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Experimental Design of Flywheel Rotor with a Flywheel Energy Storage System for Residential uses

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ABSTRACT

Flywheel energy storage system is a system that can store energy while spinning at high speed. The shape and density of materials are important parameters for energy storage in flywheels. This research aims to design a flywheel in conical disc flywheel shape, compare it with thick rim flywheel with different shape factors, and evaluate the best application efficiency. The research starts with the use of flywheel geometry manipulation. After that, the researcher designs the experiment using multiple-time series design patterns in the test. Next, an experiment to evaluate the flywheel's performance is conducted. When examining the time factor for flywheel energy storage, it is found that the conical disc flywheel has a time value of 180 seconds, whereas the thick rim flywheel has a time value of 120 seconds. At every test time, the conical disc flywheel has a faster rotational speed than the thick rim flywheel. When considering the electromotive force aspect, the speed is suitable for practical application at speeds from 1,000-1,495 rpm at a time value of 0-15 s. At the same time, it is discovered that the electromotive force produced by the conical disc flywheel generator is higher than that of the thick rim flywheel generator. In conclusion, the results show that the conical disc flywheel performs better than the thick rim flywheel in all factors. The important variables affecting the function of the flywheels are kinetic energy, angular velocity, shape factor and energy density.

1. INTRODUCTION

The energy system is very important. It can be supplied from a variety of sources and can be converted into the form of energy needed in all sectors such as public utility, industry, buildings and transportation. Currently, renewable energy such as solar energy and wind power is very important. However, this energy cannot be produced continuously depending on seasonal changes. Therefore, it is necessary to find a method for storing energy that is important to the power system [1], [2]. Energy storage can be classified in several ways [3], such as thermal energy storage, flywheel energy storage, electrochemical and batteries, thermochemical, compressed air, liquefied air, chemical and hydrogen, pumped hydro, magnetic, *etc.* [1], [4].

FESS is a popular system that can respond quickly because of many solutions in the main grid and power system [5], [6]. It is clean energy and has been used for different applications since it has special characteristics and is suitable for short and medium-term applications [5], [7]. With increasing environmental and energy issues, energy storage in the form of flywheels is, therefore, a technology that is widely used in space, vehicles, power plants [8]-[9], and advanced technology. This is because it has high efficiency and low environmental pollution. In addition, it has a long service life, easy maintenance, and efficiency at high speed [10]-[12]. The energy storage of the flywheel occurs while spinning at high speed, therefore, it can the ability to deliver high power output. The energy density and amount of energy generated by rotation are important parameters in evaluating the efficiency of energy storage [13].

A flywheel is the main piece of equipment that is important to FESS. It is a device that generates kinetic energy [14], where rotational inertia is important. It is resistant to sudden changes in rotational speed when rotating at high speeds and can release enough energy for the entire system of large machines [10], [15]-[16]. Increasing the size and density of the material will increase the energy capacity of the flywheel [17]. Important factors of flywheel energy storage technologies such as material, size and shape directly affect the amount of energy storage, specification of energy, and its suitability for application [14]. Another factor determining the energy capacity and efficiency of the flywheel storage is the geometric shape factor and the polar moment of inertia of the flywheel is proportional which causes the energy of the flywheel system [18]-[20]. It is important to analyze the shape of the flywheel to find the best fit for choosing the primary form. The shape of the conical disc flywheel can be divided into two forms. The researcher also examined

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the influence of different flywheel geometry on the efficiency of kinetic energy storage using finite element analysis. This research examines the parameters to configure each flywheel to assess the efficiency of the flywheel energy storage system. The polar moment of inertia is based on these parameters used to calculate the energy capacity of the flywheel, the form factor for each cross-section, the maximum angular velocity as well as the maximum stress. The analysis results of all parameters determine the optimal suitability of the flywheel system [19]. In the development of a series of stress, equations are used in flywheels of different shapes and are determined according to the shape of the flywheel [21]. The research focuses on using ANSYS software to analyze flywheel geometry configurations on energy storage efficiency. This examines both the analytical stress equation and the use of finite element analysis in the flywheel.

The results obtained from this research are a suitable flywheel shape with good energy storage efficiency which can be selected as a guideline for further use. The researcher further suggests that the frequency analysis of the flywheel is useful for data in the case of more complex flywheels depending on the design. Engineering research in the field of specific flywheel designs for operating speed and energy storage capacity will require an improvement in flywheel design and manufacturing processes for specialized utilization [19].

Based on the suggested guidelines in the previous research, the researcher is interested in further developing the shape of the flywheel by designing and producing it especially for the speed of operation, then testing it out to choose the best performing flywheel design for further employment. In this research, the researcher has defined the study scope as follows:

a) Designing and constructing flywheel sets with different shape factors. The flywheels used in this study are conical disc flywheels and thick rim flywheel.

b) Experimental design using the multiple-time series design format.

c) Evaluating efficiency by considering the factors of the time of flywheel energy storage, electromotive force and speed of flywheel.

The researcher designs the FESS control system for residences and communities that cannot access the official power supply. At present, solar energy has been used in the form of a solar rooftop installed above the building and around the roof of the residence. However, there are also limitations in terms of prices that are still relatively high and the limitation of solar light that occurs only during the day or according to the season.

FESS is therefore an alternative to the application without limitation in terms of power supply. It can provide energy both during the day and at night and in all seasons. Nevertheless, we must consider the performance and the cost of the system must not be very high. In Figure 1, the researcher has designed a control system for power distribution to the residence. The main equipment and basic working principles are as follows.

a) A charge controller converts AC electricity from a generator to DC electricity. The movement of electrical energy will flow in two directions. In one direction, the electric power is sent to an inverter and it is used directly. In the other direction, the electricity is sent to a battery to be charged and stored then it can be discharged to the inverter again.

b) An inverter converts the voltage to 220 ACV. It receives electricity from the charge controller one way and the other from the battery then forwards it to the main breaker to control the operation of the residence.

c) A battery is responsible for charging the electric charge from the charge controller and sending an electric voltage of 12 DCV to the inverter to convert the voltage to 220 ACV again.

d) The main breaker serves to supply the electrical system to the residence.

e) A motor control panel controls the operation of the motor unit. Electrical power used to control the motor is obtained from the battery that supplies the power to the inverter and converts the voltage from 12 V to 220 V. Then the electricity is sent to the motor control panel to operate and control the electric motor. Electrical power is used within the system which is not related to external electrical power.

f) FESS acts as a system that helps in generating electrical energy for use.

