

# **Coulomb's and Franklin's Laws Based Optimization for Nonconvex Economic and Emission Dispatch Problems**

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Abstract – The economic and emission dispatch (EED) problem addresses to minimize the fuel cost as well as the emission from the thermal power plants referring the equality and inequality constraints. Thus, the multi-objective EED problem optimizes the contradicting objectives concurrently. The non-smooth and non-convex fuel cost function such as valve point loading (VPL) effect acts as additional impediment for EED problem. These limitations drive the EED problem to be a highly nonlinear and a multimodal optimization problem. In this article, a new heuristic approach, Coulomb's and Franklin's laws based optimization (CFLBO) algorithm is bestowed to solve the nonconvex economic and emission dispatch problem. The proposed EED considers the non-smooth and nonconvex cost characteristics to ape the VPL effects. The CFLBO approach is concocted from the Coulomb's and Franklin's theories, and comprises attraction /repulsion, probabilistic ionization and contact stages. Applying these CFLBO stages has inflicted in upgrading the robustness and search proficiency of the approach, and substantially lessening the number of generations required to accomplish the optimal solution. The fuel cost and the environmental emission functions are viewed as objective functions and developed as a bi-objective EED problem. The bi-objective EED problem is tackled after converting EED problem to a solitary objective function optimization issue by weighted sum approach with price penalty factors. A fuzzy based concessive approach is employed to choose the best compromised solution from the non-dominated solution sets. To demonstrate its competence, the proposed CFLBO algorithm is employed to 10 and 40-units test systems with nonconvex characteristic. The simulation results signify that the *CFLBO* algorithm affords the best concessive solution and outruns the other compared state-of –the-art approaches.

**Keywords** – combined economic and emission dispatch, economic/emission dispatch, heuristic approach, multiobjective optimization, non-dominated solution.

## 1. INTRODUCTION

## 1.1 Research Motivation

The goal of the multi-objective Combined Economic and Emission Dispatch (CEED) issue is to estimate the best possible power distribution for every generator balancing equally the economic and emission cost meeting the demands and to operate the generator within their capacities. Many countries have developed several strategical schemes to minimize the amount of pollutant ensued from fossil fuel power generation units. These units resulted in producing toxic substances like sulfur dioxide (SO<sub>2</sub>), nitrogen oxides (NO<sub>x</sub>) and carbon dioxide (CO<sub>2</sub>).

Redressing the economic load dispatch (ELD) challenges has a substantial emphasis in the power system's operation, planning, economic scheduling, and security. The non-linear constrained ELD problem is targeted to decrease the electric power generating cost with the optimal setting of concerned generating unit outputs, meeting the demands of whole unit and system limitations. Generally, harmful emissions of fossil fuels are not handled properly by the conventional ELD. So in

<sup>1</sup>Corresponding author: Tel: +918955912345. Email: <u>vp.sakthivel@yahoo.com</u> late trends it is imperative to produce the power with least fuel cost and limit the toxin environment outflow. Considerable decrease in fuel cost could be gotten by the use of present day heuristic advancement approaches for the EED issues. From the above discussions, right now, it has been motivated that the EED issue with nonconvex fuel cost and ecological discharge as targets is unraveled.

## 1.2 Literature Survey

Many techniques have been developed to solve the EED problem with conflicting objectives which can be classified into the following three categories [1].

- The first category addresses the emission as a constraint with admissible limit. However, it refrains to ensure information about the tradeoff front.
- The second category handles the emission as a distinct objective apart from fuel cost objective. However, the EED problem considers single objective at one time to solve the optimization problem employing the linear weighted sum method and the price penalty factor. Hence, such technical proficiencies demand manifold runs to receive a set of mastered output and could not be exploited to locate the Pareto-optimal solutions for the problems redressing the nonconvex Pareto- optimal front.
- The third category deals both the fuel cost and the emission at the same time as competing and complicated objectives.

So far, many optimization approaches such as mathematical programming techniques and heuristic

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algorithms have been employed for addressing and resolving the EED issues. The conventional mathematical optimization approaches such as lambda iteration [2], Newton-Raphson [3], interior point method [4] and quadratic programming [5] have been implemented to tackle ELD and EED problems. The classical calculus-based methods failed to determine a pareto-optimal solution for EED problems due to its high constraints and non-linear features. The conventional approaches are converged prematurely into local optimum solution and sensitive to the initial starting values.

Metaheuristic optimization techniques play a decisive task in mitigating the issues of conventional approaches. Genetic algorithm (GA) [6] simulated annealing (SA) [7], differential evolution (DE) [8], particle swarm optimization (PSO) [9], ant colony optimization (ACO) [10], bacterial foraging algorithm (BFA) [11], harmony search (HS) [12], artificial bee colony (ABC) [13], [14], firefly algorithm (FFA) [15], biogeography based optimization (BBO) [16], cuckoo search (CS) [17], gravitational search algorithm (GSA) [18], bat algorithm (BA) [19], flower pollination algorithm (FPA) [20], backtracking search algorithm (BSA)[21], lightning flash algorithm (LFA) [22] and real coded chemical reaction algorithm (RCCRO) [23] have been employed to solve the CEED problem. Nevertheless, some of these approaches endure precise parameter settings and high computational effort.

Many researchers have developed multiobjective evolutionary approaches. The non-dominating sorting GA (NSGA) [24], multiobjective PSO [25], multiobjective differential evolution (MODE) [26], multiobjective quasi-oppositional teaching learning based optimization (OOTLBO) [27] and enhanced multiobjective cultural algorithm (EMOCA) [28] have been applied for solving the EED problems. Hybrid heuristic algorithms have been introduced to solve the ELD and EED problems in order to accomplish the preeminent features and performances of different algorithms [29], [30]. Maity et al. introduced bare bone TLBO (BB-TLBO) for solving EED problem addressing VPL impact and transmission losses [31]. Bhargava and Yadav proposed hybrid technique using DE and crow search algorithm (DE-CSA) for solving the EED approach for smart grid system [32]. Nevertheless, these algorithms suffer from high computational complexities. The comprehensive literature review of heuristic approaches based EED issues are summarized in Table 1.

