www.serd.ait.ac.th/reric

Optimal Power Dispatch Using Different Fuzzy Constraints in Power Systems

A.A. Abou El-Ela*, M.A. Bishr*, S.M. Allam*, and R.A. El-Sehiemy+1

Abstract – *This paper presents comparison studies between different fuzzy models to solve the fuzzy-based optimal power dispatch (OPD) problem. The proposed fuzzy-based OPD model handles fuzzy objectives and constraints and is aimed to obtain the optimal operational settings of system generation outputs. These settings minimize the total generation costs and at the same time guarantees that the power flows in critical lines are less than their maximum limit. The comparison studies are performed considering the changes in fuzzy constraints as membership models. The fuzzy constraints are modeled using two linear fuzzy models, namely triangular and trapezoidal models. Numerical studies are performed based on the fuzzy linear programming (FLP) optimization technique. These studies show that, the changes in membership models have a great effect in generation settings, elimination the overflows in the critical lines, and minimizing the total generation costs.*

Keywords – Fuzzy linear programming, linear fuzzy models, optimal power dispatch, transmission bending limits.

1. INTRODUCTION

More than four decades ago, the generalized nonlinear programming formulation of the economic dispatch problem was introduced including voltage and other operating constraints. This formulation was named as the optimal power flow (OPF) problem. The OPF problem plays an important role in power system planning and operation. The OPF problem can be viewed as a process aiming at determining the combination of generation units, which minimizes the total operational costs. Where, identifying the best generation values subject to operational and security constraints is driven by economic techniques. The conventional OPF is formulated as an optimization problem with crisp constraints. The constraints can be classified into a set of equality (power flow equations) and inequality constraints (limits and variables). The inequality constraints are the limits of the control variables and operating limits of power systems. However, in practice, there are two types of inequality constraints: hard constraints and soft constraints. For example, the limits of the generating unit outputs are hard constraints because there are physical limitations on the capacity of the generating units to produce active power. The hard constraints expression means that the physical limitation of the generation units cannot be violated. The fuzzy modeling of power generation outputs is aimed to find the optimal operational settings within their minimum and maximum operational generation limits. On the other hand, the limits for the critical transmission line power flows are soft. Small violations of these limits sometimes are acceptable, especially during stressed situations of the system (e.g. emergency or peak loaded). Identifying any transmission line as critical transmission line is based on the line sensitivity factors to different power system events, line-loading factor, and the line importance in the system operation (Line priority).

Reference [1] solved the OPF problem using the LP technique. Security studies are presented in References [2]–[4]. Lu and Unum in [5] used an interior point algorithm to solve the network constrained security control. A common trend in previous techniques has been towards utilizing fixed values, which may leads to an overestimated solution.

From an operational point of view, minimizing generation cost does not mean that a rigid minimum solution is achieved. It is more appropriate to state the OPD objectives as: to reduce the generation costs as much as possible without moving too many control settings, while satisfying the soft constraints as much as possible and enforcing the hard constraints exactly. Here, the concepts of “as much as possible” and “not too many" are fuzzy in nature. Fuzzy logic has found favour among many engineers for its ability to represent the sorts of qualitative statements employed by human. The conventional logic assumes that a variable has one precise value (it is crisp).

Recently, fuzzy set methods have been applied to obtain realistic models. Fuzzy set methods have already been used in many applications such as control, scheduling, robotics, artificial intelligence, etc. In the field of power system engineering, fuzzy set methods have been applied to some areas including OPF problems. References [6]–[10] presented the solution of the optimal power flow problem using the FLP technique. Reference [11] solved OPD problem considering multi-objective FLP technique considering preventive action constraints. Different emergency control analyses procedure using multi-objective FLP technique are presented in [12].

* A.A. Abou El Ela, Professor, Electrical Power Systems, Electrical Engineering Department, Faculty of Engineering, Menoufiya University, Shebin El-Kom, Menoufiya, Egypt.
E-mail: draaa50@hotmail.com

M.A. Bishr, Professor, Electrical Power Systems, Electrical Engineering Department, Faculty of Engineering, Menoufiya University, Shebin El-Kom, Menoufiya, Egypt.
E-mail: mbeshr52@hotmail.com

S.M. Allam, Lecturer, Electrical Power Systems, Electrical Engineering Department, Faculty of Engineering, Menoufiya University, Shebin El-Kom, Menoufiya, Egypt.
E-mail: sohierallam@yahoo.com

+ R.A. El-Sehiemy works at The Arab Contractor Company and a Ph.D. Student at Electrical Engineering Department. Menoufiya University, Home Address: P.O. Box 32735, Bakhaty, Shebin El-Kom, Menoufiya, Egypt.

1 Corresponding author; E-mail: elsehiemy@yahoo.com

This paper presents a fuzzy-based OPD procedure taking into account fuzzy modeling for both equality and inequality constraints. Two linear fuzzy models (triangular and trapezoidal models) are used to model the power system variables. The fuzzy constraints improve the OPD solution as:

- Finding the optimal operational settings of these variables within the operational generation range;
- Tuning the power systems variables;
- Ramping the power generation and power transmission lines fuzzy constraints corresponding to the amount of reserve requirements; and
- Considering the uncertainty in power systems.

