

Hence at the end of the n^{th} year, the yearly expenses accumulate to:

$$\begin{aligned} E &= \sum_{j=1}^{n} E_j = m.C_o.\sum_{j=1}^{n} (1 + r)^{j-1}.\ (1 + i)^{n-j} \\ &= m.C_o.\ [(1 + i)^n - (1 + r)^n]/(i - r). \qquad \text{. . . . . (4)} \end{aligned}$$

The annual fixed charge rate m can be determined approximately from the following data: as concerns large industrial equipment, the yearly expenses are currently estimated at 20% of the initial capital value divided by the lifetime in years, with respect to the lifetime. Hence m can be fixed at 2% to 5% of the current capital cost, including:

- taxes and insurance,
- annual active maintenance,
- passive maintenance, with replacement of parts,
- Diesel-oil consumption: 20 MWh/year at 0.25 t/MWh and 100-400 US$/t = 500 - 2000 US$/year the first year, which is at most 5% of the total yearly expenses.

A "reasonable" annual inflation rate i, according to the present economic situation, can range between 5% and 10%.

4. *Income*

The yearly income is provided through the selling price p_s of the electricity produced (about W_y = 100 MWh/year - see Section 4. 3). This price p_s is supposed to increase during the year according to the general inflation rate i. However, the income should be reinvested with an interest rate e greater than i :

- at the end of the first year, the income has been $p_s . W_y$ (p_s expressed in US\$/MWh); when reinvested, this income yields at the end of the n^{th} year:

$$I_1 = p_s . W_y .(1 + e)^{n-1};$$

- at the end of the second year, the income has been: $p_s .(1 + i). W_y$; when reinvested, it yields at the end of the n^{th} year:

$$I_2 = p_s .(1 + i).(1 + e)^{n-2};$$

- at the end of the $(n - 1)^{\text{th}}$ year, the income has been: $p_s .(1 + i)^{n-2}. W_y$; when reinvested, it yields at the end of the n^{th} year:

$$I_{n-1} = p_s .(1 + i)^{n-2}.(1 + e). W_y;$$

- at the end of the n^{th} year, it is:

$$I_n = p_s .(1 + i)^{n-1}. W_y .$$

Hence, at the end of the n^{th} year, the total income accumulates to:

$$\begin{aligned} I &= \sum_{j=1}^{n} . I_j = p_s . W_y . \sum_{j=1}^{n} (1 + i)^{j-1}.(1 + e)^{n-j} \\ &= p_s . W_y .[(1 + e)^n - (1 + i)^n]/(e - i) \qquad \ldots\ldots\ldots (5) \end{aligned}$$

The annual interest rate e can range between 8 % and 12 %. The price of electricity p_s will serve as a parameter.

5. *Absolute Economic Value—Payback Time*

At the end of the n^{th} year, the absolute economic value R of the power station is defined by:

$$R(n) = C_n + E(n) - I(n). \qquad \ldots\ldots\ldots (6)$$

The payback time is the number N of years that satisfies the equation:

$$R(N) = 0. \qquad \ldots\ldots\ldots (7)$$

This equation can be reduced to a relation between N and p_s, with the initial investment C_o as a parameter:

$$p_s . W_y . \frac{[(1 + e)^N - (1 + i)^N]}{e - i} = C_o .(1 + r)^N + m . C_o \frac{[(1 + i)^N - (1 + r)^N]}{i - r}. \qquad \ldots\ldots (8)$$

Then take the following values:

$$\begin{aligned} m &= 2\%, \\ r &= 5\%, \\ i &= 7\%, \\ e &= 12\%, \end{aligned}$$

and introduce the cost-per-peak-Watt C_p (in US\$/W-peak) as a new parameter:

$$C_o = 0.673 \times 10^6 + C_p 82 \times 10^3 \quad \text{(in US\$)}.$$

Then we obtain:

$$p_s = (0.673 \times 10^6 + C_p 82 \times 10^3) \frac{(1.07)^N}{2{,}200[(1.12)^N - (1.07)^N]} \qquad \ldots\ldots\ldots (9)$$

This equation allows one to built Table 7, giving the price of the electricity produced the first year as a function of the cost-per-peak-watt of the photovoltaic generator and of the required payback time.