#### 1.3 Contributions

In this paper, Coulomb's and Franklin's laws based optimization (CFLBO) [33] is proposed to solve the EED issues. The principle contributions of this paper are recorded as follows:

τ	Heuristic approach	Reference	Non-linear characteristics				
п	neuristic approach	Reference	Transmission losses	Prohibited operating zones			
	C A	[7]	V	N.			

Table 1. Comprehensive literature review of EED solving based on heuristic approaches.

Heuristic approach Reference		Non-inteal characteristics					
rieuristic approach	Reference	Transmission losses	Prohibited operating zones	VPL impacts			
SA	[7]	Yes	No	Yes			
DE	[8]	Yes	No	No			
ACO	[10]	Yes	No	No			
BFA	[11]	Yes	No	Yes			
HS	[12]	Yes	No	No			
ABC	[13]	Yes	No	Yes			
FFA	[15]	Yes	Yes	Yes			
BBO	[16]	Yes	No	No			
CS	[17]	Yes	No	Yes			
GSA	[18]	Yes	No	Yes			
BA	[19]	Yes	Yes	No			
FPA	[20]	Yes	No	Yes			
BSA	[21]	Yes	Yes	Yes			
LFA	[22]	Yes	No	Yes			
RCCRO	[23]	Yes	No	Yes			
NSGA	[24]	Yes	No	Yes			
MOPSO	[25]	Yes	No	No			
MODE	[26]	Yes	Yes	Yes			
QOTLBO	[27]	Yes	No	Yes			
EMOCA	[28]	Yes	Yes	Yes			
ABC-PSO	[29]	Yes	No	Yes			
Hybrid GA	[30]	Yes	Yes	Yes			
BB-TLBO	[31]	Yes	Yes	Yes			
DE-CSA	[32]	Yes	No	No			
CFLBO	[33]	No	No	No			
Fuzzified CFLBO	Suggested approach	Yes	No	Yes			



Fig. 1. Schematic overview of the suggested CFLBO based EED approach.

- A new physics inspired meta-heuristic optimization approach known as CFLBO which is used to solve a multi-objective EED optimization problem having multifaceted non-convex characteristics with intense equality and inequality constraints is proposed. The performance of CFLBO is improved in the accompanying aspects compared with the existing heuristic approaches.
  - So as to expand the learning capacity of populace, the attraction/repulsion strategy is acquainted to update the position of each individual.
  - In CFLBO, each dimension in current solution can be refreshed independently because of the ionization probability. This probabilistic ionization phase improves the global search ability and quickens the convergence speed of the suggested approach.
  - To prevent premature convergence and increase the diversity of populace, the probabilistic contact phase is adopted in the algorithm.
- ii) The fuzzy decision making approach is employed in the CFLBO approach to choose the best compromise solution of fuel cost and emission.
- iii) In order to fortify the felicitousness of the proposed CFLBO algorithm, two power systems including 10 and 40 generating units are considered and the results are compared with the other heuristic optimization techniques (HOTs) stated in recent literature.

The schematic overview of the CFLBO based EED approach is displayed in Figure 1.

#### 1.4 Organization of the Research Manuscript

The remainder of this paper is composed as follows. Section 2 describes the formulation of EED issue including constraints. Sections 3 and 4 explore the CFLBO algorithm and fuzzy based concessive approach for nonconvex problem. The application of CFLBO approach to deal with the EED issue is proposed in Section 5. Section 4 gives the case studies of the 10-unit and 40-unit test systems, and demonstrates the effectiveness of CFLBO in managing the EED issues compared with other heuristic approaches. Section 5 abridges several conclusions and gives some future research areas.

#### 2. FORMULATION OF THE NONCONVEX ECONOMIC AND EMISSION DISPATCH

The goal of the EED problem is to find an optimal power generation schedule while minimizing fuel costs and emissions simultaneously.

#### 2.1 Objectives

#### 2.1.1 Economic load dispatch

The problem with ELD is formulated as follows:

$$Minimize \ F = \sum_{i=1}^{ng} F_i(P_i) \tag{1}$$

The generator's quadratic fuel cost function is defined by:

$$F_{i}(P_{i}) = a_{i} + b_{i}P_{i} + C_{i}P_{i}^{2}$$
(2)

The sequential valve opening in multi-valve steam turbines generates rippling effect on the fuel cost curve of the generator. To model an accurate and practical ELD solution, this VPL effects should be included in the fuel cost function. Then the fuel cost function of each generating unit is expressed in the non-convex form as follows:

$$F_i(P_i) = a_i + b_i P_i + C_i P_i^2 + \left| d_i \times sin\left(e_i \times \left(P_{i,min} - P_i\right)\right) \right|$$
(3)

Figure 2 displays the fuel cost curve with and wiithout VPL impacts.

#### 2.1.2 Economic emission dispatch

The thermal power plants release emissions such  $NO_x$  to the atmosphere while burning the fossil fuels. The emission of these pollutants can be illustrated as the sum of quadratic and exponential functions as follows:

$$Minimize \ E = \sum_{i=1}^{ng} E_i(P_i)$$
(4)

The generator's quadratic emission function with VPL effects is defined by:

$$E_i(P_i) = \alpha_i + \beta_i P_i + \gamma_i P_i^2 + \eta_i \exp(\delta_i P_i)$$
 (5)



#### Fig. 2. Fuel cost curve.

### 2.1.3 Economic and emission dispatch

The EED problem can be formulated as bi-objective function in which the fuel cost and the emission as rivaling objectives. This bi-objective function can be transferred to a single objective function as follows:

$$Minimize \ F_{EED} = w \times F + h \times (1 - w) \times E \tag{6}$$

The above equation becomes ELD objective function when w = 1 and becomes EED objective function when w = 0. w is a main function of rand [0,1] which compromises the fuel cost and emission objectives.

The price penalty factor (PPF) is expressed as follows:

$$h_i = \frac{F_i(P_{i,max})}{E_i(P_{i,max})} \tag{7}$$

The accompanying advances are utilized to determine the PPF value for a specific load demand:

- i. Estimate the proportion between most extreme fuel cost and greatest discharge of every generator.
- ii. Orchestrate the estimations of PPF in ascending manner.