2. PROBLEM FORMULATION

Conventional Optimal Power Dispatch

The objective of the conventional OPD problem is to minimize the total generation costs under various system and operational constraints. The OPD problem is formulated as:

$$Min \;\; C = \sum_{i=1}^{NG} F_i\left(PG_i\right) \tag{1}$$

Subject to:

- Power balance constraint.

$$\sum_{i=1}^{NG} PG_i = PD \tag{2}$$

- Power flow constraint

$$PF_k^{\min} \le PF_k \le PF_k^{\max} \tag{3}$$

The power flow PF_k can be computed from:

$$PF_k = \sum_{i=1}^{NG}\left(D_{k,i}.PG_i\right), \; for \; i = 1,.....,NG \tag{4}$$

Where, $D_{k,i}$ is the generalized generation distribution factor for line k due to generator *i* [13].

- Power generation limits constraints.

$$PG_i^{\min} \le PG_i \le PG_i^{\max} \tag{5}$$

Fuzzy Optimal Power Dispatch Problem

The fuzzy-based OPD problem is formulated as:

$$Min \;\; C = \sum_{i=1}^{NG} F_i\left(\tilde{P}G_i\right) \tag{6}$$

Subject to:

$$\sum_{i=1}^{NG} \tilde{P}G_i = \tilde{P}D \tag{7}$$

$$PF_k^{\min} \le \tilde{P}F_k \le PF_k^{\max} \tag{8}$$

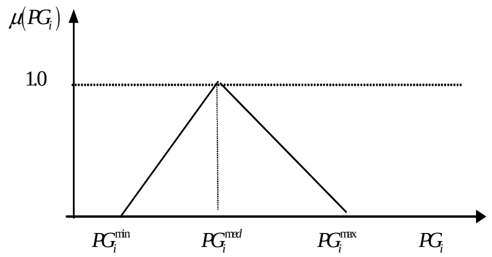
$$PG_i^{\min} \le \tilde{P}G_i \le PG_i^{\max} \tag{9}$$

Linear Fuzzy Models Formulation

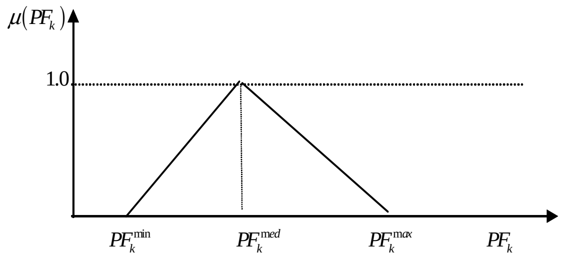
Before starting with the fuzzy modeling of constraints, it is important to define the meaning of the considered models, which are the trapezoidal model and the triangular model. Let the symbol P be used to express one of these constraints. For instance through a linguistic declaration as "power P may occur between P_1 and P_4 MW but likely to be between P_2 and P_3. This can be translated into trapezoidal fuzzy model at which the uncertainty through interval. If $P_2 = P_3$, the resulted model will be define the triangular model of power constraints. The next two sections deal with the two fuzzy models of the power system constraints.

Triangular fuzzy modeling

The triangular fuzzy modeling for the active power generation at bus *i* is shown in Figure 1a. It is seen that, a membership function is equal to 1 assigned to PG_i^{med} . The triangular fuzzy modeling for the power flow in critical line k is shown in Figure 1b. It is seen that, a membership function is equal to 1 assigned to PF_k^{med} . The triangular membership functions, for generation limit at bus i and for the power flow in critical transmission line k, are presented in Equations 10 and 11, respectively.



(a) Power generation membership for unit *i*

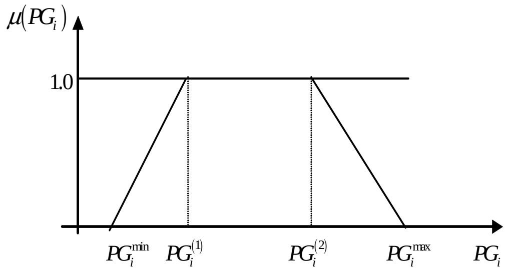


(b) Power flow membership in critical line *k*

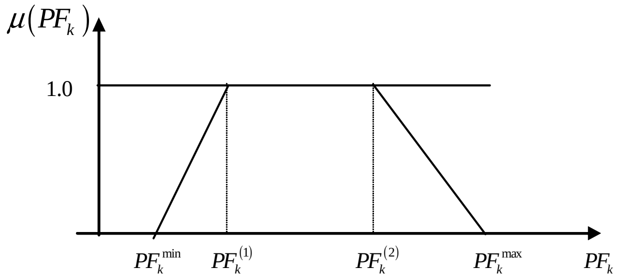
Fig. 1. Triangular membership model

Trapezoidal fuzzy modeling

The trapezoidal fuzzy modeling of the power generation and the power flow in critical lines constraints are presented in Figure 2. The trapezoidal membership functions of the power generation at bus i and the power flow in the critical transmission line k, the violated transmission line, considered as critical line, are described and shown in Equations 12 and 13, respectively.



