Development of a Mathematical Model for Heat Pump Fruit Drying

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ABSTRACT

A mathematical model for papaya glacé drying using heat pump has been developed. It comprises of drying rate equation, mass and energy balance equations, thermo-physical property of papaya glacé equations and heat exchanger equations. Successive substitution method was used for finding the solution. It was found that the model was fairly accurate for predicting final moisture content, air temperature at various parts in the system and temperature of refrigerant especially at low moisture level of papaya glacé.

The mathematical model was then used to find out strategy for drying papaya glacé with a specific air flow rate of 29.8 kg/h-kg dry papaya glacé. Simulated results showed that drying time and energy consumption decreased when the drying air temperature increased. In addition, the appropriate by-pass air ratio was in the range of 86%-90%.

1. INTRODUCTION

Britnell, et al.[1] studied drying of ginger and potato by using heat pump. They found that quality of agricultural product, dried by heat pump, in terms of smell, taste and color was better than those dried by rotary drying. Clement, et al. [2] studied continuous drying of rubber using heat pump. They found that COP increased with air relative humidity and suitable by-pass air ratio was 60%-70%. Meyer and Greyvenstein[3] studied economic grain drying using heat pump. They found that heat pump drying which operated along with other purpose such as heating water was more economical than drying using electric hot wire and diesel fuel. Soponronnarit, et al. [4] studied suitable strategy for papaya glacé drying in thermal tunnel and developed a mathematical model. Main assumption of the model was heat equilibrium between moist air and papaya glacé. Simulation result showed that the simulated drying rate was slightly higher than the experimental one.

The objective of this research is to develop a mathematical model for papaya glacé drying using heat pump. Successive substitution method was used for finding solution. Strategy of drying was studied by adjusting the suitable by pass air ratio with consideration of drying time and energy consumption.

2. MATERIALS AND METHODS

2.1 Development of Mathematical Model

Soponronnarit, et al. [4] developed a mathematical model for predicting papaya glacé drying in tunnel. Assumption of the model was thermal equilibrium between moist air and product. In this research, the model was further developed for predicting papaya glacé drying using heat pump. The additional assumption was that temperature at the surface of evaporator tube equaled to refrigerant temperature in the evaporator and by-pass factor depended on type of evaporator. Refrigerant temperature at condenser and evaporator was the average value between inlet and outlet temperatures of both equipment. Temperature difference at inlet and outlet of recuperator was 4°C.

Calculation of Papaya Glacé Moisture Content

Diffusion equation of cubic material was used to calculate papaya glacé moisture content. The equation is as follows:

$$MR(t) = (8/\pi^2)^3 \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} [\{1/(2i+1)^2\} \{1/(2j+1)^2\} \{1/(2k+1)^2\}]$$
$$\exp [- \{ (2i+1)^2 / L^2_x + (2j+1)^2 / L^2_y + (2k+1)^2 / L^2_z \} \pi^2 Dt] \quad (1)$$

where: $$MR(t) = (M(t) - M_{eq})/(M_{in} - M_{eq}) \quad (2)$$

Diffusion coefficient of Achariyaviriya and Soponronnarit [5] was used for predicting drying rate. It is written as follows:

$$D = 0.000917 \exp [-2877.49/(T_{mix} + 273)] \quad (3)$$

Equation (3) can be used for temperature of 40°C to 80°C and initial moisture content not higher than 50% dry basis. Achariyaviriya and Soponronnarit [5] found that equilibrium moisture content of papaya glacé and air relative humidity can be calculated as follows:

$$M_{eq} = CM_{in}RH/[100^{*} (1.01 + CRH + 2RH - CRH^2 + RH^2)] \quad (4)$$

$$C = 163.15 \exp (-0.0647\, T_{mix}) \quad (5)$$

$$M_{in} = 3.1987 + 0.14077\, T_{mix} \quad (6)$$

where:

$MR(t)$	=	moisture ratio, fraction
$M(t)$	=	average moisture content of papaya glacé at any time, decimal dry basis
M_{in}	=	average initial moisture content of papaya glacé, decimal dry basis
M_{eq}	=	equilibrium moisture content of papaya glacé, decimal dry basis
D	=	diffusion coefficient, m^2/h
RH	=	relative humidity of drying air, fraction
T_{mix}	=	drying air temperature, °C
t	=	drying time, h
L	=	dimension of papaya glacé, m

From Eq. 1, average moisture content changing at any time can be calculated from differential of Eq. 1 to time, using Runge-Kutta method.

Calculation of Air Condition after Drying

Considering CV. 1 in Fig.1, from conservation of mass principle, it is found that air moisture change is equal to evaporated water from papaya glacé. From this equation, W_f can be determined.

$$W_f = (M_i - M_f)\,R + W_{mix} \tag{7}$$

where: $R = M_p \,/\, m_{mix}\Delta t$

From conservation of energy principle, it shows that enthalpy change of air flow including change of internal energy in drying material is equal to total heat transfer between system and environment. From this equation, T_f can be determined.

$$T_f = (C_a T_{mix} + W_{mix}(h_{fg} + C_v T_{mix}) + RC_{pw}\theta - W_f h_{fg} - Q_1) \,/\, (C_a + W_f C_v + RC_{pw}) \tag{8}$$