- iii. Include the greatest limit of every unit ( $P_{i,max}$ ) each in turn, beginning from the littlest  $h_i$ , until  $\sum P_i \ge P_D$ .
- iv. Now,  $h_i$  which related with the last unit right now is the approximate PPF for the given load.

#### 2.2 System Constraints

#### 2.2.1 Power balance constraints

The generators' power output must be equal to the sum of power requirements and complete transmission losses and is provided by:

$$\sum_{i=1}^{ng} P_i = P_D + P_L \tag{8}$$

The transmission loss is expressed as:

$$P_L = \sum_{i=1}^{ng} \sum_{j=1}^{ng} P_i B_{ij} P_j + \sum_{i=1}^{ng} B_{0i} P_i + B_{00}$$
(9)

#### 2.2.2 Generator Capacity Constraints

Each unit's output power needs to be restricted by limiting inequality between its limits. This constraint is represented by:

$$P_{i,min} \le P_i \le P_{i,max} \tag{10}$$

#### **3. CFLBO ALGORITHM**

CFLBO is a metaheuristic algorithm which is introduced by Ghasomi *et al.* in 2018 [33]. This algorithm simulates the Coulomb's and Franklin's theories.

The following concepts of laws are utilized in the CFLBO algorithm.

*Coulomb's Law:* The relationship between two different point charges is determined by the magnitude of electrostatic force of attraction (or) repulsion.

*Franklin's Law:* Each object consists of equal positive and negative charges.

CFLBO algorithm uses different objects (populations) of points charges (X) which moves around different areas in an exploring space to recognize the global optimum solution. The initial objects are formed by various groups of point charges are randomly generated in the Search space. Each point charge comprised of D quantized charges x and each point charge corresponds to a candidate solution of the problem.

The mathematical model of CFLBO is a repetitious process, which comprises four phases, namely:

- Initialization phase
- Attraction / repulsion phase
- Probabilistic ionization phase
- Probabilistic contact phase

## 3.1 Initialization Phase

Consider an object formed by a population of m charges with dimension D. The objects, populations and each individual are represented by:

$$O = [O_1, O_2, \dots O_n]$$

$$X = \begin{bmatrix} X_1, X_2, \dots X_m \end{bmatrix}$$
$$X_{ij} = \begin{bmatrix} x_{i1}, x_{i2}, \dots x_{iD} \end{bmatrix}$$

The initial populations of point charges are generated as follows:

$$x_{ij} = U\left(x_j^{min}, x_j^{max}\right) \tag{11}$$

for 
$$i = 1, 2, ..., m$$
 and  $j = 1, 2, ..., D$ 

where U is a vector of uniformly distributed random numbers between  $x_j^{min}$  and  $x_j^{max}$ .

Then, the initial population is sorted and distributed into several objects  $(O_1, \dots, O_n)$ .

#### 3.2 Attraction / Repulsion Phase

The displacement of point charge is influenced by attraction and repulsion forces acting on them. The net force acting on a point charge  $(X_i)$  is equal to its cost value  $(F_i)$ . The CFLBO algorithm is used to minimize the net force (cost) acting on them. For each object, the location of point charges is updated by

$$x_{j}^{new} = x_{ij}^{old} + \left|\cos\theta_{j}^{new}\right|^{2} \times \left(x_{j}^{best} - x_{j}^{Worst}\right) + \left|\sin\theta_{j}^{new}\right|^{2} \times \left(mean \begin{pmatrix}a_{max} \\ \sum \\ n=1 \end{pmatrix} x_{jn} - mean \begin{pmatrix}r_{max} \\ \sum \\ n=1 \end{pmatrix} x_{jn} \end{pmatrix}$$
(12)

where,  $\theta_{j}^{initial} = U(0, 2\pi)$ 

$$\theta_j^{new} = \theta_j^{old} + U\left(0, \frac{3}{2}\pi\right)$$

The  $a_{max}$  and  $r_{max}$  are determined by the following equations:

$$a_{max} = a_0 \times (1 + \cos \theta) \tag{13}$$

$$r_{max} = r_0 \times (1 - \cos \theta) \tag{14}$$

#### 3.3 Probabilistic Ionization Phase

Due to the influence of probabilistic ionization energy, there is a possibility in the displacement of location of elementary charge  $x_j$  and can be mathematically modelled by the following equation.

$$x_{j}^{new} = x_{j}^{Best} + x_{j}^{Worst} - x_{j}^{old} \text{ if }$$

$$rand(i) \le p_{i}$$
(15)

The control variable 'j' is chosen as

$$j = round(unifrnd(1, D))$$
(16)

where, rand (*i*) is the  $i^{\text{th}}$  point charge of a uniform random number generation within [0, 1].

#### 3.4 Probabilistic Contact Phase

If the objects are in contact with each other, then each object passes its best and worst point charges to its neighbour. The probabilistic contact phase is modelled as follows:



Fig. 3. Flowchart of CFLBO algorithm.

If  $rand_c \leq p_c$ , then

$$x_{j}^{Best_{Obj1}} = x_{j}^{Best_{Objn}}, \dots x_{j}^{Best_{Objn}} = x_{j}^{Best_{Objn-1}}$$
(17)  
$$x_{j}^{Worst_{Obj1}} = x_{j}^{Worst_{Objn}}, \dots x_{j}^{Worst_{Objn}} = x_{j}^{Worst_{Objn-1}}$$

where,  $rand_c$  is uniform number generation within [0, 1]. The processes of CFLBO are shown in Figure 3.

#### 4. FUZZY BASED CONCESSIVE APPROACH FOR NONCONVEX EED PROBLEM

In multi-objective economic emission dispatch problem, the two objective functions namely, economic and emission dispatch functions are to be simultaneously considered and consequently it is tricky to compare two solutions. If solution vector  $X_1$  and  $X_2$  are Pareto optimal, then neither set of vectors must be superior to other. It is because if  $X_1$  offers superior result for one objective then, X2 would provide better result for the other. One and the other sets are rivaling or nondominating solutions in nature. In multi-objective EED problem, it is difficult to find the best solution from many non-dominated solutions. In order to compare these outcomes and get the best compromised solution, a certain mechanism is essential to combine both the objectives in conformity with the decision maker's preference.