(a) Power generation membership for unit *i*



(b) Power flow membership in critical line k

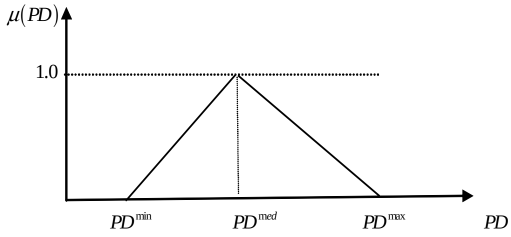
Fig. 2. Trapezoidal membership models

Fuzzy Modeling of Load Demand

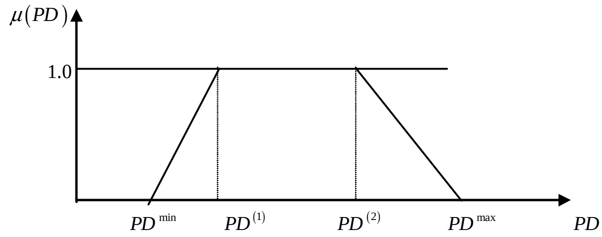
Similarly, for the fuzzy modeling of load demand, Figures 3a and 3b show the triangular and the trapezoidal membership for load demand, respectively. The triangular and trapezoidal membership functions for the load demand are described in Equations 14 and 15, respectively.

Fuzzy Modeling of Objective Function

The objective function which is considered in the proposed procedure minimized the generation cost function as much as possible. The fuzzy modeling of the generation cost function is shown in Figure 4. The fuzzy membership function of the cost, which is less than or equals the permissible cost, is described in Equation 16.



(a) Triangle membership model



(b) Trapezoidal membership model

Fig. 3. Fuzzy modeling of the load demand

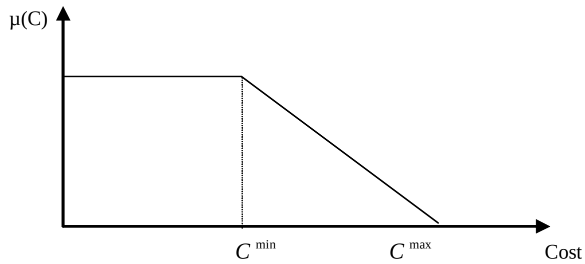


Fig. 4. Fuzzy membership function for the generation cost function

$$\mu_{PG_i}(PG_i)=\begin{cases}0 & PG_i < PG_i^{med}\\ \left(PG_i - PG_i^{\min}\right)/\left(PG_i^{med} - PG_i^{\min}\right) & PG_i^{\min} \le PG_i \le PG_i^{med}\\ \left(PG_i^{\max} - PG_i^{med}\right)/\left(PG_i^{\max} - PG_i^{med}\right) & PG_i^{med} \le PG_i \le PG_i^{\max}\\ 0 & PG_i > PG_i^{\max}\end{cases} \tag{10}$$

$$\mu_{PF_k}\left(PF_k\right)=\begin{cases}0 & PF_k < PF_k^{\min}\\ \left(PF_k - PF_k^{\min}\right)/\left(PF_k^{med} - PF_k^{\min}\right) & PF_k^{\min} \le PF_k \le PF_k^{med}\\ \left(PF_k^{\max} - PF_k\right)/\left(PF_k^{\max} - PF_k^{med}\right) & PF_k^{med} \le PF_k \le PF_k^{\max}\\ 0 & PF_k > PF_k^{\max}\end{cases} \tag{11}$$

$$\mu_{PG_i}(PG_i)=\begin{cases}0 & PG_i < PG_i^{\min}\\ \left(PG_i - PG_i^{\min}\right)/\left(PG_i^{(1)} - PG_i^{\min}\right) & PG_i^{\min} \le PG \le PG^{(1)}\\ 1 & PG_i^{(1)} \le PG_i \le PG_i^{(2)}\\ \left(PG_i^{\max} - PG_i\right)/\left(PG_i^{\max} - PG_i^{(2)}\right) & PG_i^{(2)} \le PG_i \le PG_i^{\max}\\ 0 & PG_i > PG_i^{\max}\end{cases} \tag{12}$$

$$\mu_{PF_k}\left(PF_k\right)=\begin{cases}0 & PF_k < PF_k^{\min} \\ \left(PF_k - PF_k^{\min}\right)/\left(PF_k^{(1)} - PF_k^{\min}\right) & PF_k^{\min} \le PF_k \le PF_k^{(1)} \\ 1 & PF_k^{(1)} \le PF_k \le PF_k^{(2)} \\ \left(PF_k^{\max} - PF_k\right)/\left(PF_k^{\max} - PF_k^{(2)}\right) & PF_k^{(2)} \le PF_k \le PF_k^{\max} \\ 0 & PF_k > PF_k^{\max}\end{cases} \tag{13}$$