Table 7. Price of the produced electricity (in US\$/MWh) as a function of C_p and payback time

C_p (US\$/W-peak)	N					
	5	9	13	17	21	25
1	1338	675	423	292	213	161
2	1483	748	469	324	236	178
5	2500	968	607	419	306	231
7	2200	1115	699	483	352	266
10	2646	1335	837	578	422	318
15	3370	1701	1067	737	538	406

These values indicate that the price of solar electricity is still four to five times more expensive than the current prices (about 50 US\$/MWh), even in the case where the cost of photovoltaic generators is reduced by 80%, as expected in the next 5 to 10 years.

However, considering that the installation of a solar power plant is a part of the regular electricity network extension, the absolute economic value has to be compared with the absolute economic value of the conventional power plant that could have been installed, a Diesel-run power plant for example. This leads to a consideration of the relative economic value.

6. *Relative Economic Value and Conclusion*

In the case of this same example, a Diesel power plant should have the following characteristics:

- Rated power: 50 kW (5 units 10 kW each).
- Annual production: 110 MWh/year.
- Investment (including installation and miscellaneous), estimated at 1,500 US\$/kW: 75,000 US\$.
- Diesel-oil consumption: 250 kg/MWh: 30 t/year at 100 to 400 US\$/t: 3,000 to 12,000 US\$ the first year.
- Annual fixed charge rate: m = 5% because the Diesel-run alternator is employed much more than the auxiliary Diesel motor in the photovoltaic system.
- Income: same form as above.

The absolute economic value of this replaced power plant is: $S(n)$ = Investment + Annual charges + Diesel-oil consumption-Income. With the same figures as the previous calculations, it is given by:

$$\begin{aligned} S(n) &= 7.5 \times 10^4 (1.05)^n + 18.75 \times 10^4 [(1.07)^n - (1.05)^n] \\ &\quad + A.10^4 [(1.07)^n - 1] - p_D.2200.[(1.12)^n - (1.07)^n] \qquad \ldots\ldots\ldots (10) \end{aligned}$$

where: A = 4.29 for a 100 US\$/t Diesel-oil price

A = 17.14 for a 400 US\$/t Diesel-oil price

This equation stresses that in the first case (easy access, Diesel-oil at 100 US$/t), the Diesel plant is paid back in less than 15 years, while in the second case (difficult access, Diesel-oil at 400 US$/t), it is paid back in more than 20 years, with the current price of electricity (0.05 US$/kWh = 50 US$/MWh).

The relative economic value of the solar project is:

$$T(n) = R(n) - S(n). \qquad \ldots\ldots\ldots (11)$$

Its actual payback time is the number N of years that satisfies the equation:

$$T(N) = 0. \qquad \ldots\ldots\ldots (12)$$

Taking: p_D = 50 US$/MWh, similarly to the above computation, one obtains Tables 8a and 8b, giving the price of the electricity produced as a function of the cost-per-peak - Watt of the photovoltaic generator and of the required payback time, for each of the two cost conditions of the Diesel-oil.

The difference between Tables 7 and 8 is very slight, which is explained by the fact that the amount of money represented by the economic value is much larger for the solar investment than for the Diesel investment. Nevertheless, the economic situation may change, and the non-renewable energy reserves are not unlimited, giving a chance to systems paid back in 30 to 35 years to be installed successfully. The main conclusion is that photovoltaic applications are limited to small-scale electricity production (power less than 100 kW), the large-scale being reserved for thermodynamic power plants employing more conventional techniques.

Table 8. Price of the produced electricity (in US$/MWh) as a function of C_p and payback time by the method of relative economic value

a - Diesel - oil cost: 100 US$/t

C_p (US$/W-peak)	N					
	5	8	11	14	17	20
1	1215	716	493	369	290	237
2	1360	800	549	410	322	262
5	1797	1054	722	535	417	337
7	2087	1223	836	618	481	387
10	2523	1477	1007	743	576	462
15	3250	1899	1293	951	735	586

b - Diesel - oil cost: 400 US$/t

C_p (US$/W-peak)	N					
	5	8	11	14	17	20
1	1150	661	446	329	256	208
2	1296	745	503	370	288	233
5	1732	999	675	495	383	308
7	2022	1168	789	578	447	358
10	2458	1421	960	703	542	433
15	3185	1844	1246	911	701	557

But obviously, these prices are definitely not "chiseled-in-stone" figures. They depend strongly on the load factor of the power plant and on the weather conditions. However their magnitude is accurate enough to give a precise idea on what can be expected from photovoltaic devices.

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