Fuzzy set theory is repeatedly used by researchers to get the best compromised solution from many uncontrolled solutions. As both the targets of fuel cost and emission are contrary inherently, it is not feasible to get the least fuel cost and to attain the least emission at the same time. But it is feasible and practicable to get a dispatch option that can reduce both fuel cost and emission as far as possible. Degree of agreement (DA) to each objective is assigned by fuzzy membership functions, where DA reflects the merit of their objective in a linear scale of 0 - 1 (worst to best). If  $F_j$  is a solution in the Pareto-optimal set in the *j*th objective function and is represented by a membership function as,

$$F_{i} = \begin{cases} 1 & \text{if } F_{j} \leq F_{j}^{min} \\ F_{j}^{max} - F_{j} & \text{if } F_{i}^{min} \leq F_{i} \leq F_{i}^{max} \end{cases}$$
(1)

$$\mu(F_j) = \left\{ \begin{array}{l} \frac{1}{F_j^{max} - F_j^{min}} & \text{if } F_j^{max} \le F_j \le F_j^{max} \\ 0 & \text{if } F_j \ge F_j^{max} \end{array} \right\}$$
(18)

For each non-dominated solution, the normalized membership function  $\mu_D^k$  can be calculated as,

$$\mu_{D}^{k} = \frac{\sum_{i=1}^{2} \mu(F_{i}^{k})}{\sum_{k=1}^{D} \sum_{i=1}^{2} \mu(F_{i}^{k})}$$
(19)

The solution that contains the maximum of  $\mu_D^k$  based on cardinal priority ranking is the best compromised solution.

$$Max\left\{\mu_D^k: k=1,2,\dots,M\right\}$$
(20)

## 5. APPLICATION OF CFLBO ALGORITHM TO NONCOVEX EED PROBLEM

The step by step procedure of CFLBO algorithm applied to solve EED problem is described as follows:

#### 5.1 Representation of the Point Charge $(x_i)$

Since the optimization of variables for EED problem are real power outputs of the generators, they are represented by individual point charge. For EED problem, each point charge is presented as:

$$X_{i} = [x_{i1}, x_{i2}, \dots, x_{iD}] = P_{ij} = [P_{i1}, P_{i2}, \dots, P_{ing}]$$
  
Where *j*=1, 2, ..., *n*<sub>g</sub>

#### 5.2 Initialization of the Point Charge

Each individual of the object matrix, *i.e.*, each quantized element x of a given point charge set X, is generated randomly within the lower and upper limits of power generations.

#### 5.3 Evaluation of Net Acting Force

In nonconvex EED problem, the net acting force of each point charge set is represented by the total fuel cost of generation and emission for all the generators.

The steps of CFLBO algorithm to solve nonconvex EED problem are given below.

- Step 1. Read the number of generators units  $(n_g)$ , number of objects and point charges, population size, maximum iteration number (iter<sub>max</sub>), minimum and maximum capacities of each generator, power demand, fuel and emission coefficients and the CFLBO parameters ( $a_0$  and  $r_0$ ).
- Step 2. Initialize the iteration counter and the weight factor *W* as zero.
- Step 3. Initialize each quantized element of a given point charge set of  $x_i$  matrix and satisfy the equality power balance constrains of each point charge set in  $x_i$  matrix.
- *Step 4.* Calculate the objective value (net acting force) for each point charge set of all objects using Equation 6.
- Step 5. Identify the best and worst point charge set of each object based on the objective values.
- Step 6. Update the location of each point charge set using Equation 12.Generate random numbers rand  $(i) \in [0, 1]$ . If rand (i) is lesser than ionization probabilistic constant  $P_i$ , select any quantized element randomly of the  $i^{th}$  point charge and relocate its location using Equation 15.
- Step 7. Authenticate the viability of each newly generated point charge set. Each quantized element of the modified point charge set must satisfy the operating limits and power balance constraints. If any quantized element violates any of the operating limits, then fix its corresponding limit value.

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- *Step 8.* Evaluate the objective value for the new point charge set using Equation 6 and update the best and worst point charge set of all objects.
- Step 9. Generate a random number  $rand_c \in [0,1]$ . If  $rand_c \leq P_{i}$ , then move the best and worst point charge set of each object to its adjacent object by Equation 16.
- Step 10. Repeat steps 6 -10 until stopping criterion is not met.
- Step 11. Increment the weight factor in step of 0.5 and repeat step 6-11, until the weight factor reaches unity.
- Step 12. Best compromising solution: Determine the membership value for each non-dominated

solution sets which are acquired for different weight factors using Equation 18. The point charge set that procures maximum membership value is chosen as the best compromising solution for the EED problem.

## 6. CASE STUDIES

To show the effectiveness of the proposed CFLBO algorithm, two case studies with nonconvex fuel cost functions are considered for solving the EED problems and compared with various HOTs available in the literature.

Table 2. Comparison of the best	economic and	environmental solutions obtained by	various <b>H</b>	HOTs for 10-unit system.	•
			_		

Unit	Best eco		omic solution		Best environmental solution			
(MW)	EMOC [28]	RCCRO [23]	BSA [21]	CFLBO	EMOC [28]	RCCRO [23]	BSA [21]	CFLBO
P <sub>1</sub>	55.00	55.00	55.00	54.535565	55.00	55.00	55.00	54.842943
$P_2$	80.00	79.99	80.00	78.329740	80.00	80.00	80.00	79.764640
P <sub>3</sub>	109.42	106.92	106.93	107.650316	76.60	81.13	81.13	79.731052
$P_4$	93.23	100.54	100.57	102.665828	81.52	81.36	81.36	81.364232
<b>P</b> <sub>5</sub>	80.51	81.52	81.50	82.390970	160.00	160.00	160.00	158.275709
$P_6$	91.17	83.05	83.02	83.050644	240.00	240.00	240.00	239.259623
$\mathbf{P}_7$	300.00	299.99	300.00	299.614028	300.00	294.48	294.48	294.571044
$P_8$	337.65	339.99	340.00	339.265905	293.05	297.27	297.27	299.699395
P <sub>9</sub>	470.00	469.99	470.00	469.420855	398.87	396.76	396.76	395.045105
$\mathbf{P}_{10}$	470.00	469.99	470.00	469.377327	396.61	395.57	395.57	397.431551
Total generation	2086.97	2087.03	2087.0388	2086.301178	2081.64	2081.59	2081.5952	2079.985295
Cost (\$/h)	111,509.4	111,497.632	111497.631	111480.8469	116,418.8	116412.444	116412.444	116237.5234
Emission (lb/h)	4528.08	4571.9552	4572.1939	4572.464947	3934.54	3932.2433	3932.2433	3931.128912

Table 3. Comparison of the best concessive solutions for EED obtained by various HOTs for 10-unit system.