$$\mu_{PD}\left(PD\right)=\begin{cases}0 & PD < PD^{\min} \\ \left(PD - PD^{\min}\right)/\left(PD^{med} - PD^{\min}\right) & PD^{\min} \le PD \le PD^{med} \\ \left(PD^{\max} - PD\right)/\left(PD^{\max} - PD^{med}\right) & PD^{med} \le PD \le PD^{\max} \\ 0 & PD > PD^{med}\end{cases} \tag{14}$$

$$\mu_{PD}\left(PD\right)=\begin{cases}0 & PD < PD^{\min} \\ \left(PD - PD^{\min}\right)/\left(PD^{(1)} - PD^{\min}\right) & PD^{\min} \le PD \le PD^{(1)} \\ 1 & PD^{(1)} \le PD \le PD^{(2)} \\ \left(PD^{\max} - PD\right)/\left(PD^{\max} - PD^{(2)}\right) & PD^{(2)} \le PD \le PD^{\max} \\ 0 & PD > PD^{\max}\end{cases} \tag{15}$$

$$\mu_C\left(C\right)=\begin{cases}1 & C < C^{min} \\ \left(C^{max} - C\right)/\left(C^{max} - C^{min}\right) & C^{min} \le C \le C^{max} \\ 0 & C > C^{max}\end{cases} \tag{16}$$

3. PROPOSED PROCEDURE FOR OPTIMAL POWER DISPATCH PROBLEM

Linearization of the Generation Cost Function

The OPD with quadratic form of generation cost functions is formulated as nonlinear optimization problem. The solution of the OPD problem using FLP technique requires linear objective function.

The quadratic generation cost function of the form:

$$F_i\,(PG_i) = a_i.PG_i^{\,2} + b_i.PG_i + c_i \tag{17}$$

The generation cost function, of unit i, in linear form for small variation in unit i power generation output can be written with the help of as the basics of derivative as:

$$F_i = \left.\frac{dF_i}{dPG_i}\right|_{PG_i = PG_i^{\,0}} .PG_i \tag{18}$$

$$F_i = \left.\left(2a_i PG_i + b_i\right)\right|_{PG_i = PG_i^{\,0}} .PG_i \tag{19}$$

$$F_i = \left(2a_i PG_i^{\,0} + b_i\right).PG_i \tag{20}$$

Then, the approximate form of total generation cost function is written as:

$$F_i = \sum_{i=1}^{NG} (2a_i\, PG_i^{(0)} + b_i\,).PG_i \tag{21}$$

FLP Optimization Model

The FLP optimization technique is used to solve the fuzzy-based OPD problem (5-8). The degree of satisfaction the fuzzy objectives and constraints, Equations 10–16, can be represented by a membership variable λ. The variable λ is defined as the minimum of all membership functions of the fuzzy objectives constraints. The fuzzy-based optimal OPD solution maximizes satisfaction variable λ. Then, the relationship between the satisfaction factor λ and other membership functions can be written as the minimum of all membership functions. In this section the fuzzy symbols appeared as the degree of membership function. The mathematical model is:

$$\max_{PG_i\,,\lambda} \lambda, \tag{22}$$

s. t. $\lambda \le \mu_m(.), m = 1, ..., NC$ (23)

Rewriting the mathematical model of the proposed procedure gives:

$$\max_{PG_i, \lambda} \lambda' \tag{24}$$

s. t.: $\lambda \le \mu_c(C(PG_i))$ (25)

$$\lambda \le \mu_{PG_i}(PG_i), i = 1, 2, ..., NG \tag{26}$$

$$\lambda \le \mu_{PF_k}(PF_k(PG_i)), k = 1, 2, ..., NL \tag{27}$$

$$\lambda \le \mu_{PD}(PD(PG_i)) \tag{28}$$

and $0 \le \lambda \le 1$ (29)

The generation cost constraint in Equation 25 can be rewritten as:

$$C + (C^{\max} - C^{\min})\lambda \le C^{\max} \tag{30}$$

The triangular fuzzy model of power system variables in Equations 26–28 can be rewritten as:

For power generation units:

$$-PG_i + (PG_i^{med} - PG_i^{\min})\lambda \le -PG_i^{\min}, \text{i=1, 2,......., NG} \tag{31}$$

$$PG_i + (PG_i^{\max} - PG_i^{med})\lambda \le PG_i^{\max}, \text{i=1, 2,......, NG} \tag{32}$$

For critical transmission lines:

$$-PF_k + (PF_k^{med} - PF_k^{\min})\lambda \le -PF_k^{\min}, \text{k =1, 2,........, NL} \tag{33}$$

$$PF_k + (PF_k^{\max} - PF_k^{med})\lambda \le PF_k^{\max}, \text{k =1, 2,........, NL} \tag{34}$$

For power demand:

$$-PD + (PD^{med} - PD^{\min})\lambda \le -PD^{\min} \tag{35}$$

$$PD + (PD^{\max} - PD^{med})\lambda \le PD^{\max} \tag{36}$$

The trapezoidal fuzzy model of power system variables in Equations 26–28 can be rewritten as:

For power generation units:

$$-PG_i + (PG_i^{(1)} - PG_i^{\min})\lambda \le -PG_i^{\min}, \text{k =1, 2,........, NG} \tag{37}$$

$$PG_i + (PG_i^{\max} - PG_i^{(2)})\lambda \le PG_i^{\max}, \text{k =1, 2,........, NG} \tag{38}$$

For critical transmission lines:

$$-PF_k + (PF_k^{(1)} - PF_k^{\min})\lambda \le -PF_k^{\min}, \text{k =1, 2,........, NL} \tag{39}$$

$$PF_k + (PF_k^{\max} - PF_k^{(2)})\lambda \le PF_k^{\max}, \text{k =1, 2,........, NL} \tag{40}$$

For power demand:

$$-PD + (PD^{(1)} - PD^{\min})\lambda \le -PD^{\min} \tag{41}$$

$$PD + (PD^{\max} - PD^{(2)})\lambda \le PD^{\max} \tag{42}$$

Procedure Steps

The procedure steps, for certain studied condition are:

1. Simulating the operating condition.
2. Computing the initial generation settings and the related power flows in transmission lines.
3. Identifying the violated transmission lines as critical transmission lines.
4. Preparing the fuzzy modeling of different system variables based on the initial state.
5. Solving the OPD problem using the proposed procedure.
6. Ensuring the power flows in all transmission lines within their permissible limits.
7. If there is not a violation, print results else modify the critical transmission lines and (Go to step 3).

4. APPLICATIONS

Test System

The IEEE 30-bus test system (6-generation units, 41-lines) [14] is used to extensively study the OPD problem using the FLP technique for different fuzzy models. The bus data of the six generation units are presented in Table 1 while, the data for 8-critical transmission line is presented in Table 2. These lines are lines No. 1, 2, 4, 5, 6, 9, 11, and 14. The power flow computations are performed using the MATPOWER package [15].

Table 1. Bus generation data

Generator No.	Bus No.	Min. Limits (MW)	Max. Limits (MW)	Initial PG_i (MW)	Cost Function ($/hr)
1	1	50	200	152.98	$2P_1+0.00375\ P_1^2$
2	2	20	80	57.56	$1.75P_2+0.0175\ P_2^2$
3	5	15	50	24.56	$P_3+0.0625\ P_3^2$
4	8	10	35	31.404	$3.25P_4+0.00834\ P_4^2$
5	11	10	30	13.43	$3P_5+0.025\ P_5^2$
6	13	12	40	16.846	$3P_6+0.025\ P_6^2$

Table 2. Critical lines data

Line No.	Connection		Max. Limits. PF_k (MW)	Initial PF_k (MW)
	From Bus No.	To Bus No.		
1	1	2	75	100.21
2	1	3	50	52.54
4	3	4	44	49.17
5	2	6	50	60.9
6	3	4	36	41.49
9	5	7	35	35.94
11	6	8	10	11.13
14	6	28	30	30.5
40	27	30	3.35	3.51

Membership Modeling Studied Cases

The fuzzy-based OPD problem presented in Equations 6 to 9 is solved using the FLP procedure. Choosing LP instead of other optimization methods is based on the following:

i) The proposed fuzzy models are linear models
ii) The cost objective function is modeled as a piecewise quadratic function and can be approximated as a piecewise linear function.

Table 3 shows the possible fuzzy membership models for power generations, power flows in critical transmission lines, and load demand. These models may be triangular and/or trapezoidal models for one or more of the system variables. So, there are eight possible cases under consideration. The mathematical model for power system variables (Case 1) is presented in Equations 31 to 36. The FLP solution of the OPD problem considers the objective function (Equation 24) subjected to the satisfaction factor limits constraint (Equation 29), generation cost constraint (Equation 30), and the triangular linear fuzzy model Equations 31 to 36. Other cases introduced different fuzzy membership functions for power generation, flows, and power demand. In Case 2, two linear constraints corresponding to the trapezoidal model of load demand are introduced as Equations 40 and 41 instead of Equations 35 and 36 in Case 1. The same procedure is followed to ramp the system constraints for satisfying the other cases. Case 8 considers Equations 37 to 41 instead of Equations 31 to 36 in the fuzzy-based OPD problem.

Table 3. Fuzzy membership models

Variables	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8
Generation	*	*	*	*	**	**	**	**
Power Flow	*	*	**	**	*	*	**	**
Load	*	**	*	**	*	**	*	**
Where (*) Refers to use the triangle model and (**) refers to use the trapezoidal model								

Intermediate points of membership functions

The OPD solution is dependent on the choice of PG_i^{med}, PF_k^{med} and PD^{med} in the case of triangular fuzzy modeling. In this paper, the med-points of power generation units and the power demand equal to the initial generation and demand values. While, the med-points of the power flows in transmission lines are considered at 90% of the maximum limit of these lines. Also, The OPD solution is very significant to the choice of the intermediate points of the trapezoidal model of power generation at unit i ($PG_i^{(1)}$ and $PG_i^{(2)}$), power flow in critical transmission line k ($PF_k^{(1)}$ and $PF_k^{(2)}$), and the total power demand ($PD^{(1)}$ and $PD^{(2)}$). These intermediate points are adjusted at 10% and 90 % of the variable range, while the variable membership degree is varied from (1–0). The optimal values of these intermediate points are computed according to their effects for achieving the problem objectives and constraints.