From reviewing the previous research, the researcher has not found that the FESS system is used for residential purposes. Only solar energy is used. Therefore, the researcher has an idea to develop the FESS system to be applied to residences. For this research, the focus is on FESS.

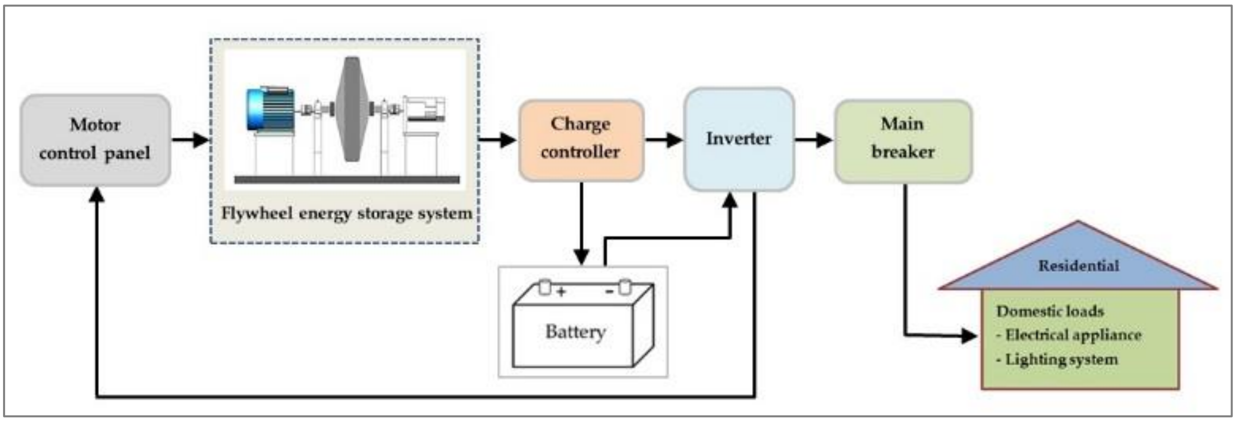

Fig. 1. Architecture of application FESS in residential.

2. METHODOLOGY

2.1 Flywheel Design Procedures and Experiments

The research starts by choosing the model of the flywheel to be studied by considering the shape factor that affects the energy density. The types of flywheels considered are conical disc flywheels and thick rim flywheel. The study also manipulates the geometry of the flywheel in both forms. The next process is to design an experiment in the form of multiple-time series design.

It involves collecting multiple data at different intervals and having a control box in the experimental process. The variables used in the study are independent, controlled variables, and dependent variables. The final process is to evaluate the performance of the two types of flywheels which are the time of energy storage, speed of the flywheel, and electromotive force, as shown in Figure 2.

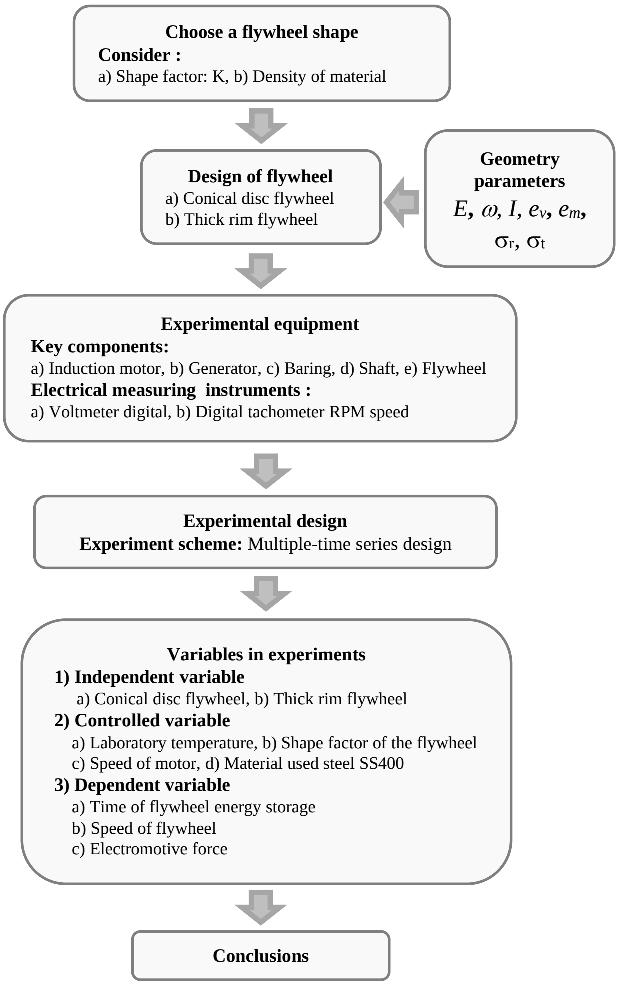

Fig. 2. Flywheel design procedures and experiments to determine the influence factors.

2.2 Flywheel Structural Model

Research and development of the flywheel design are essential in energy storage systems. The geometric parameters are the basis for the design and analysis of the flywheel. Since the flywheel operates at high speeds it is necessary to have high mechanical strength, high energy density and dynamic properties [22]. The flywheel can energy storage or distribution kinetic energy through inertia due to rotation. Three key areas can determine the capacity and efficiency of energy storage:

a) The strength of the material to withstand extreme stresses. Therefore, it can be processed at higher speeds.

b) Geometric details of the shape factor and pole moment of inertia are proportional to the energy production of the flywheel system.

c) The rotational speed is the square of the speed which is proportional to the energy production of the flywheel system [18], [19]. An important design factor is the shape of the flywheel which affects the energy density. [19], [20].

The researcher, therefore, considered two different flywheel shapes with different shape factors to design and conduct an experiment to compare the best performance. From the previous research, it is found that the conical disc flywheel has a shape factor of 0.806. On the other hand, the thick rim flywheel has a shape factor of 0.305. Accordingly, the thick rim flywheel is selected to use in this study because it has the lowest shape factor and is easy to build. Moreover, it has recently been a popular research model, as shown in Table 1.

Table 1. Different flywheel shape factors [20], [21].