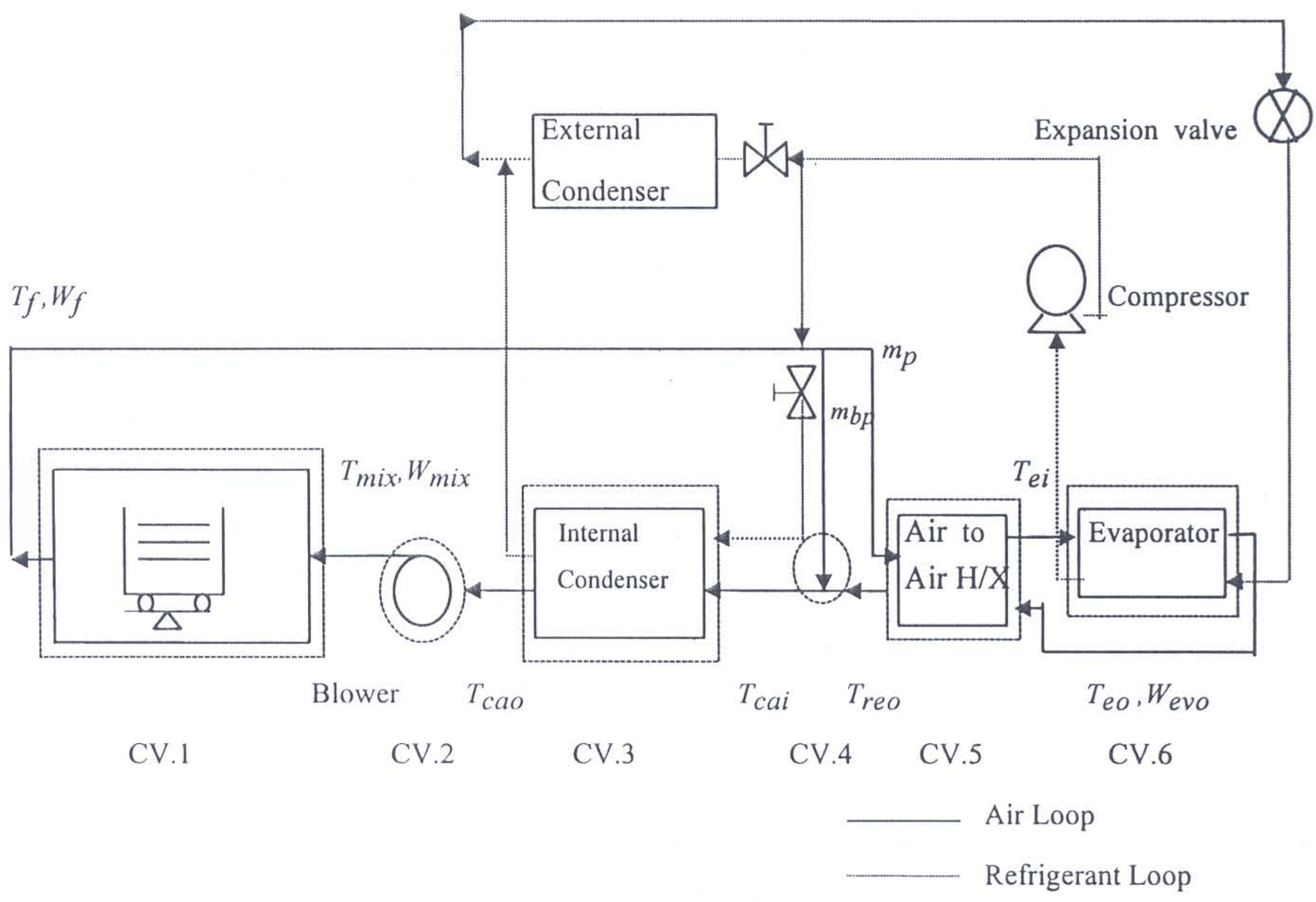

Fig. 1. Control volumes of the heat pump drying system.

where: m = dry air mass flow rate, kg/h
h_{fg} = latent of vaporization, kJ/kg
C = specific heat, kJ/kg °C
θ = papaya glace' temperature before drying, °C
Q_l = heat loss, kJ/kg dry air
M_p = dry mass of product, kg
M = average moisture content of papaya glacé, decimal dry basis
T = temperature, °C
W = humidity ratio, kg water/kg dry air
Subscripts:
a = dry air
f = after drying
I = before drying
pw = papaya glacé
v = water vapor
mix = mixed air before entering to drying chamber

Calculation of Mixed Air Temperature and Recycled Air

Considering CV. 4 in Fig. 1, from conservation of mass principle, it is found that mass of water in hot air before drying is equal to total mass of water in reduced moisture air from evaporator and mass of water in recycled hot air. From this equation, W_{mix} can be determined.

$$W_{mix} = RCW_f + (1 - RC)W_{evo} \tag{9}$$

where: RC = m_{bp}/m_{mix}
W_{evo} = humidity ratio of air leaving from evaporator, kg water/kg dry air
m_{bp} = mass flow rate of bypass air, kg/h

From conservation of energy principle, enthalpy of air flow entering to CV. 4 is equal to enthalpy of air flow leaving from CV. 4 when heat loss between system and environment is neglected. From this equation, T_{cai} can be determined.

$$T_{cai} = \{RC\,[C_aT_f + W_f(h_{fg} + C_vT_f)] + (1 - RC)\,[T_{reo}(C_a+W_{evo}C_v) + W_{evo}h_{fg}] - W_{mix}h_{fg}\}/(C_a+W_{mix}C_v) \tag{10}$$

where: T_{reo} = outlet air temperature of recuperator, °C
T_{cai} = inlet air temperature of condenser, °C