5. RESULTS AND COMMENTS

The proposed fuzzy models are applied for solving the OPD problem for normal operation and emergency conditions. The objective functions for normal conditions are minimizing the generation costs and maximizing the power reserve in the critical transmission lines. For the emergency conditions, the previous objectives should be satisfied and the overflows in transmission lines must be eliminated.

The studied conditions are summarized as follows:

Normal operation

Table 4 shows the proposed OPD results for different fuzzy modeling cases at $\sum_{i=1}^{NG} PG_i$ =268.9 MW. The generation costs are minimized for all different fuzzy models cases compared with conventional LP solution. The maximum reduction in the generation costs was obtained with savings of 48.93 $/hr which occured in cases 5 and 6. While, the minimum reduction in generation cost (40.95 $/hr) occured in cases 3 and 4. The savings in generation costs in cases 1 and 2 was 42.37 $/hr. In cases 7 and 8 the total generation costs decrease by 48.07 $/hr. It is clear that, the fuzzy-based OPD results minimized the total generation costs compared with the conventional LP result for all fuzzy modeling cases. Table 5 shows the corresponding power flows in the critical transmission lines. The overflows in the critical transmission lines are fully removed. These tables present different reserve levels obtained for power generation units and from transmission lines. For example, for Line No. 1, the maximum reserve level for this line occurred at the conventional LP solution (30.454 MW). While, the minimum reserve level (2.383 MW) occurred at cases 5 and 6. Cases 1 to 4, 7, and 8 presented different reserve levels for this line as shown in Table 6. It is clear that, the fuzzy-based OPD results increased the total generation costs in proportional manner to the amount of reserves from critical transmission lines.

Effect of load demand variations

Different studied cases are introduced to discuss the load demand as a judgment in the OPD problem. Tables 6 show the total generation costs of the proposed fuzzy-based OPD model at different loading levels with variant maximum transmission limits of critical transmission lines. Table 6 show that, the trapezoidal representation, i.e. case 6, is the best fuzzy model of load demand over the loading interval between 150 to 300 MW. The best generation costs occurred at case 2 for power demand of 150 MW and are at both cases 4 and 8 at loading point of 200 MW. In cases 2, 4, and 8, the trapezoidal representation of power demand was proposed. The power demands at cases 5 and 6 have equal effects. Case 6 is the preferable one for load representation with trapezoidal model. The proposed fuzzy models lead to minimized total generation costs in a manner less than the generation costs that results from the conventional LP. It is seen that, the main benefit of trapezoidal membership model over the triangular fuzzy model is the good distribution of power generation and power demand.

Effect of transmission bending limit variations

The transmission lines power flow ranges in the model were treated as fuzzy constraints. The decrease in transmission lines limits helped us to model the stressed system cases. Transmission limit variations were presented to show these effects in the stressed cases. Table 7 shows the effects of bending limit variation for transmission lines. In this table, the maximum limit of the power flows in critical transmission lines are allowed to increase from 44 MW to 46, 48, 50, and 52 MW. The increase of bending limit did not have an affect on the fuzzy results. The results of fuzzy-based modeling were obtained by fine-tuning of the generation settings. Then, the fuzzy-based solution is still at the best economic level. The LP solution was improved with increasing the bending level as the generation costs were decreased from 861.41 $/hr to 859.3 $/hr when increasing the bending limit from 46 to 52 MW.

Tables 4-7 lead to the following comments:

1. The proposed method validated for both normal and emergency conditions as system contingencies and increase in power demand tunes the searching of economic generation settings while the power flows in TLs are away from their bending limits.
2. The generation costs are minimized compared with the conventional LP case for all studied fuzzy cases.

3. The use of trapezoidal membership model for the power generations and power demand constraints lead to minimized generation costs. In terms of efficiency, the trapezoidal fuzzy model may be the suitable one for power generation and power demand. The main benefit of trapezoidal membership model over the triangular fuzzy model is the good distribution of power generation and power demand. The trapezoidal membership function achieves the physical operation of generation units. Representation that is more accurate was found based on the trapezoidal model, which suits the modeling of hard constraints of generation units.
4. In terms of efficiency, the trapezoidal fuzzy model may be the suitable membership for power generation and power demand. The main benefit of trapezoidal membership model over the triangular fuzzy model is the good distribution of power generation and power demand. The trapezoidal membership function achieves the physical operation of generation units.
5. Also, in terms of efficiency, the use the triangular fuzzy model in the case of transmission power flows leads to more effective use of transmission lines.
6. The use of triangular model for the power flows in critical lines reduces the total generation cost compared to the use of trapezoidal model.
7. The proposed fuzzy modeling leads to very variety degree in dealing with different power systems variables.
8. The optimal operational settings of these variables within the operational generation range are obtained.
9. Fine-tuning of power system variables reduce the generation cost than the conventional techniques.
10. Different reserve levels from power generation units and critical transmission lines are satisfied corresponding to the fuzzy models of generation and transmission lines fuzzy constraints.