Flywheel Geometry	Cross Section	Shape Factor K
Disc		1.000
Modified Constant Stress		0.931
Conical Disc		0.806
Flat Unpierced Disc		0.606
Thick Rim		0.305
Thin Rim		0.500
Shaped Bar		0.500
Rim with web		0.400
Single Bar		0.333

2.2.1 Thick rim flywheel

The flywheel serves to store energy in the form of kinetic energy. This is done by rotating at the designed speed. It is attached to a shaft that is applied as a spindle and connected to a generator. The moment of inertia and energy storage depends on the rotation speed and shape of the flywheel. The design of the flywheel geometry can be determined based on its energy density. The thick rim flywheel is shown in Figure 3 and the shape factor is 0.305, as shown in Table 1. It appears in the shape of a hollow cylindrical shape with an inside radius (ri) = 30 mm, an outer radius (ro) = 400 mm, and the thickness (t) = 70 mm.

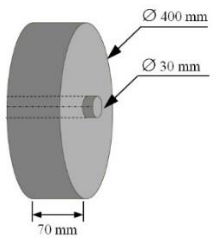

Fig. 3. Thick rim flywheel.

The kinetic energy is proportional to the mass and square of the rotational speed. The calculation formulas are as follows [10]:

$$E_k = \frac{1}{2} I \omega^2 \tag{1}$$

Where: ω is the angular velocity and I is the moment of inertia. The moment of inertia depends on the mass and shape of the flywheel. For the flywheel with the hollow cylinder shape, as shown in Figure 4.

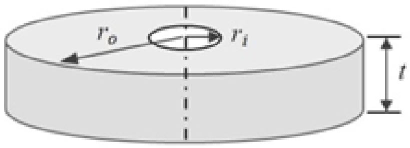

Fig. 4. Hollow circular cylinder.

The moment of inertia is calculated as:

$$I = \frac{1}{2} r^2 M = \frac{1}{2} r^4 \rho \pi t \tag{2}$$

Where: r is radial, t is the thickness of the hollow circular cylinder, M is the mass, and ρ is the density. The mass of a hollow cylinder is as follows [13], [20]:

$$M = \rho \pi t (r_o^2 - r_i^2) \tag{3}$$

So the moment of inertia form is as follows:

$$I = \frac{1}{2} \rho \pi t (r_o^4 - r_i^4) \tag{4}$$

The angular velocity is calculated as:

$$\omega = \frac{2\pi N}{60} \tag{5}$$

Thus, the kinetic energy in the hollow cylindrical flywheel is:

$$E = \frac{1}{4} \rho \pi t (r_o^4 - r_i^4) \omega^2 \tag{6}$$

Maximum energy density relative to volume and mass is:

$$e_v = k\sigma \tag{7}$$

$$e_m = k \frac{\sigma}{\rho} \tag{8}$$

Where: e_v is the energy per unit volume and e_m is the energy per unit mass. K is the shape factor [10], [23], σ is the maximum stress, and ρ is the density of the material.

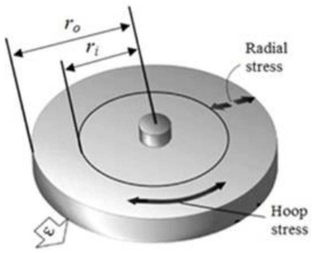

Fig. 5. Radial and hoop stress rotating [10].

In a three-dimensional object, there is a reaction to the stress of the material. For flywheels constructed with non-isotropic materials such as composite reinforced fibers, the stress relationship is limited to practical dimensions. One thing to keep in mind is safety. The resulting flywheel design must be based on a hollow cylindrical shape.

When designing, two main stresses are important . These are the radial and hoop stresses, as shown in Figure 5 and an isotropic material is expressed as:

$$\sigma_r = \frac{3+v}{8} \rho \omega^2 \left(r_o^2 + r_1^2 - \frac{r_0^2 r_i^2}{r^2} - r^2 \right) \tag{9}$$

Where: ρ is the density, ω is the angular velocity, v is the Poisson 's ratio, r_o is the outer radius, r_i is the inside radius and r represents any radius within the flywheel. The hoop stress is expressed as follows:

$$\sigma_t = \frac{3+v}{8} \rho \omega^2 \left[r_o^2 + r_i^2 + \frac{r_o^2 r_i^2}{r^2} - \frac{1+3v}{3+v} r^2 \right] \tag{10}$$

The details of the values obtained from the calculated thick rim flywheel are listed in Table 3.

2.2.2 Conical disc flywheel

In Figure 6 the conical disc flywheel, is set as follows: inside radius (r_i) = 30 mm, outer radius (r_o) = 400 mm, core length = 130 mm, thickness of flywheel (t) = 70 mm, which has a shape factor of = 0.826, as shown in Table 1.

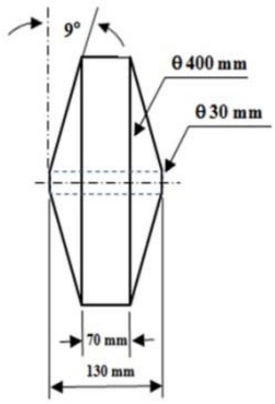

Fig. 6. Conical disc flywheel.

The analysis was performed by extracting the flywheel profile shape, as shown in Figures 7 and 8. The thick-edged flywheel in Figure 7 has the greatest mass concentration in the outer radius region, as shown in Figure 8 which reverses the shape shown in Figure 7 from Equation 11 and is the representation of the mathematical formula for the shape thickness for both figures. The development of solving a series of stress formulas for flywheel shapes is shown in Figures 7 and 8. The shape, as shown in Figure 8, is a template for estimating stress Equation 11 and can be manipulated and explained in Figure 7 for compatibility with the strain. From shape 1 the value of s is positive and shape 2 is negative, from the Equations 12 and 13 depending on the shape [19], [21].

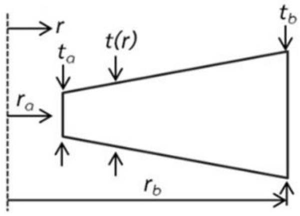

Fig. 7. Flywheel profile shape 1.