Calculation of Humidity Ratio and Outlet Air Temperature of Evaporator

Considering CV. 6 in Fig. 1, it is assumed that humidity ratio and temperature of the outlet air of evaporator can be calculated by the following equations :

$$W_{evo} = BFW_f + (1 - BF)\,W_{er} \tag{11}$$

$$T_{eo} = BFT_{ei} + (1 - BF)\,T_e \tag{12}$$

where: BF = fraction of air which does not touch refrigerant coil tube with value of 30%
T_{ei} = inlet air temperature of evaporator, °C
T_{eo} = outlet air temperature of evaporator, °C
T_e = refrigerant temperature at evaporator, °C
W_{er} = saturated humidity ratio at evaporator surface, kg water/kg dry air

Electricity Consumption

Considering CV. 2 in Fig. 1, from conservation of energy principle, it is found that enthalpy change rate of air flow is equal to shaft power of main blower which can be written as follows :

$$W_s = m_{mix}(C_a + W_{mix}C_v)(T_{mix} - T_{cao}) \tag{13}$$

where: T_{cao} = outlet temperature of condenser, °C
W_s = shaft power of blower, kJ/h

Heat Exchangers

Considering CV. 3 in Fig. 1., from conservation of energy principle, it is found that rate of heat rejected by condenser is equal to enthalpy change rate of air flow, when heat loss between environment and system is neglected.

$$Q_{in,cond} = m_{mix}(C_a + W_{mix}C_v)(T_{cao} - T_{cai}) \tag{14}$$

where: $Q_{in,cond}$= heat received from internal condenser, kJ/h

From conservation of energy principle in CV. 5, rate of heat exchanged at recuperator or air to air heat exchanger is equal to enthalpy change rate of air flow.

$$\text{Hot current } Q_{re} = m_p(C_a + W_f C_v)(T_f - T_{ei}) \tag{15}$$
$$\text{Cold current } T_{reo} = [Q_{re} / m_p(C_a + W_{evo}C_v)] + T_{eo} \tag{16}$$

where Q_{re} = heat exchange, kJ/h
m_p = air flow rate via evaporator, kg/h

Assumption of calculation is that the difference between inlet and outlet temperature is 4°C, i.e.

$$T_{ei} = T_f - 4 \tag{17}$$

Calculation of Refrigerant Temperature

Heat transfer equation at heat exchanger with phase change of media substance can be expressed as follows :

At the condenser :

$$T_c = [T_{cai} - T_{cao} \exp((T_{cao} - T_{cai})UA_c/Q_{in,cond})] / [1 - \exp((T_{cao} - T_{cai})UA_c/Q_{in,cond})] \tag{18}$$

At the evaporator :

$$T_e = [T_{ei} - T_{eo}\exp((T_{ei} - T_{eo})UA_e/Q_{ev})] / [1 - \exp((T_{ei} - T_{eo})UA_e/Q_{ev})] \quad (19)$$

Calculation of Heat Pump Equipment

Assumption of calculation is that the performance of compressor and evaporator are related to refrigerant temperature at evaporator and condenser. Mathematical function in terms of polynomial is as follows:

$$P = A_1 + A_2T_e + A_3T_e^2 + A_4T_c + A_5T_c^2 + A_6T_eT_c + A_7T_e^2T_c + A_8T_eT_c^2 + A_9T_e^2T_c^2 \quad (20)$$

$$Q_{ev} = B_1 + B_2T_e + B_3T_e^2 + B_4T_c + B_5T_c^2 + B_6T_eT_c + B_7T_e^2T_c + B_8T_eT_c^2 + B_9T_e^2T_c^2 \quad (21)$$

where *A* and *B* are constants from manufacturing company.

2.2 Calculation Procedure

To start, W_f is assumed and bypass factor is set according to the design of evaporator for calculating W_{evo} value from Eq. 11. W_{evo} is used to calculate W_{mix} from Eq. 9, and then value of T_{mix} is used to calculate *RH* value and M_{eq} value is calculated from Eq. 4. Then M_f value is calculated from derivative of Eq. 1 to time. New value of W_f is calculated from Eq. 7 and T_f is calculated from Eq. 8. T_{mix} and new W_f are used to calculate relative humidity of hot air leaving from dryer(RH_f) and then RH_f is checked. If RH_f is not acceptable ($RH_f > 1$), subprogram for moisture condensation will be calculated. But if RH_f is acceptable ($RH_f < 1$), assumed T_e and T_c are used to calculate *P* and Q_{ev} from Eq. 20 and Eq. 21, respectively. Then Q_c is calculated. Assumed W_f value and W_{evo} value are used to find T_{ei}, T_{eo}, T_{cai}, T_{cao}, $Q_{in,cond}$, Q_{re} and T_{reo} from Eq. 17, Eq. 12, Eq. 10, Eq. 13, Eq. 14, Eq. 15 and Eq. 16, respectively. The new values of T_c and T_e are calculated from Eq. 18 and Eq. 19, respectively. W_f, T_c, and T_e values are checked. If different value between new W_f and assumed W_f is more than acceptable value (0.00001) and the different value between new T_c, T_e and assumed T_c, T_e are more than acceptable value (0.1), recalculation by using new value of W_f, T_c and T_e in place of assumed value will be conducted. If those values are less than acceptable value, the following calculation will be done: energy consumption of main blower and compressor are calculated. Then final moisture content of papaya glace' is checked whether it is less than determined value or not. If it is less than or equal to determined value, program will be stopped. Figure 2 shows block diagram of calculation and Fig. 3 shows computer simulation flow chart.

3. RESULTS AND DISCUSSIONS

3.1 Comparison between Experimental Drying Rate and Drying Rate from Mathematical Model

From comparison between results of calculation from mathematical model and experimental results at temperature of 50°C with papaya glacé dimension of 1.1 x 1.2 x 1.3 cm³, initial moisture content of 38.3% dry basis, final moisture content of 23% dry basis and bypass air ratio of 63% as shown in Fig. 4, it was found that average moisture content of papaya glace' from calculation was nearly the same as those from experiment. Air temperature at various points as shown in Fig. 5 show that inlet and outlet air temperature of any equipment such as condenser, evaporator and recuperator calculated by mathematical model was close to experimental data. Refrigerant temperature at evaporator and condenser as shown in Fig. 6 show that simulated value with mathematical model was in average value of experimental results.