Table 4. Power generations and generation cost of the OPD for different fuzzy models ($\sum_{i=1}^{NG} PG_i$ =268.9 MW)

Variables	Initial Condition	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	LP Solution
PG_1	152.98	114.1	114.1	113.52	113.52	117.1	117.1	116.67	116.67	**85.877**
PG_2	57.56	51.469	51.469	51.32	51.32	51.786	51.786	51.563	51.563	**66.595**
PG_3	24.56	33.122	33.122	33.642	33.642	30.19	30.19	30.513	30.513	**39.28**
PG_4	31.404	32.362	32.362	32.362	32.362	32.358	32.358	32.359	32.359	**31.775**
PG_5	13.404	16.475	16.475	16.22	16.22	17.649	17.649	17.527	17.527	**21.553**
PG_6	16.846	21.368	21.368	21.832	21.832	19.819	19.819	20.273	20.273	**23.82**
Generation Costs $/hr	**818.4352**	**752.93**	**752.93**	**754.35**	**754.35**	**746.37**	**746.37**	**747.23**	**747.23**	**795.3**

Table 5. Power flows in critical lines of the OPD for different membership models ($\sum_{i=1}^{NG} PG_i$ = 269.8 MW)

Lines	Max. Limits	Initial Condition	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	LP Solution
1	75	100.21	69.988	69.988	69.491	69.491	72.617	72.617	72.26	72.26	44.546
2	50	52.54	43.791	43.791	43.694	43.694	44.225	44.225	44.137	44.137	40.502
4	44	49.17	40.601	40.601	40.501	40.501	41.049	41.049	40.959	40.959	37.2
5	50	60.9	46.659	46.659	46.327	46.327	48.412	48.412	48.173	48.173	42.124
6	36	41.49	30.605	30.605	30.448	30.448	31.223	31.223	31.054	31.054	27.432
9	35	35.94	30.253	30.253	30.06	30.06	31.459	31.459	31.37	31.37	28.518
11	10	11.13	6.1291	6.1291	6.1857	6.1857	5.7265	5.7265	5.6989	5.6989	2.4189
14	30	30.5	27.995	27.995	27.796	27.796	28.766	28.766	28.617	28.617	29.363

Table 6. Generation costs of the different modeling cases for different loading points

Load (MW)	Generation costs ($/hr)								
	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	LP solutions
150	390.14	**376.2**	389.12	376.81	384.88	376.95	385.14	376.86	**390.31**
175	446.65	442.11	448.26	**441.41**	448.43	442.11	450.96	**441.41**	**475.59**
200.0	515.59	514.36	518.84	**513.41**	516.94	514.36	521.8	**513.41**	**564.57**
225	591.34	591.34	592.96	591.73	589.38	589.38	590.76	**588.78**	**657.9**
250	683.56	683.56	684.19	684.19	**675.58**	**675.58**	676.4	676.4	**736.34**
268.9	752.93	752.93	754.35	754.35	**746.37**	**746.37**	747.23	747.23	**795.3**
285	817.22	817.22	818.74	818.74	**811.66**	**811.66**	812.13	812.13	**847.09**
295	862.54	862.54	862.75	862.75	**861.3**	**861.3**	861.44	861.44	**882.18**

Table 7. Effects of variant bending limit of critical transmission line no. 4

PF_4 bending limits (MW)	Generation costs $/hr								
	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	LP solution
46	835.18	835.18	835.6	835.6	833.01	833.01	833.29	833.29	861.41
48	835.18	835.18	835.6	835.6	833.01	833.01	833.29	833.29	860.26
50	835.18	835.18	835.6	835.6	833.01	833.01	833.29	833.29	859.67
52	835.18	835.18	835.6	835.6	833.01	833.01	833.29	833.29	859.3

6. CONCLUSIONS

This paper presented an efficient and a reliable procedure to solve optimal power dispatch problem in power systems. The proposed procedure minimized the total generation costs and at the same time, eliminated the overflows in critical transmission lines. Comparison studies between the two linear fuzzy models, trapezoidal and triangular membership models, covering many system conditions, helped the programmer in choosing the best model that the operator may use. Trapezoidal membership for modeling both of power generation and power demand constraints lead to minimized generation cost. While, the use of triangular membership model for modeling the power flows in the critical transmission lines leads to minimized generation cost. The proposed procedure can help the operator to ramp the system constraints corresponding to the amount of reserve requirements. The proposed procedure leads to allocate both responsibility and security action payments to system individuals

NOMENCLATURE

Control Variables

PG_i	generation outputs of unit i (MW).
$PG_i^{\min}$	minimum limit of generation for unit i (MW).
$PG_i^{\max}$	maximum limits of generation for unit i (MW).
PG_i^{med}	a point within the operational range of generation unit i (MW).
$PG_i^{(0)}$	initial power generator output i (MW).
$PG_i^{(1)}$	a point within the operational range of generation unit i (MW).
$PG_i^{(2)}$	a point within the operational range of generation unit i (MW).
NG	number of generation buses.