Unit (MW)	GSA [18]	EMOC [28]	RCCRO [23]	TLBO [27]	QOTLBO [27]	LFA 22]	CFLBO
$\mathbf{P}_1$	54.9992	55.0000	55.0000	55.0000	55.0000	54.9920	54.136951
$P_2$	79.9586	80.0000	80.0000	80.0000	80.0000	78.7689	79.419606
<b>P</b> <sub>3</sub>	79.4341	83.5594	85.6453	83.9202	84.8457	87.7168	81.254051
$P_4$	85.0000	84.6031	84.1259	82.8342	83.4993	78.1055	79.679627
<b>P</b> <sub>5</sub>	142.1063	146.5632	136.5034	132.0131	142.9210	140.6272	137.906152
P <sub>6</sub>	166.5670	169.2481	155.5801	173.9880	163.2711	157.0936	158.599723
$\mathbf{P}_7$	292.8749	300.0000	300.0000	299.7099	299.8066	299.9954	296.662357
$P_8$	313.2387	317.3496	316.6746	317.9684	315.4388	309.2219	321.681429
P <sub>9</sub>	441.1775	412.9183	434.1252	427.0166	428.5084	439.3243	441.671891
$P_{10}$	428.6306	434.3133	436.5724	431.3955	430.5524	438.6947	431.462758
Cost (\$/h)	113490	113444.85	113355.7454	113471	113460	1132460	113124.857889
Emission (lb/h)	4111.4	4113.98	4121.0684	4113.5	4110.2	4139.89	4148.996574
FCPI	40.54	39.4227	37.81	40.1262	39.9267	-	34.5622
ECPI	28.01	30.2322	29.52	28.7276	27.9654	-	33.9709
Difference	12.53	9.1906	8.29	11.3986	11.9613	-	0.5913

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The CFLBO algorithm is implemented in Matlab 7.1 and executed on an Intel core i3 processor with 4GB RAM personal computer. The proposed approach is executed for 20 independent trials on each case study to appraise the solution quality and convergence characteristics. The number of objects, population size and maximum iteration number of CFLBO algorithm are chosen as 5, 20 and 100 respectively.

#### 6.1 Case Study 1

A 10-unit system with VPL effects and NO<sub>x</sub> emission are considered. The input data for this test system is described in Appendix A and the load demand is assumed as 2000 MW. Table 2 summarizes the results for solving the fuel cost minimization and emission minimization independently by the proposed CFLBO algorithm, EMOC, RCCRO and BSA approaches. The CFLBO approach reduces the cost by 28.58 \$/h, 16.79 \$/h 16.78 \$/h for fuel cost minimization and the emissions by 181.307 lb/h, 174.92 lb/h and 174.92 lb/h for emission minimization in comparison with EMOC [28], RCCRO [23] and BSA [21] respectively.

The performance indices of CEED problem such as fuel cost performance index (FCPI) and emission cost performance index (ECPI) are ascertained as follows:

$$FCPI = \frac{F_{bcs} - F_{min}}{F_{max} - F_{min}} \times 100$$
(20)

$$ECPI = \frac{E_{bcs} - E_{min}}{E_{max} - E_{min}} \times 100$$
(21)

Table 3 outlines the comparison of best concessive solutions for CEED obtained by GSA [18], EMOC [28], RCCRO [23], TLBO [27], QOTLBO [27], LFA [22] and CFLBO approaches. From the table, it is clear that the CFLBO approach gives lesser performance indices deviation and better concessive solution. The fuel cost and emission convergence behaviors of the suggested CFLBO approach for CEED problem are shown in Figures 4 and 5, respectively. It is clear that the proposed CFLBO algorithm converges to its global best solution (fuel cost and emission) in less number of iterations. Figure 6 illustrates the Pareto optimal fronts (POF) acquired by the suggested approach. The results obviously transpire that the obtained solutions are very much disseminated and secured the whole Pareto front of the CEED issue.



Fig. 4. Fuel cost convergence behavior of CFLBO approach for case study 1.



Fig. 5. Emission convergence behavior of CFLBO approach for case study 1.