3.2 Using a Mathematical Model for Finding Appropriate Drying Condition

Appropriate drying conditions were investigated by using the developed mathematical model. Suitable bypass air ratio was calculated by using criteria of minimum specific energy consumption and near minimum drying time as shown in Fig. 3. The quantity of papaya glacé was varied in three values as follows: 56.5 kg, 80.1 kg and 101.1 kg. Figures 7 to 9 show the results of simulation at various quantity of papaya glacé. From these figures, it was found that the suitable bypass air ratio was 90%, 88% and 86% with quantity of papaya glacé of 56.5 kg, 80.1 kg and 101.1 kg, respectively. At these conditions, specific energy consumption or drying time is at the minimum values.

Simulation result with varying drying air temperature showed that drying time and specific energy consumption decreased when drying air temperature increased. It was because high drying temperature caused low relative humidity of drying air. Therefore, moisture content of papaya glacé decreased quickly and resulted in low energy consumption. Suitable drying temperature in the range of study was 50°C as shown in Fig. 10.

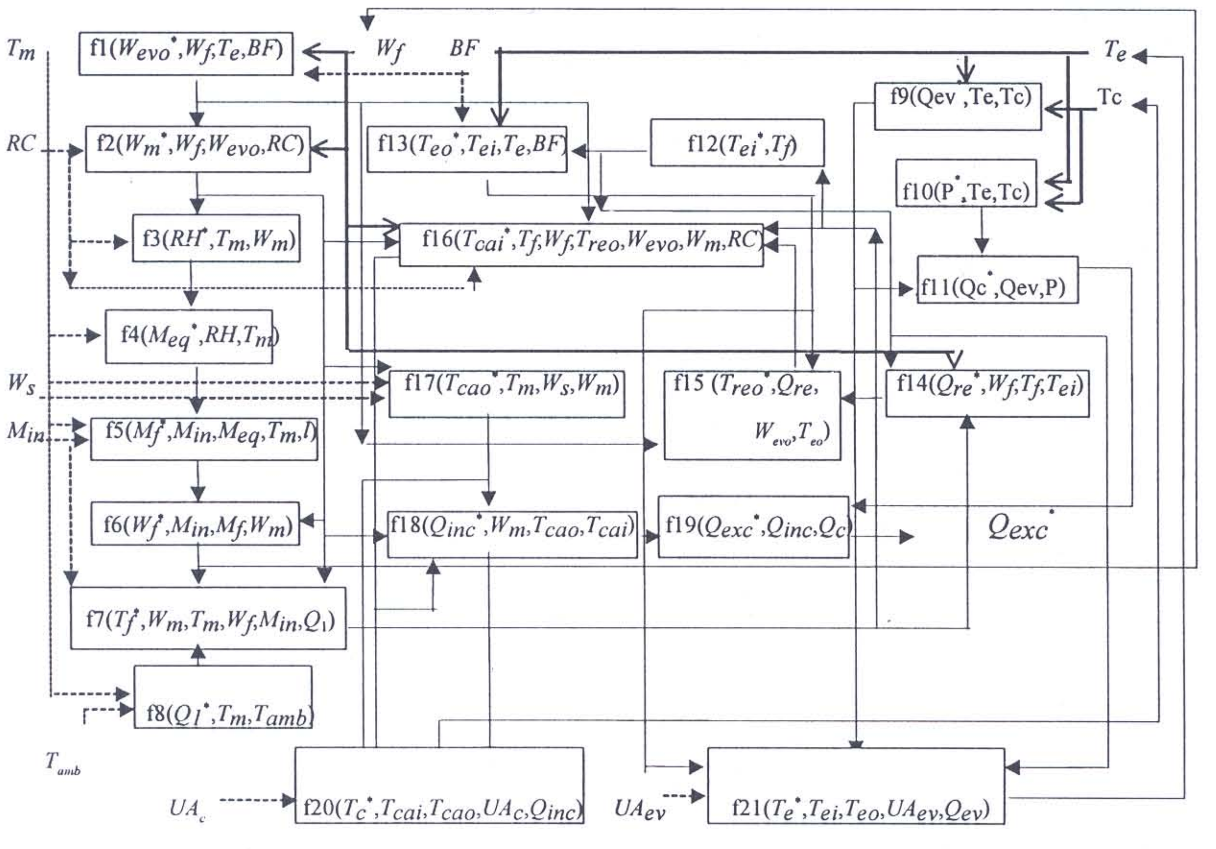

Fig. 2. Block diagram of calculation.