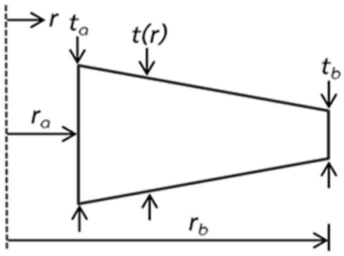

Fig. 8. Flywheel profile shape 2.

$$t(r) = hr^{-s} \tag{11}$$

Where: *t(r)* is the thickness at radius, *h* is the constant of the disk profile, and *r* is the radius of the flywheel.
Where:

$$h = \frac{t_b}{r_a^{-s}} \tag{12}$$

Where:

$$s = \frac{\ln \frac{t_b}{t_a}}{\ln \frac{r_b}{r_a}} \tag{13}$$

In Equations 12 and 13, the mathematical definitions of the constants are given in Equation 11. The details of the values from the calculated shapes 1 and 2 are listed in Table 2.

Table 2. Design parameters.

Shape	r_a	r_b	t_a	t_b	h	s	$t(r)$
Flywheel profile shape #1	0.035	0.185	0.07	0.13	0.069	0.37	0.128
Flywheel profile shape #2	0.065	0.185	0.13	0.07	0.189	-0.58	0.071

The calculation of the hoop and radial stresses from Equation 14 for the stress distribution control in variable-thickness discs [21] .

$$\frac{d}{dt}\left[t(r) r \sigma_r\right] - t(r)\sigma_\theta + t(r)\rho\omega^2 r^2 = 0 \tag{14}$$

In Equation 14 is valid as long as profile, t(r), satisfies the plane stress hypothesis, for example, tb<<rb [24]. Equation 14 is fully obtained in [19], [21], and the final solutions for the hoop and radial stresses are expressed in Equations 15 and 16.

$$\sigma_r = \frac{c_1}{h} r^{m_1+s-1} + \frac{c_2}{h} r^{m_2+s-1} - \frac{3+v}{8-(3+v)s}\rho\omega^2 r^2 \tag{15}$$

$$\sigma_t = \frac{c_1}{h} m_1 r^{m_1+s-1} + \frac{c_2}{h} m_2 r^{m_2+s-1} - \frac{1+3v}{8-(3+v)s}\rho\omega^2 r^2 \tag{16}$$

Where: c_1 and, c_2 is the constants of integration from Equations 17 and 18, ρ is the density of the material, ω is the angular velocity, *v* is the Poisson's ratio, and E is the Young's modulus, as shown in Table 4 [21].

$$c_1 = \rho\omega^2 \frac{(t_b^2 + t_a^2)}{E} \frac{(1-v)(3+v)}{8} \tag{17}$$

$$c_2 = \rho\omega^2 \frac{(t_b^2 t_a^2)}{E} \frac{(1-v)(3+v)}{8} \tag{18}$$

Where:

$$m_1, m_2 = -\frac{s}{2} \pm \sqrt{\left(\frac{s}{2}\right)^2 + (1+vs)} \tag{19}$$

The energy stored and released in a flywheel in Equation (20).

$$E = \frac{1}{2} J\omega^2 \tag{20}$$

Where:

$$J = \int_{r_a}^{r_b} 2\pi\rho r^3 t(r) dr \tag{21}$$

In Equation 21 is the polar definition of the moment of inertia is dependent on the shape factor and usually has a value between 0.3 and 1.0 [18]. The results are shown in Table 1, where the shape factor is related to performance. This is because since when 100% energy density from a material can be converted to energy density from mass and shape, In Equations 22 and 23 for energy density is as follows [10]:

$$e_v = k\sigma \tag{22}$$

$$e_m = k\frac{\sigma}{\rho} \tag{23}$$

The calculation results for the thick rim flywheel and conical disc flywheel are listed in Table 3. Calculated parameters of the flywheel from the calculation, the energy stored in the thick rim flywheel is 16.89KJ, and in the conical disc flywheel is 73.45 KJ. The energy density of the thick rim flywheel is 1.91 MJ and that of the conical disc flywheel is 2.54 MJ. In addition, the maximum hoop stress of the thick rim flywheel is 6.28 MPa and the conical disc flywheel is 1.71 MPa and the maximum radial stress of thick rim flywheel is 2.68 MPa and the conical disc flywheel is 3.16 MPa.

Based on the calculations used in the analysis of the suitability of shape 1 and shape 2, the researcher selects the main model, shape 1, which is more efficient because it has higher kinetic energy and higher energy density than shape 2.

Table 3. Calculated parameters of the flywheel.

Item	Type of flywheel: Thick Rim Flywheel	Conical Disc Flywheel
Weight (kg)	68	87.6
Volume (m^3)	0.0087	0.01164
Energy (KJ)	16.89	73.45
Moment of Inertia ($kg.m^2$)	1.38	6
Energy Density (MJ)	1.91	2.54
Shape Factor (K)	0.305	0.806
Maximum Hoop Stress: σ_t (MPa)	6.28	1.71
Maximum Radial Stress: σ_r (MPa)	2.68	3.16

3. STRUCTURE AND COMPONENTS OF FESS

3.1 Flywheel Material and Configurations

There are two materials used for making flywheels: a very popular heavy-duty ferrous material which is suitable for low rpm applications, and a composite material which is light, strong and suitable for high-speed applications, typically for magnetic bearings. Comparing prices, high-speed flywheels cost five times more than low-speed flywheels. [25]-[29].

According to research related to flywheels, there are two ways to increase the power content and power capacity of the flywheel. One way is to increase the speed of rotation of the flywheel. And the second way is to increase the moment of inertia [30]. The maximum speed of the flywheel is determined by the tensile strength. For this reason, extensive research is carried out on materials with tensile strength and high strength to increase the amount of energy storage. The problem with lightweight materials such as carbon fiber composites is that they interfere with absorption. In the field of advancements in flywheel technology [31], research has been carried out to develop medium-speed flywheels using existing tools and equipment such as ferrous materials.

The medium-speed flywheels benefit from low costs and sufficiently high-power densities. They have also been developed regarding the use of potential coated steel and compactness [25], [32]. In the future, steel flywheels are expected to be able to operate at higher speeds than composites in a safe and stress-free manner [23]. The steel materials have the expediency of low-cost production. This is because the material is readily available and the production route is accepted. As a result, the overall cost of the flywheel is further reduced. In addition, iron is easy to recycle compared to batteries. However, it is not necessary to recycle because it has a longer service life [24]. According to prior studies, there are two types of flywheels commonly used in industrial storage systems: steel flywheels operating at speeds below 6,000 rpm and composite flywheels operating at high speeds between 10,000 to 100,000 rpm. Nevertheless, there are some disadvantages to flywheels in terms of safety, including fatigue due to stopping and starting, flywheel operation at high speeds and at low speeds, stress fluctuations and vibration caused by flywheel imbalance in working at high speed [19], [26].