Unit	Best economic solution			vironmental lution	Best economic and environmental solution			
(MW)	BFA [11]	CFLBO	BFA [11]	CFLBO	GSA [18]	MODE [26]	TLBO [27]	CFLBO
P <sub>1</sub>	114.0000	110.580925	114.0000	114.000000	113.9989	113.5295	114.0000	110.4068
$P_2$	110.8035	110.805109	114.0000	114.000000	113.9896	114.0000	114.0000	113.4505
P <sub>3</sub>	97.4002	97.662057	120.0000	120.000000	119.9995	120.0000	91.9893	108.4061
$P_4$	179.7333	179.614740	169.3671	169.217094	179.7857	179.8015	177.4467	177.7379
$P_5$	87.8072	87.991188	97.0000	97.000000	97.0000	96.7716	97.0000	88.3691
$P_6$	140.0000	140.000000	124.2630	124.150800	139.0128	139.2760	140.0000	121.9143
$P_7$	259.6004	259.827209	299.6931	299.779034	299.9885	300.0000	300.0000	285.3091
$P_8$	284.6002	284.675862	297.9093	297.920586	300.0000	298.9193	283.7368	299.3117
P <sub>9</sub>	284.6006	284.248949	297.2578	297.196340	296.2025	290.7737	300.0000	289.0739
P <sub>10</sub>	130.0000	130.000000	130.0007	130.000000	130.3850	130.9025	130.0000	130.0000
P <sub>11</sub>	168.7999	94.000000	298.4210	298.438971	245.4775	244.7349	318.1965	240.4698
P <sub>12</sub>	168.7998	94.000000	298.0264	298.045747	318.2101	317.8218	241.5727	243.3303
P <sub>13</sub>	214.7598	214.800523	433.5590	433.617924	394.6257	395.3846	391.9916	395.5716
P <sub>14</sub>	304.5195	394.052534	421.7360	421.746907	395.2016	394.4692	394.4501	395.2566
P <sub>15</sub>	394.2794	394.410721	422.7884	422.899795	306.0014	305.8104	394.3549	394.2189
P <sub>16</sub>	394.2794	394.841086	422.7841	422.765580	395.1005	394.8229	394.0597	396.0000
P <sub>17</sub>	489.2794	489.177253	439.4078	439.311814	489.2569	487.9872	490.5281	447.4039
P <sub>18</sub>	489.2794	489.215035	439.4132	439.466355	488.7598	489.1751	484.2049	495.1025
P <sub>19</sub>	511.2795	511.286120	439.4111	439.458462	499.2320	500.5265	423.9535	478.8628
P <sub>20</sub>	511.2795	511.350412	439.4155	439.704233	455.2821	457.0072	507.3859	424.4995
P <sub>21</sub>	523.2794	523.371235	439.4421	439.527102	433.4520	434.6068	438.5029	499.9355
P <sub>22</sub>	523.2794	523.378942	439.4587	439.555181	433.8125	434.5310	433.6163	512.4599
P <sub>23</sub>	523.2796	523.194564	439.7822	439.667526	445.5136	444.6732	434.1238	500.6126
P <sub>24</sub>	523.2794	523.214651	439.7697	439.734941	452.0547	452.0332	446.0748	456.7811
P <sub>25</sub>	523.2795	523.478172	440.1191	440.277492	492.8864	492.7831	437.2666	440.8122
P <sub>26</sub>	523.2796	523.286486	440.1219	440.118508	433.3695	436.3347	433.3886	438.6621
P <sub>27</sub>	10.0001	10.000000	28.9738	28.899275	10.0026	10.0000	10.2118	10.9679
P <sub>28</sub>	10.0002	10.000000	29.0007	28.845728	10.0246	10.3901	11.1608	10.4538
P <sub>29</sub>	10.0002	10.000000	28.9828	28.822147	10.0125	12.3149	10.2531	10.4108
P <sub>30</sub>	89.5070	87.735345	97.0000	97.000000	96.9125	96.9050	97.0000	89.5072
P <sub>31</sub>	190.0000	190.000000	172.3348	172.209561	189.9689	189.7727	190.0000	183.3655
P <sub>32</sub>	190.0000	190.000000	172.3327	172.270108	175.0000	174.2324	190.0000	183.0703
P <sub>33</sub>	190.0000	190.000000	172.3262	172.347423	189.0181	190.0000	190.0000	173.0104
P <sub>34</sub>	164.8026	164.708636	200.0000	200.000000	200.0000	199.6506	200.0000	199.7548
P <sub>35</sub>	164.8035	194.045083	200.0000	200.000000	200.0000	199.8662	200.0000	199.4690
P <sub>36</sub>	164.8292	200.000000	200.0000	200.000000	199.9978	200.0000	200.0000	199.3909
P <sub>37</sub>	110.0000	110.000000	100.8441	100.956032	109.9969	110.0000	110.0000	105.0895
P <sub>38</sub>	110.0000	110.000000	100.8346	100.826994	109.0126	109.9454	110.0000	96.2228
P <sub>39</sub>	110.0000	110.000000	100.8362	100.932138	109.4560	108.1786	110.0000	96.33841
P <sub>40</sub>	511.2795	511.059386	439.3868	439.333571	421.9987	422.0682	459.5306	458.0239
Cost (\$/h)	121415.65	121414.8434	129995.0	129995.4326	125782	125792	125602	125404.06
Emission (ton/h)	356424.5	357404.9693	176682.3	176681.9764	210932.9	211190	206648.3	229799.4

Table 4. Comparison of the best economic, environmental, and combined economic and environmental solutions obtained by various HOTs for 40-unit system.

#### 6.2 Case Study 2

In this case study, the larger test system of 40-unit is considered to test the effectiveness of the proposed CFLBO algorithm for solving the EED problem. The cost and emission coefficients with generators limits are given in Appendix B. The power demand is 10500 MW.

The optimal scheduling results of the CFLBO algorithm are compared to BFA for best economic/environmental situations in Table 4. As observed in Table 4, the CFLBO reduces the fuel cost and  $NO_x$  emissions than the BFA approach [11]. The

best concessive solutions obtained by GSA [18], MODE [26], TLBO [27] and CFLBO are also provided in the same Table. It can again be dissected that the proposed CFLBO approach is proficient of finding the best compromise non-dominated solutions by successfully solving the EED problem. Nevertheless, EED performance indices by the aforementioned approaches are given in Table 5. It is figured out that CFLBO achieves lower deviation between the FCPI and ECPI corroborating its consistency and supremacy with other HOTs in solving the multi objective EED problem.



Fig. 6. POF curve of CFLBO approach for case study 1.



Fig. 7. Fuel cost convergence behavior of CFLBO approach for case study 2.







150000 121414.8 123414.8 125414.8 127414.8 129414.8 Fuel cost (\$/h)

Fig. 9. POF curve of CFLBO approach for case study 2.

Table 5. Comparison of performance indices obtained by various HOTs for 40-unit system.
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200000

Performance indices	GSA [18]	MODE [26]	TLBO [27]	CFLBO
FCPI	50.8866	51.0031	47.3812	46.4912
ECPI	18.6922	18.8341	15.9463	29.3916
Difference	32.1943	32.1690	31.4348	17.0996

The convergence behaviors of fuel cost and emission minimizations are depicted in Figures 7 and 8 respectively. It is worth noting that the CFLBO approach converges swiftly. The CFLBO approach acquires optimal solutions at iterations 28 and 37 for fuel cost and emission minimizations respectively. The POF curve procured by the proposed approach is viewed in Figure 9. It leads to the conclusion that the proposed approach is competent for determining the Pareto front by adequately tackling the issue when all the imperatives are addressed.