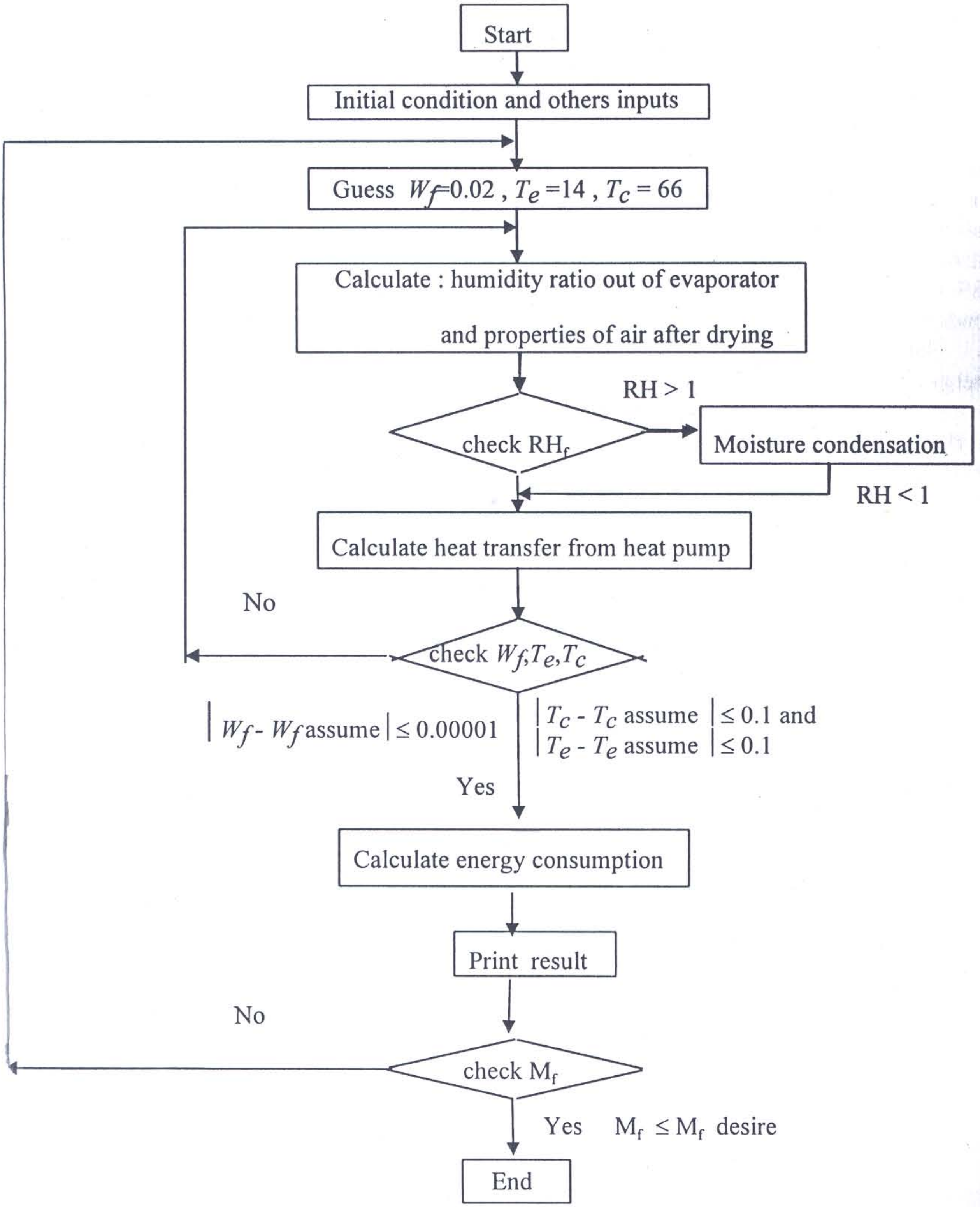

Fig. 3. Computer simulation flow chart.

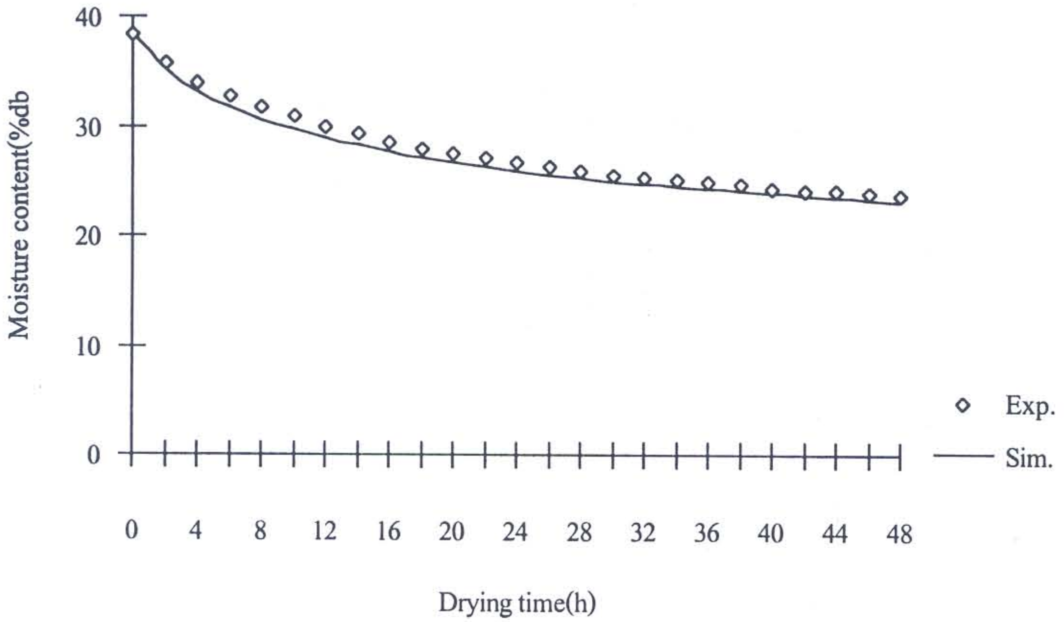

Fig. 4. Evolution of simulated and experimental moisture content (Test 3/2]
[By-pass air ratio = 63%, Temperature = 50°C,
Specific air flow rate = 29.83 kg/h-kg dry papaya glacé].