Steel, cast iron, aluminium alloy, and titanium are the most commonly used materials for the manufacture of flywheels. Cast iron is the most popular because of its long-term durability, and its design is easily adaptable [27].

The design of the flywheel involves, two decisions: flywheel material and shape. According to the material composition and manufacturing process used to form the final shape, not all the desired shapes can be created. This includes the symmetry of the flywheel when it is done because the flywheel must be rotated at high speed, so the workpiece must have a good rotational balance. Determining the form factor of each flywheel and factors related to the maximum working power σ (Pa), density ρ (kg/m^3), and energy per unit mass E (J/kg) are considered [28]

In this study, the researcher focuses on the production of steel flywheels operating at low speeds to reduce stress fluctuations and vibration as well as increase safety of flywheel operation while rotating. Three materials were considered in this study: SS400, ASTM A283 steel, and high-tensile steel AISI 4340. The mechanical properties of the materials are presented is shown in Table 4.

Table 4. Mechanical properties of the material [29]

Raw Material	Material of Flywheel: SS400	ASTM A283 Steel	High tensile Steel AISI 4340	Shaft: S45C
Density (kg/m^3)	7,860	7,800	7,830	7,700
Young's Modulus (GPa)	190	200	210	190
Tensile Strength (MPa)	400	310	850	569
Yield Strength (MPa)	205	165	635	343
Poisson's Ratio	0.26	0.25	0.29	0.27-0.3

When considering all three materials, stress, density, and energy per unit mass, the researcher selected SS400 steel round shaft material to produce the flywheel for testing. This type of steel round shaft is a white steel shaft that is strong, durable, and round. These properties are suitable for use in various industrial applications. Furthermore, it can be purchased in the market, and its price is not very high compared to other materials.

3.2 Induction Motor

The electric motor is responsible for converting electrical energy into mechanical energy. The operation of an electric motor is due to the interaction between the magnetic field of the magnet in the motor and the magnetic field generated by the current in the windings, causing the attraction and repulsion of the two magnetic fields. The induction motor is shown in Figure 9 and its specifications are single-phase 220 V, motor power is 3.7 Kw, horsepower is 5HP and pole of the motor is 4 pole.

Fig. 9. Induction motor (Mitsubishi brand) [33].

3.3 Generator

A generator is a tool used to convert mechanical energy into electrical energy, which is based on the working principle when the magnetic field moves through the coil or the moving coil cuts through a magnetic field to obtain electricity, as shown in Figure 10 the specifications are AC generator of single-phase 220 V, power generation is 3.5 kW.

Fig. 10. AC Generator single-phase (Fox brand).

3.4 Bearings

An important part of the FESS process is bearing design, if poorly designed it will lead to increased friction and more losses, as well as higher costs and maintenance [34]-[35], Bearings are important mechanical components. They are divided into two main categories: ball bearings and magnetic bearings. Ball bearings are used for low-speed flywheels and have disadvantages such as high friction, high loss, and low service life therefore need to be lubrication and maintenance for magnetic, they do not come into contact with the rotor, therefore do not wear and do not require lubrication. While in operation, it floats around the rotor shaft with no physical contact and low friction, thereby reducing system support losses at high speed [14], [36]-[39], however, they have a complicated control system. Magnetic bearings are permanent magnet types [10], or magnetic fields, from a current-carrying coil to support the weight of the flywheel [40]-[41]. There are three types of magnetic bearings: permanent magnetic bearings (PMB), superconducting magnetic bearings (SMB) and active magnetic bearings (AMB) [42]-[47]. When comparing the advantages of magnetic bearings, permanent magnetic bearings (PMB) are characterized by high rigidity, low cost of construction and installation as well as low loss [25], [47]-[49]. They are permanent magnets rather than electromagnets. Therefore, if choosing to use them with other types of ball bearings, they may not be able to support the operation [50]. From past research studies, the researcher can summarize the advantages and disadvantages of ball bearings and magnetic bearings in different types as shown in Table 5.

Table 5. Comparison of advantages and disadvantages of bearings.

Type of Bearings	Advantages	Disadvantages
Ball	- suitable for use at low and medium speeds - simple - economical price - compact	- need maintenance - need lubrication - high friction
Magnetic	- suitable for use at high speeds - high load-bearing capacity - longer service life - faster response - low loss	- high cost - complex control system

Comparing the types of ball bearings and magnetic bearings, it is found that magnetic bearings have properties that are better suited to the application, but using a magnetic bearing will result in a relatively high cost and complex operating system. The researcher designs the system to be used at low speed and needs to reduce costs. Therefore, ball bearings are selected. Bearings are shaft and flywheel support devices that help reduce friction. In this study, the rotational speed of the flywheel is less than 1,500 rpm. Therefore, the bearings used must be able to support the rotation at that speed, which can withstand the weight and forces that occur. Ball bearings come in a variety of shapes and sizes, depending on the purpose. The bearings used in this experiment are double-direction thrust ball bearings, which are sturdy and can sustain high speed. They are frequently found in industrial equipment, as shown in Figure 11.

Fig. 11. Double-direction thrust ball bearings [51], [52].

3.5 Flywheel Rotor Shaft

The shaft holds the flywheel and bearings. The shaft end is connected to the generator using coupling and is responsible for transmitting torque from the generator to the flywheel. The material used to build the shaft was made of S45C, with a diameter of 30 mm, and a length of 530 mm. The shaft layout is illustrated in Figure 12. The mechanical properties of the material are listed in Table 4.

4. EXPERIMENTAL

4.1 Experimental Design

Experimental design is a model, procedure, or process in an experiment to study the influence of independent variables, control variables, and dependent variables. There are many types of experimental schemes.

The experimental scheme used in this study is a multiple-time series design. The data are collected multiple times at different time intervals. There is a control box in the experimental process, but nothing is done on the control group. The symbols used are as follows: E stands for the experiment group, C for the control group, X for treatment, O_1 for pre-observation, and O_2 for post-observation. The experimental model [53] is shown in Figure 13.