Dependent Variables

PF_k	power flow in line k (MW).
PD	total power demand (MW).
$F_i(PG_i)$	generation cost of unit i ($/hr).
$a_i, b_i, and\ c_i$	generation cost coefficient of unit i ($/hr).
C	total generation costs of all generation units ($/hr).
$PF_k^{\min}$	minimum limit of power flow in critical line k (MW).
$PF_k^{\max}$	maximum limit of power flow in critical line k (MW).
PF_k^{med}	a point in the operational range of critical line k limits (MW).
$PF_k^{(1)}$	a point in the operational range of critical line k (MW).
$PF_k^{(2)}$	a point in the range of the power flow in critical line k (MW).
NL	number of critical transmission lines.
$PD^{\min}$	minimum limit of permissible load demand (MW).
$PD^{\max}$	maximum limit of permissible load demand (MW).
PD^{med}	intial value of load demand (MW).
$PD^{(1)}$	a point within the loading range of the total system demand unit i (MW).
$PD^{(2)}$	point within the loading range of the total system demand (MW).
$C^{\min}$	minimum permissible generation cost ($/hr).
$C^{\max}$	maximum permissible generation cost ($/hr).

Fuzzy Variables

$\widetilde{PG}_i$	fuzzy active power generation (MW).
$\widetilde{PD}$	fuzzy load demand included power losses (MW).
$\widetilde{PF}_k$	fuzzy active power transmission line flow in line k (MW).
$\mu_{PG_i}(PG_i)$	lower fuzzy membership function for generator i.
$\mu_{PF_k}(PF_k)$	lower fuzzy membership function for critical line k.
$\mu_{PD}(PD)$	lower fuzzy membership function for load demand.
$\mu_C(C)$	fuzzy membership function for objective cost function.
NC	number of fuzzified constraints.

REFERENCES

[1] Alsac, O.; Bright, J.; Prais, M.; and Stott, B. 1990. Further Development in LP-based optimal power flow. *IEEE Transactions. on Power Systems* 5 (3): 697–711.

[2] Stott, B. and Hobson, E. 1978. Power system security control calculations using linear programming, Part I and II. *IEEE Transactions on Power Systems* PAS-97: 1713–1731.

[3] Stott, B.; Alsac, O.; and Monticelli, A. 1987. Security analysis and optimization. *IEEE Special*

Issues on Computers in Power Systems (75): 1623–1644.

[4] Olofsson, M.; Andersson, G.; and Söder, L. 1996. Linear programming based optimal power flow using second order sensitivity. *IEEE Transactions on Power Systems*. (10) 1: 1691–1697.

[5] Lu, C.N. and Unum, M.R. 1993. Network Constrained security control using an interior point algorithm. *IEEE Transactions on Power Systems.* (8) 3: 1068–1076.

[6] Tomsovic, K. 1992. A fuzzy linear programming approach to the reactive power/voltage control problem. *IEEE Transactions on Power Systems.* (7) 1: 287–293.

[7] Abdul–Rahman K.H., and Shahidehpour, S.M. 1993. A fuzzy based optimal reactive power control. *IEEE Transactions on Power Systems.* (8) 2: 662–670.

[8] Xiaohong, G.; Liu, W.H.E.; and Papalexopoulos, A.D. 1995. Application of a fuzzy set method in an optimal power flow. *Electric Power Systems Research.* (34) 1: 11–18.

[9] Abdul–Rahman, K.H. and Shahidehpour, S.M. 1995. Static security in power system operation with fuzzy real load conditions. *IEEE Transactions on Power Systems.* (10) 1: 77–87.

[10] Guan X. and Liu, W.H. 1996. Fuzzy constraints enforcement and control action curtailment in an optimal power flow. *IEEE Transactions on Power Systems.* (11) 2: 639 - 645.

[11] Abou EL–Ela, A.A.; Bishr, M.; Allam, A.S.; and El–Sehiemy, R. 2005. Optimal preventive control actions using multi-objective fuzzy linear programming technique. *Electric Power System Journal (EPSR).* Vol. 74: 147-155.

[12] Abou EL–Ela, A.A.; Bishr, M.; Allam, S.; and El–Sehiemy, R. 2007. Emergency control analysis procedure using multi-objective fuzzy linear programming technique. *International Energy Journal.* (8) 2: 117-124.

[13] Wai Y.N. 1981. Generalized generation distribution factors for power system security evaluations. *IEEE Transaction.* Vol. PAS-100: 1001-1005.

[14] Power Systems Test Case Archive. Washington University, USA. [Online]. Available at: http://www.ee.washington.edu/research/pstca/pf30/pg_tca30bus.htm

[15] Zimmerman, R.D. and Gan, D.D. 1997. MATPOWER version 2.0, a Matlab™ Power System Simulation Package [Online]. Available at: http:// www.pserc.cornell.edu/matpower.