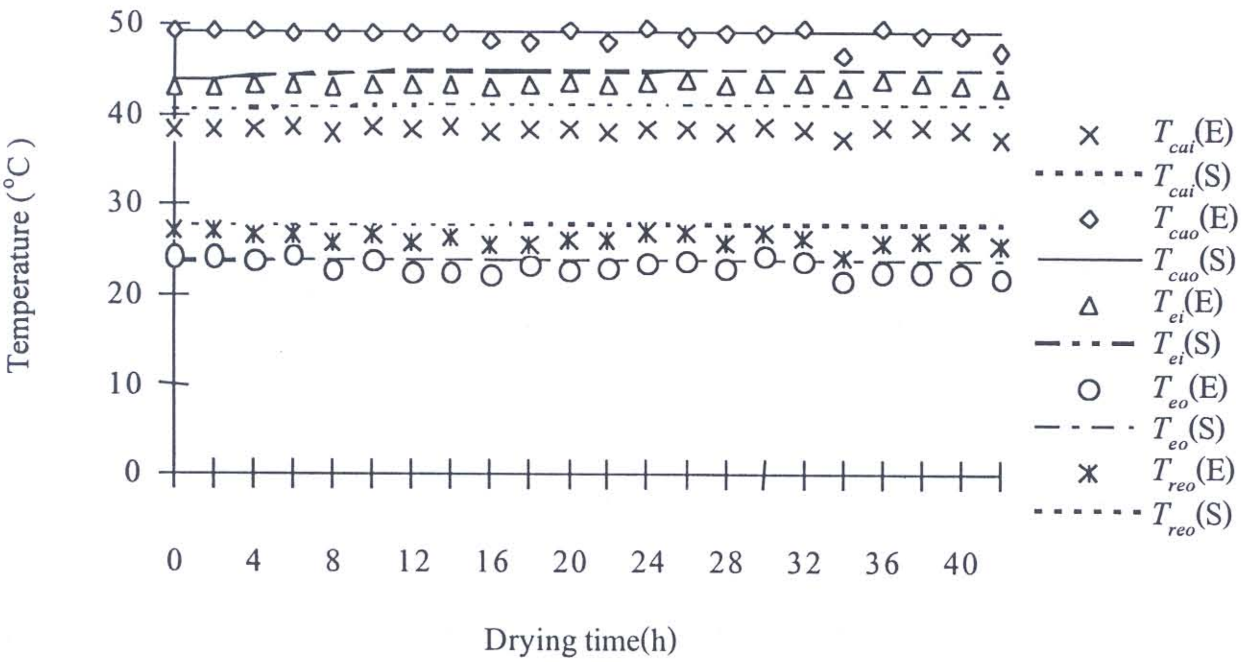

Fig. 5. Evolution of air temperature at various parts in system (Test 3/2]
[By-pass air ratio = 63%, Temperature = 50°C,
Specific air flow rate = 29.83 kg/h-kg dry papaya glacé].

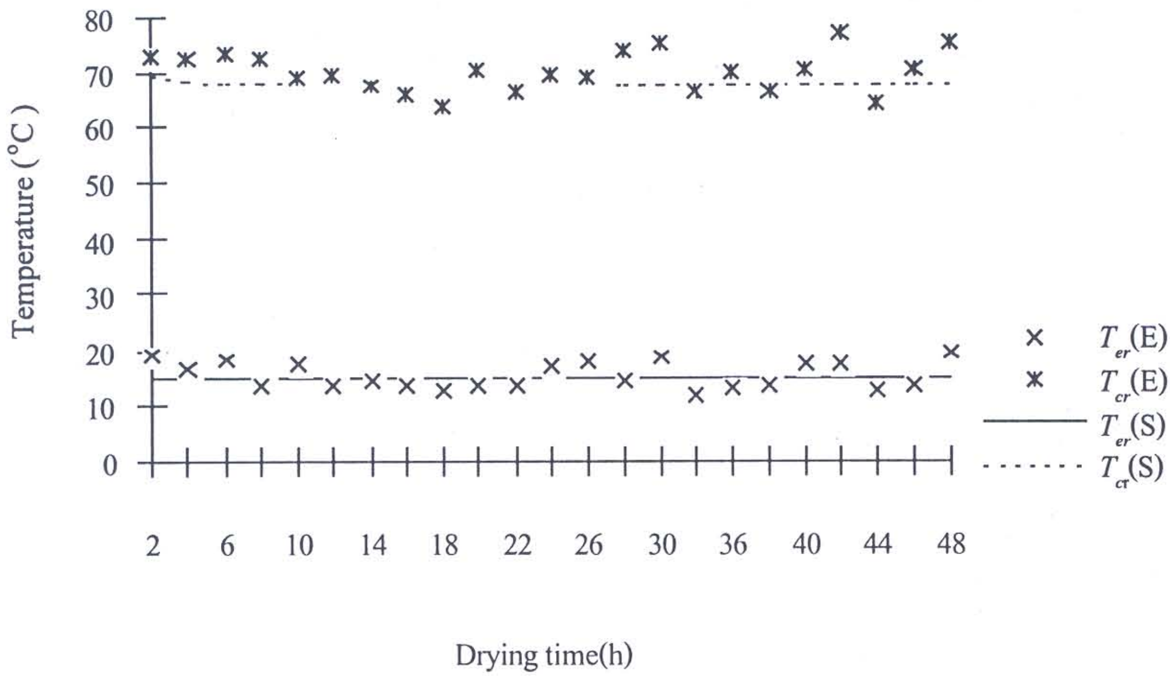

Fig. 6. Temperature of refrigerant at condenser and evaporator (Test 3/2]
[By-pass air ratio = 63%, Temperature = 50°C,
Specific air flow rate = 29.83 kg/h-kg dry papaya glacé].