4.2 The Variables used in the Experiment

In the design of the experiment, the researcher defined of variables used in this study from Figure 14. The details of the study are as follows:

1) Independent variables
   a) Thick rim flywheel
   b) Conical disc flywheel
2) Control variables
   a) Laboratory temperature for testing at 25^{0}C.
   b) Speed motor starts at 1,500 rpm
   c) Shape factor of flywheel is 0.305 for thick rim flywheel and 0.806 for conical disc flywheel
   d) Material used steel SS400
3) Dependent variables
   a) Time of flywheel energy storage
   b) Speed of flywheel
   c) Electromotive force

4.3 The Experimental Equipment

The main equipment related to the experimental equipment includes an induction motor, flywheel, generator, shaft, and bearing. The installation of the experimental equipment is shown in Figure 15. The specifications of the test equipment are presented in Table 6.

Table 6. Specifications of the test equipment.

Specifications	
1. Induction Motor (Mitsubishi Model)	Single-phase 220V 5HP (3.7kW) 4 Pole
2. Generator (Fox Model)	AC generator single-phase 3.5 kW
3. Shaft	Steel material S45C
4. Flywheel	Steel material SS400
5. Baring	Double-direction thrust ball
6. Coupling	Aluminum material

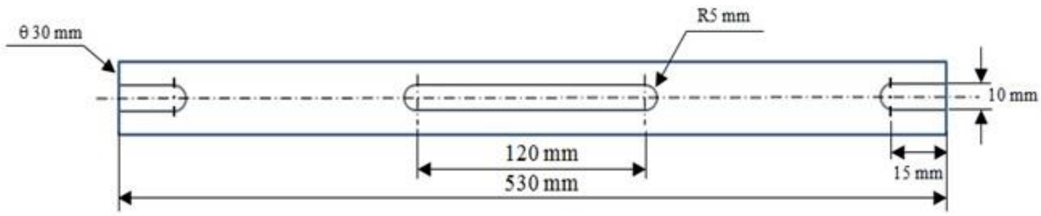

Fig. 12. Shaft of the flywheel.

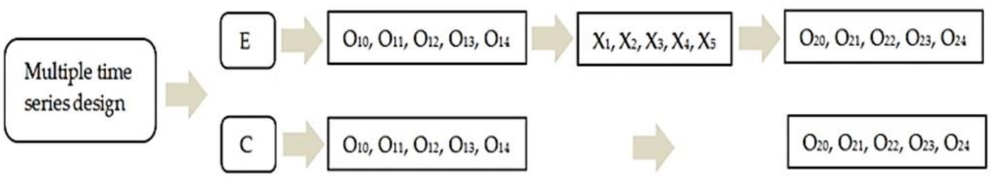

Fig. 13. Experimental design model.

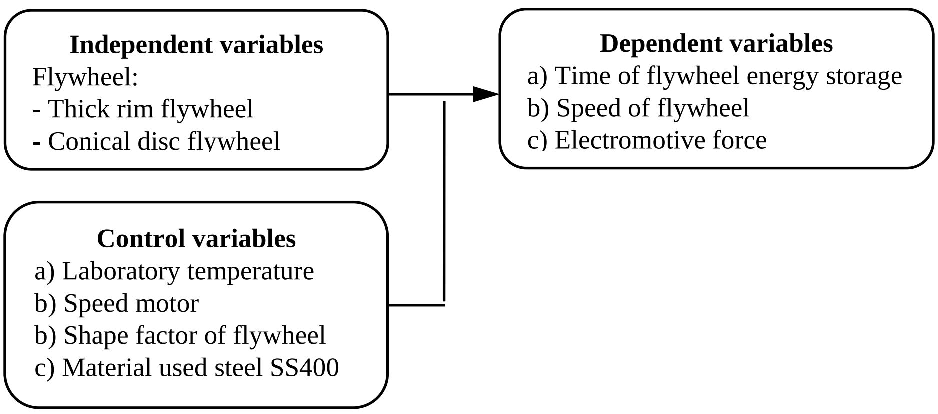

Fig. 14. Assumptions in the experiment.

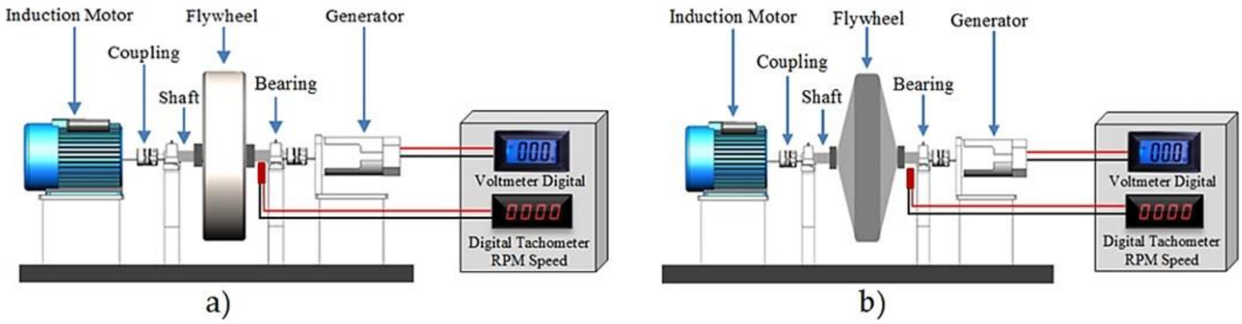

Fig. 15. Experimental equipment for without load. a) Testing of thick rim flywheel; b) Testing of conical disc flywheel.

4.4 The Experimental Procedure

When testing the production system of electromotive force with a flywheel energy storage system (FESS) under no-load conditions, the flywheel installation is directly connected to the electric motor and the generator, without the powertrain, to eliminate frictional losses due to power transmission. At the beginning of the work, the motor spins and directs power to the flywheel when the switch is in the ON position. This causes the flywheel to rotate according to the speed of the motor, and the flywheel transmits rotational power to the generator, causing the generator to produce an electromotive force. When the motor is spinning at 1,500 rpm, the switch is turned off to allow the motor to stop working. Simultaneously, the flywheel continues to spin with the inertia of the flywheel assembly. The generator can also generate electricity because the flywheel is still spinning until the inertia of the flywheel decreases and stops working . Therefore, the generator could not produce electricity. The factors used in this study were the time of flywheel energy storage, speed of the flywheel, and electromotive force. The test was conducted in a laboratory with temperature control at 25^{0}C. The researcher ran a total of 5 iterations to confirm that the experimental results from data collection were more accurate. In each test result, the measured value was the same, and the measuring instrument was calibrated after the end of each experiment. The timers were used for timing the flywheel rotation, a digital voltage meter was used to read the electromotive force a digital tachometer RPM speed to read the speed of the flywheel.