## 6.3. Comparison of Computational Effect and Solution Quality

The comparison of computation efficiencies acquired by the TLBO and CFLBO are shown in Figure 10. From Figure 10, it is obvious that the CPU time of the CFLBO is lesser in comparison with the TLBO approach.

The statistical performances of CFLBO algorithm for 20 independent trials are presented in Table 6. It can be evident that the occurrence of attaining the best solutions is about 87.5%. Thus the CFLBO algorithm is more robust and stable in accomplishing the best compromise solutions.

#### 6.4 Multi-objective Performance Indicators

In order to dissect the quality of the suggested approach, the two distinctive multi-objective performance indicators, the ratio of non-dominated individuals (RNI) and spacing metric (s-metric) are assessed. RNI is defined as the proportion of number of non-dominated solutions for the populace size. The higher the RNI measure, the better the solution quality. The s-metric estimates the distance between the variance of neighboring points in the POF curve. The lower the spread value, the better the dissemination of solutions.



algorithm for different test systems.

#### Table 6. Statistical analysis of CFLBO approach for EED problem.

Case study	Best economic solution	Best environmental solution	No. of hits to optimal solution
1	113124.858	4148.9966	17
2	125404.063	229799.3845	18



Fig.11. Comparison of RNI for the test systems.



Fig. 12. Comparison of s-metric for the test systems.

problem with contending fuel cost and ecological effect

The RNI and S-metric indicators are determined for 20 independent trials which are shown in Figures 11 and 12, respectively. From the figures, it is indeed obvious that the suggested approach is proficient of delivering well RNI index and spacing between points on the POF curve.

#### 7. CONCLUSION

A new heuristic approach based on Coulomb's and Franklin's laws based optimization (CFLBO) algorithm has been bestowed for solving the economic and emission dispatch problem with non-smooth and nonconvex characteristics. More complex fuel cost characteristic such as VPL impacts are addressed. The EED issue is detailed as a bi-objective optimization destinations. The bi-objective problem is transferred into single objective function by weighted sum approach with price penalty factor. The fuzzy based concessive approach is employed to choose the best compromised solution from the non-dominated solution sets. To test the performance of the proposed CFLBO algorithm, 10unit and 40-unit test systems have been favored. Simulation results show that the CFLBO approach is competent of offering a better concessive solution for the EED problem. The non-dominated solutions acquired by the suggested approach are all around dispersed and have great convergence attributes. The fuzzy concessive strategy adopted in the suggested CFLBO approach comprehends the EED issue with low

emanation. Nevertheless, the EED performance indices namely FCPI and ECPI are ascertained for the test systems which elucidate the aptness of the proposed CFLBO algorithm. Accordingly, CFLBO approach is a propitious approach for tackling the confounded power system optimization problems. The future work is dedicated to tackle the multi-area ELD with multi-fuel alternatives and hybrid multi-area power system optimization issues because of its promising exhibitions.

#### **NOMENCLATURE**

$F_i$	fuel cost of the generator <i>i</i>	
$a_{i}$ , $b_{i}$ , $c_{i}$	cost coefficients of generator i	[4]
$n_g$ $d_i$ , $e_i$	total number of generating units cost coefficients of the VPL effect of generator <i>i</i>	
$E_i$	emission of the generator <i>i</i>	[5]
$\alpha_{_i}$ , $\beta_{_i}$ , $\gamma_{_i}$	emission coefficients of generator i	
$\eta_{_i}$ , $\delta_{_i}$	emission coefficients of the VPL effect of generator <i>i</i>	[6]
h	price penalty factor in \$/h	
W	weight or compromise factor	[7]
$P_D$	power demand	
$P_L$	transmission losses	
$\mathbf{B}_{ij}$	line loss coefficients	
$P_{i,min}$ , $P_{i,max}$	minimum and maximum generation of unit <i>i</i>	[8]
k	index of prohibited zone	
nz	total number of POZs	501
$P_{i,k}^L$ , $P_{i,k}^U$	lower and upper power outputs of the <i>k</i> th prohibited zone of the <i>i</i> th generator	[9]
n	maximum number of objects	
m x <sub>ij</sub>	population size of each object <i>j</i> <sup>th</sup> elementary charge of the <i>i</i> <sup>th</sup> point charge	[10
$x_{i}^{min}$ and $x_{i}^{max}$	lower and upper limits of variable <i>j</i>	
$a_{max}$ and $r_{max}$	maximum number of positive and negative charges respectively	[11
$a_0$ and $r_0$	initial values for positive and negative charges respectively	
$p_i$	ionization probabilistic constant.	F1 <b>2</b>
$P_c$	contact phase probabilistic constant	[12
$F_j^{max}$ and $F_j^{min}$	maximum and minimum values of <i>j</i> th objective function respectively	
М	number of non-dominated solutions	[13
$F_{\rm bcs}$ and $E_{\rm bcs}$	fuel cost and emission attained by CEED	-
$F_{\min}$ and $E_{\max}$	fuel cost and emission attained by ELD minimization, respectively	
$F_{\rm max}$ and $E_{\rm min}$	fuel cost and emission attained by EED minimization, respectively	[14

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APPENDIX 2	A
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Unit	P <sub>i</sub> <sup>min</sup> MW	$P_i^{max}$ <b>MW</b>	a <sub>i</sub> (\$/h)	b <sub>i</sub> (\$/MWh)	c <sub>i</sub> (\$/MWh)	d <sub>i</sub> (\$/h)	e <sub>i</sub> (rad/MW)	α <sub>i</sub> (lb/h)	β <sub>i</sub> (lb/MWh)	γ <sub>i</sub> (lb/MW <sup>2</sup> h)	η <sub>i</sub> (lb/h)	δ <sub>i</sub> (1/MW)
1	10	55	1000.403	40.5407	0.12951	33	0.0174	360.0012	-3.9864	0.04702	0.25475	0.01234
2	20	80	950.606	39.5804	0.10908	25	0.0178	350.0056	-3.9524	0.04652	0.25475	0.01234
3	47	120	900.705	36.5104	0.12511	32	0.0162	330.0056	-3.9023	0.04652	0.25163	0.01215
4	20	130	800.705	39.5104	0.12111	30	0.0168	330.0056	-3.9023	0.04652	0.25163	0.01215
5	50	160	756.799	38.5390	0.15247	30	0.0148	13.8593	0.3277	0.00420	0.24970	0.01200
6	70	240	451.325	46.1592	0.10587	20	0.0163	13.8593	0.3277	0.00420	0.24970	0.01200
7	60	300	1243.531	38.3055	0.03546	20	0.0152	40.2669	-0.5455	0.00680	0.24800	0.01290
8	70	340	1049.998	40.3965	0.02803	30	0.0128	40.2669	-0.5455	0.00680	0.24990	0.01203
9	135	470	1658.569	36.3278	0.02111	60	0.0136	42.8955	-0.5112	0.00460	0.25470	0.01234
10	150	470	1356.659	38.2704	0.01799	40	0.0141	42.8955	-0.5112	0.00460	0.25470	0.01234