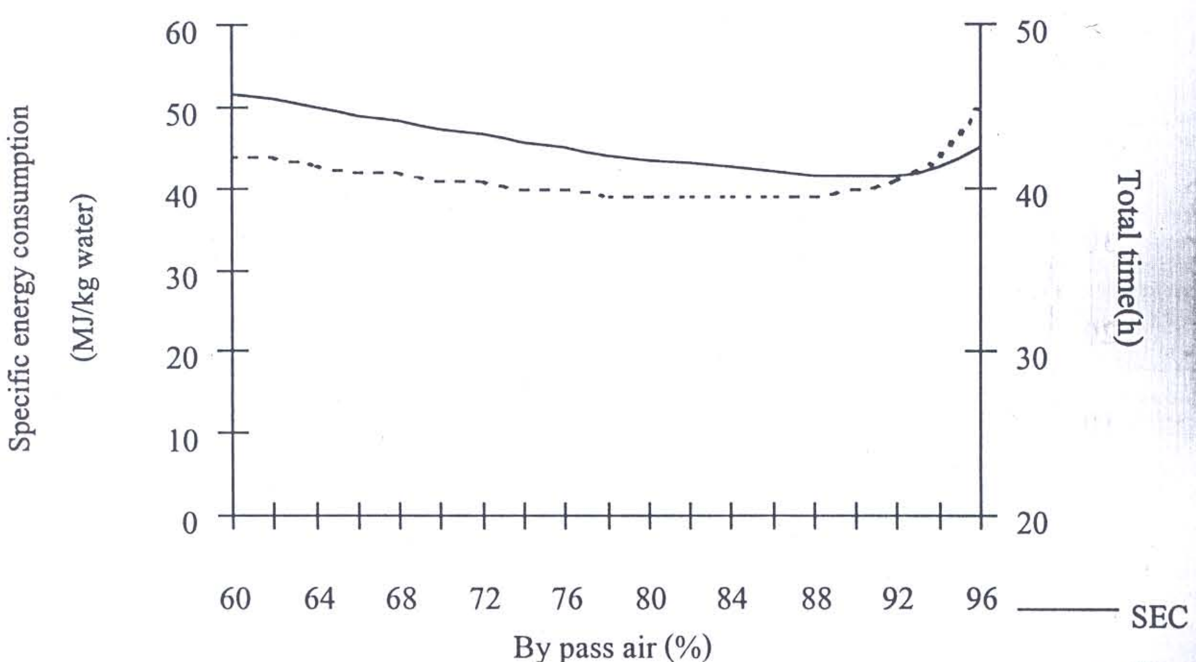

Fig. 7. Effect of by-pass air ratio on specific energy consumption, papaya glacé 56.5 kg
[Temperature = 50°C, size = 1.1x1.2x1.3 cm³, initial moisture content = 38.3% dry basis, final moisture content = 23.6 %dry basis].

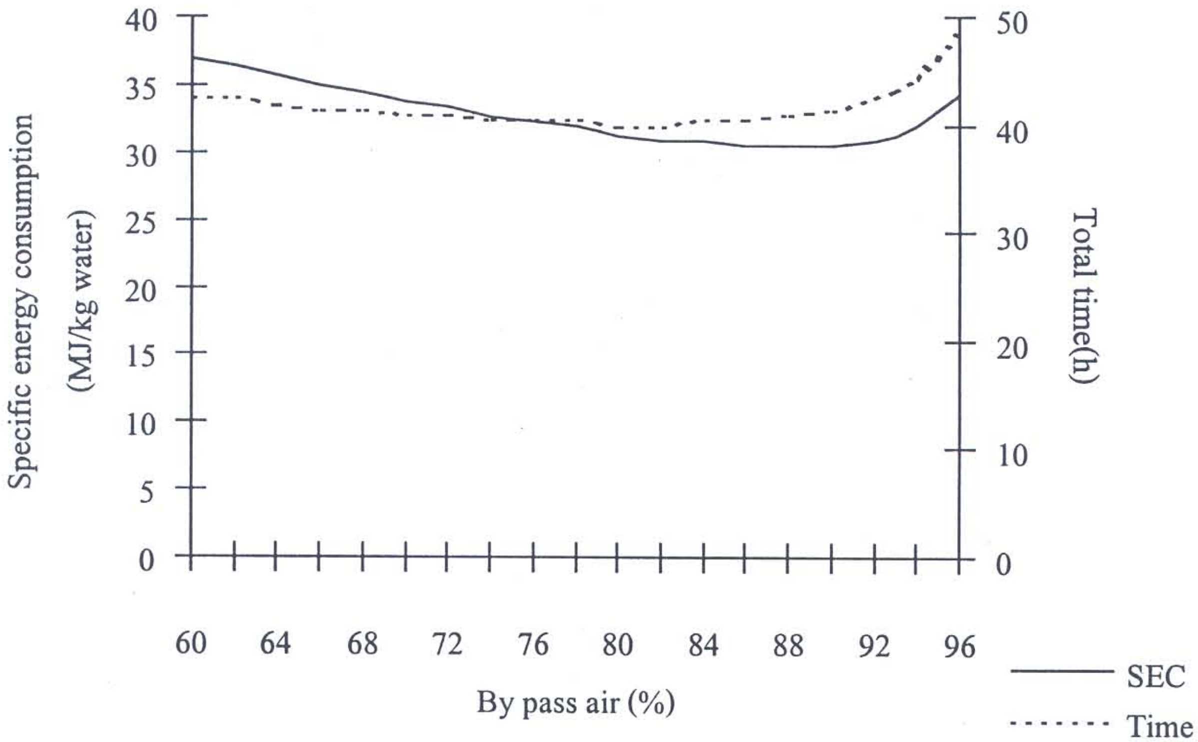

Fig. 8. Effect of by-pass air ratio on specific energy consumption, papaya glacé 80.1 kg [Temperature = 50°C, size = 1.1x1.2x1.3 cm³, initial moisture content = 38.3 % dry basis, final moisture content = 23.6 % dry basis].