5. EXPERIMENTAL RESULTS

5.1 Experimental Results of Thick Rim Flywheel

The test findings can be summarized as follows from Table 7. The flywheel rotates for 120 s, from start to finish. The flywheel's speed decreases over time. At 0-10 s, the rotating speed acceptable for using electrical energy is 1,237-1,495 rpm. The highest electromotive force is 253 V, and it steadily decreases with the flywheel's spinning speed. The direct application requires an electromotive force of 195.4-253 V, a time of 0-10 s, and a speed of 1,237-1,495 rpm.

5.2 Experimental Results of Conical Disc Flywheel

From Table 8, the test findings can be summarized as follows. The flywheel rotates for 180 s, from start to finish. The flywheel's speed lowers over time. At 0-15 s, the rotational speed that is adequate for delivering electric power is 1,219-1,495 rpm. The highest electromotive force is 253 V, and it steadily decreases with the flywheel's spinning speed. The direct application requires an electromotive force of 191.2-253 V, a time of 15 s, and a speed of 1,219-1,495 rpm.

Table 7. Experimental results compare of time of flywheel energy storage, speed of flywheel and electromotive force of thick rim flywheel.

Time of flywheel energy storage (second)	Experimental results of thick rim flywheel											
	Speed of flywheel (rpm)						Electromotive force (volt)					
	Test 1	Test 2	Test 3	Test4	Test 5	Mean	Test 1	Test 2	Test 3	Test 4	Test 5	Mean
1	1,495	1,495	1,495	1,495	1,495	1,495	253	253	253	253	253	253
5	1,350	1,348	1,352	1,350	1,350	1,350	217	216	219	217	217	217.2
10	1,238	1,235	1,235	1,237	1,238	1,237	196	194	195	196	196	195.4
15	1,197	1,193	1,195	1,197	1,197	1,196	182	180	181	182	182	181.4
20	1,122	1,120	1,121	1,122	1,122	1,121	170	168	170	170	170	169.6
25	1,040	1,038	1,040	1,040	1,040	1,040	148	145	148	148	148	147.4
30	976	970	975	976	976	975	132	128	131	132	132	131
35	915	914	915	915	915	915	117	115	117	117	117	116.6
40	706	704	706	706	706	706	98	97	98	98	98	97.8

45	517	516	516	517	517	517	75	75	75	75	75	75
50	472	470	472	472	472	472	63	60	63	63	63	62.4
55	433	430	432	432	433	432	48	45	47	48	48	47.2
60	393	392	393	393	393	393	38	38	38	38	38	38
65	354	352	353	354	354	353	25	23	25	25	25	24.6
70	314	312	313	314	314	313	10	10	10	10	10	10
75	282	278	280	282	282	281	8	8	8	8	8	8
80	242	247	243	242	242	243	7	7	7	7	7	7
85	211	208	210	211	211	210	7	7	7	7	7	7
90	179	175	173	176	179	176	5	5	5	5	5	5
95	145	141	142	144	145	143	4	4	4	4	4	4
100	120	118	120	120	120	120	3	3	3	3	3	3
105	89	87	89	89	89	89	3	3	3	3	3	3
110	65	63	64	65	65	64	3	3	3	3	3	3
115	40	40	40	40	40	40	3	3	3	3	3	3
120	24	24	24	24	24	24	3	3	3	3	3	3
125	0	0	0	0	0	0	0	0	0	0	0	0

5.3 Comparison of Experimental Results Between Thick Rim Flywheel and Conical Disc Flywheel

The researcher calculated the mean of both types of flywheels using the test findings in Tables 7 and 8, and concluded in Table 9. The following is a summary of the situation.

The total flywheel rotation period of a conical disc flywheel is 180 s, which is longer than a thick rim flywheel's total flywheel rotation time of 120 s. The conical disc flywheel has a 60 s, greater energy retention period than the thick rim flywheel, according to the findings. When the speed of the conical disc flywheel and the thick rim flywheel are compared at the same moment, the conical disc flywheel has a higher speed. When the electromotive force of the conical disc flywheel and the thick rim flywheel are compared at the same time, the conical disc flywheel has a larger electromotive force.

Table 8. Experimental results compare of time of flywheel energy storage, speed of flywheel and electromotive force of the conical disc flywheel.

Time of flywheel energy storage (second)	Experimental results of the conical flywheel											
	Speed of flywheel (rpm)						Electromotive force (volt)					
	Test 1	Test 2	Test 3	Test4	Test 5	Mean	Test 1	Test 2	Test 3	Test 4	Test 5	Mean
1	1,495	1,495	1,495	1,495	1,495	1,495	253	253	253	253	253	253
5	1,386	1,382	1,386	1,385	1,386	1,385	226	224	226	226	226	225.6
10	1,288	1,283	1,285	1,285	1,285	1,285	206	205	206	206	206	205.8
15	1,218	1,218	1,220	1,220	1,220	1,219	190	190	192	192	192	191.2
20	1,160	1,160	1,162	1,160	1,160	1,160	178	178	182	178	178	178.8
25	1,107	1,107	1,110	1,107	1,107	1,108	164	164	166	164	164	164.4
30	1,049	1,049	1,051	1,049	1,049	1,049	152	152	153	152	152	152.2
35	1,005	1,005	1,005	1,005	1,005	1,005	140	140	140	140	140	140
40	956	963	964	956	956	959	125	127	127	127	127	126.6
45	914	914	914	914	914	914	115	115	115	115	115	115
50	876	876	877	876	876	876	102	102	104	102	103	102.6
55	670	670	670	670	670	670	92	92	92	92	92	92
60	521	521	523	521	521	521	85	85	86	85	85	85.2
65	489	489	489	489	489	489	74	74	74	74	74	74
70	464	464	465	464	464	464	63	63	63	63	63	63
75	437	437	437	437	437	437	52	52	52	52	52	52
80	414	416	416	414	414	415	40	43	43	43	43	42.4
85	393	392	393	393	393	393	35	34	35	35	35	34.8
90	370	373	373	372	370	372	26	27	27	27	27	26.8