## **APPENDIX B**

Unit	P <sub>i</sub> <sup>min</sup> MW	$P_i^{max}$ <b>MW</b>	a <sub>i</sub> (\$/h)	b <sub>i</sub> (\$/MWh)	c <sub>i</sub> (\$/MWh)	d <sub>i</sub> (\$/h)	e <sub>i</sub> (rad/MW)	α <sub>i</sub> (ton/h)	β <sub>i</sub> (ton/MWh)	$\gamma_i$ (ton/MW <sup>2</sup> h)	η <sub>i</sub> (ton/h)	$\delta_i$ (1/MW)
1	36	114	94.705	6.73	0.00690	100	0.084	60	-2.22	0.0480	1.3100	0.05690
2	36	114	94.705	6.73	0.00690	100	0.084	60	-2.22	0.0480	1.3100	0.05690
3	60	120	309.540	7.07	0.02028	100	0.084	100	-2.63	0.0762	1.3100	0.05690
4	80	190	369.030	8.18	0.00942	150	0.063	120	-3.14	0.0540	0.9142	0.04540
5	47	97	148.890	5.35	0.01140	120	0.077	50	-1.89	0.0850	0.9936	0.04060
6	68	140	222.330	8.05	0.01142	100	0.084	80	-3.08	0.0854	1.3100	0.05690
7	110	300	287.710	8.03	0.00357	200	0.042	100	-3.06	0.0242	0.6550	0.02846
8	135	300	391.980	6.99	0.00492	200	0.042	130	-2.32	0.0310	0.6550	0.02846
9	135	300	455.760	6.60	0.00573	200	0.042	150	-2.11	0.0335	0.6550	0.02846
10	130	300	722.820	12.9	0.00605	200	0.042	280	-4.34	0.4250	0.6550	0.02846
11	94	375	635.200	12.9	0.00515	200	0.042	220	-4.34	0.0322	0.6550	0.02846
12	94	375	654.690	12.8	0.00569	200	0.042	225	-4.28	0.0338	0.6550	0.02846
13	125	500	913.400	12.5	0.00421	300	0.035	300	-4.18	0.0296	0.5035	0.02075
14	125	500	1760.400	8.84	0.00752	300	0.035	520	-3.34	0.0512	0.5035	0.02075
15	125	500	1760.400	8.84	0.00752	300	0.035	510	-3.55	0.0496	0.5035	0.02075
16	125	500	1760.400	8.84	0.00752	300	0.035	510	-3.55	0.0496	0.5035	0.02075
17	220	500	647.850	7.97	0.00313	300	0.035	220	-2.68	0.0151	0.5035	0.02075
18	220	500	649.690	7.95	0.00313	300	0.035	222	-2.66	0.0151	0.5035	0.02075
19	242	550	647.830	7.97	0.00313	300	0.035	220	-2.68	0.0151	0.5035	0.02075
20	242	550	647.810	7.97	0.00313	300	0.035	220	-2.68	0.0151	0.5035	0.02075
21	254	550	785.960	6.63	0.00298	300	0.035	290	-2.22	0.0145	0.5035	0.02075
22	254	550	785.960	6.63	0.00298	300	0.035	285	-2.22	0.0145	0.5035	0.02075
23	254	550	794.530	6.66	0.00284	300	0.035	295	-2.26	0.0138	0.5035	0.02075
24	254	550	794.530	6.66	0.00284	300	0.035	295	-2.26	0.0138	0.5035	0.02075
25	254	550	801.320	7.10	0.00277	300	0.035	310	-2.42	0.0132	0.5035	0.02075
26	254	550	801.320	7.10	0.00277	300	0.035	310	-2.42	0.0132	0.5035	0.02075
27	10	150	1055.100	3.33	0.52124	120	0.077	360	-1.11	1.8420	0.9936	0.04060
28	10	150	1055.100	3.33	0.52124	120	0.077	360	-1.11	1.8420	0.9936	0.04060
29	10	150	1055.100	3.33	0.52124	120	0.077	360	-1.11	1.8420	0.9936	0.04060
30	47	97	148.890	5.35	0.01140	120	0.077	50	-1.89	0.0850	0.9936	0.04060
31	60	190	222.920	6.43	0.00160	150	0.063	80	-2.08	0.0121	0.9142	0.04540
32	60	190	222.920	6.43	0.00160	150	0.063	80	-2.08	0.0121	0.9142	0.04540
33	60	190	222.920	6.43	0.00160	150	0.063	80	-2.08	0.0121	0.9142	0.04540
34	90	200	107.870	8.95	0.00010	200	0.042	65	-3.48	0.0012	0.6550	0.02846
35	90	200	116.580	8.62	0.00010	200	0.042	70	-3.24	0.0012	0.6550	0.02846
36	90	200	116.580	8.62	0.00010	200	0.042	70	-3.24	0.0012	0.6550	0.02846
37	25	110	307.450	5.88	0.01610	80	0.098	100	-1.98	0.0950	1.4200	0.06770
38	25	110	307.450	5.88	0.01610	80	0.098	100	-1.98	0.0950	1.4200	0.06770
39	25	110	307.450	5.88	0.01610	80	0.098	100	-1.98	0.0950	1.4200	0.06770
40	242	550	647.830	7.97	0.00313	300	0.035	220	-2.68	0.0151	0.5035	0.02075