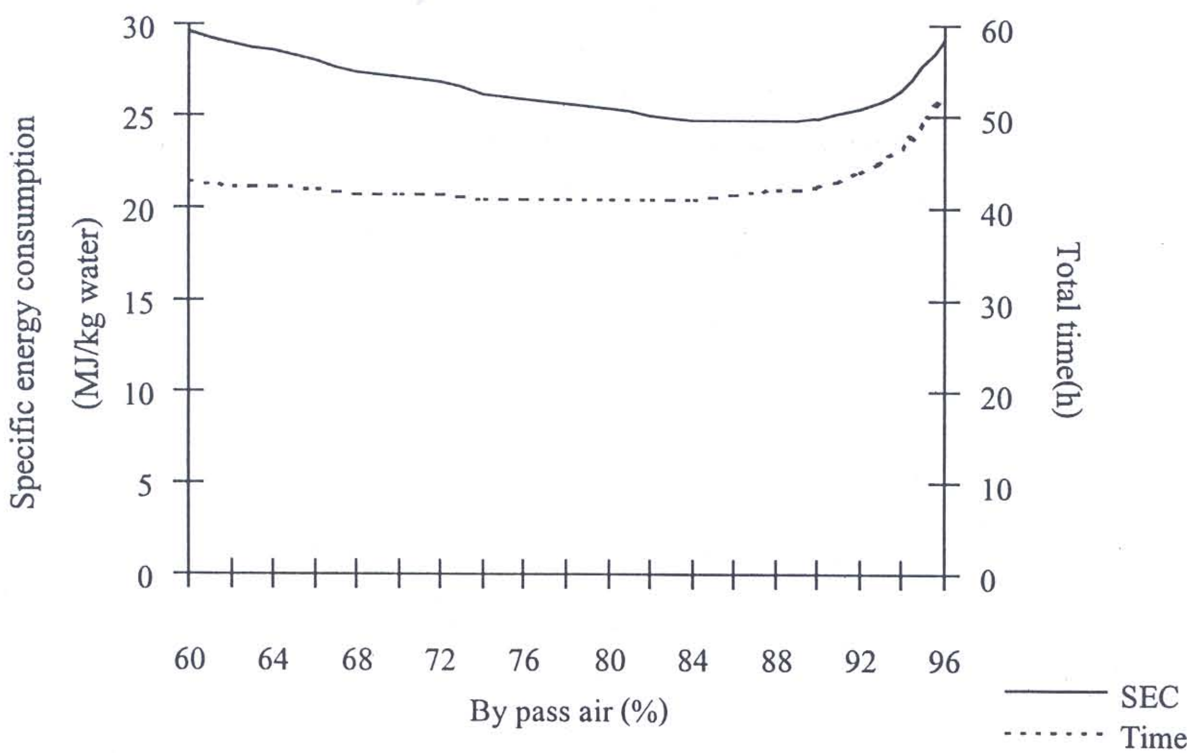

Fig. 9. Effect of by-pass air ratio on specific energy consumption, papaya glacé 101.1 kg [Temperature = 50°C, size = 1.1x1.2x1.3 cm³, initial moisture content = 38.3 % dry basis, final moisture content = 23.6 % dry basis].

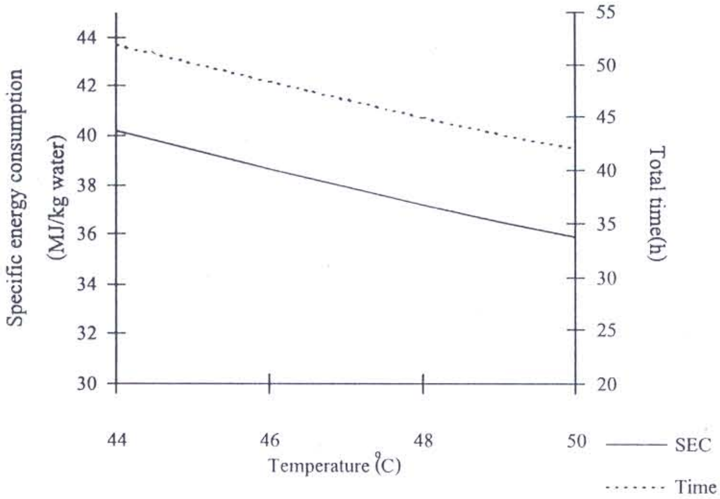

Fig. 10. Effect of drying air temperature on specific energy consumption [Temperature = 50°C, size = 1.1 x 1.2 x 1.3 cm³, initial moisture content = 38.3% dry basis, final moisture content = 23.6 % dry basis, by-pass air ratio = 30%].

4. CONCLUSION

Mathematical model of papaya glacé drying using heat pump provided good prediction for drying at low initial moisture content. The model also provided good prediction for air temperature at various points in the system. Drying time and energy consumption should be considered in drying. From mathematical model, it was found that adjustment tendency of suitable by-pass air ratio which yielded minimum specific energy consumption depended on quantity of papaya glacé to be dried. If quantity of papaya glace' was large, by-pass air ratio decreased. The suitable by-pass air ratio in this study was in range of 86%-90%.

5. ACKNOWLEDGMENT

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