95	348	348	350	348	348	348	16	16	17	16	16	16.2
100	329	329	332	329	329	330	10	10	12	10	10	10.4
105	305	305	308	305	305	306	10	10	10	11	11	10.4
110	288	288	289	288	288	288	8	8	8	8	8	8
115	266	265	266	266	266	266	8	7	8	8	8	7.8
120	246	247	251	246	246	247	7	7	8	8	8	7.6
125	205	204	207	205	205	205	5	5	5	5	5	5
130	190	194	196	190	190	192	5	5	5	5	5	5
135	165	168	168	165	165	166	4	4	4	4	4	4
140	148	148	150	148	148	148	4	4	4	4	4	4
145	129	129	132	129	129	130	4	4	4	4	4	4
150	113	113	115	113	113	113	3	3	3	3	3	3
155	95	94	95	95	95	95	3	3	3	3	3	3
160	80	80	80	80	80	80	3	3	3	3	3	3
165	61	61	61	61	61	61	3	3	3	3	3	3
170	45	46	46	45	45	45	2	2	2	2	2	2
175	37	37	37	37	37	37	2	2	2	2	2	2
180	22	22	22	22	22	22	2	2	2	2	2	2

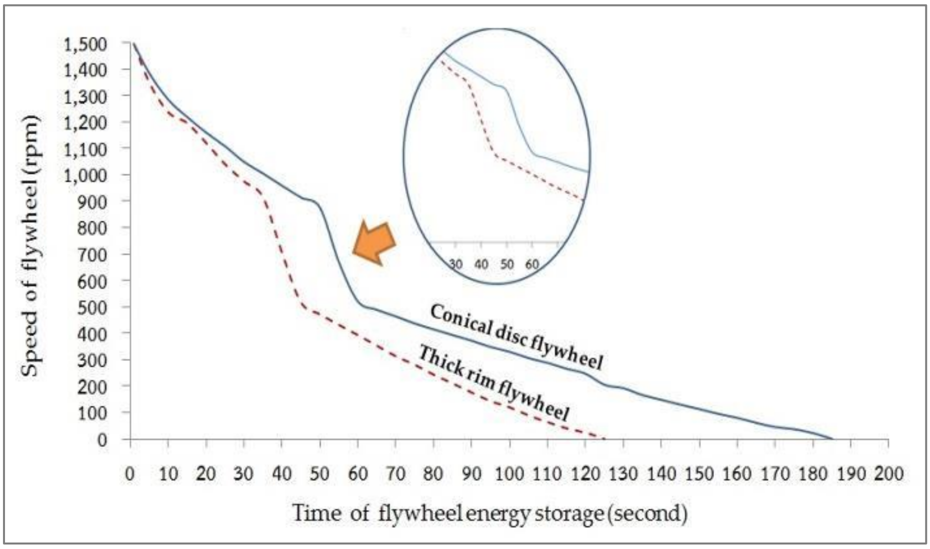

Fig. 16. Comparison of time of flywheel energy storage and speed of flywheel between thick rim flywheel with conical disc flywheel.

5.4 Comparison with the Time of Flywheel Energy Storage and Speed of Flywheel

When comparing the time of flywheel energy storage with the speed of the flywheel between the thick rim flywheel and the conical disc flywheel, it is found that the time of flywheel energy storage of conical disc flywheels is 180 s, and that of the thick rim flywheel is 120 s. During the same test period, the conical disc flywheel has a higher flywheel speed than the thick rim flywheel. The results of the comparison show that the time of flywheel energy storage of the conical disc flywheel is 60 s, longer than that of the thick rim flywheel. The rotating speed of both types of flywheels quickly drops in the range of rotational speed of the flywheel, about 500-900 rpm. It is also found that the rotation of the flywheel is slightly out of balance in that speed range (500-900 rpm), but when beyond that speed point, the flywheel spins normally. As a result, the researcher hypothesizes that it is caused by the functioning of double-direction thrust ball bearings that act independently of the flywheel's speed at that speed point, resulting in a large speed reduction.

The shape factor that influences the energy density and the weight of the flywheel that affects the rotating inertia force of the flywheel are the variables that result in greater speed of the conical disc flywheel, as shown in Figure 16.

5.5 The Comparison with the Time of Flywheel Energy Storage and Electromotive Force

When the generator is working, it produces electricity and an electromotive force. The results of testing to compare the time of flywheel energy storage with the electromotive force, according to the findings, the electromotive force of the conical disc flywheel is higher than that of the thick rim flywheel at all times of operation. The comparison shows that, the conical disc flywheel has a longer retention time to store energy to produce electromotive force and simultaneously has a higher voltage than energy from the thick rim flywheel. The variable that results in a higher electromotive force of the conical disc flywheel than that of the thick rim flywheel at every moment is the flywheel rotational speed, as shown in Figure 17.

Table 9. Experimental results comparison of thick rim flywheel with conical disc flywheel.

Time of flywheel energy storage (second)	Mean of the speed of flywheel and electromotive force			
	Speed of flywheel (rpm)		Electromotive force (volt)	
	Thick rim flywheel	Conical disc flywheel	Thick rim flywheel	Conical disc flywheel
1	1,495	1,495	253	253
5	1,350	1,385	217.2	225.6
10	1,237	1,285	195.4	205.8
15	1,196	1,219	181.4	191.2
20	1,121	1,160	169.6	178.8
25	1,040	1,108	147.4	164.4
30	975	1,049	131	152.2
35	915	1,005	116.6	140
40	706	959	97.8	126.6
45	517	914	75	115
50	472	876	62.4	102.6
55	432	670	47.2	92
60	393	521	38	85.2
65	353	489	24.6	74
70	313	464	10	63
75	281	437	8	52
80	243	415	7	42.4
85	210	393	7	34.8
90	176	372	5	26.8
95	143	348	4	16.2
100	120	330	3	10.4
105	89	306	3	10.4
110	64	288	3	8
115	40	266	3	7.8
120	24	247	3	7.6
125	0	205	0	5
130	0	192	0	5
135	0	166	0	4
140	0	148	0	4
145	0	130	0	4
150	0	113	0	3
155	0	95	0	3
160	0	80	0	3
165	0	61	0	3
170	0	45	0	2
175	0	37	0	2
180	0	22	0	2
185	0	0	0	0

5.6 The Comparison with the Time of Flywheel Energy Storage and Electromotive Force

The results of the test to compare the speed of flywheel and electromotive force between the thick rim flywheel and the conical disc flywheel show that the electromotive force depends on the speed of flywheel. Besides, in the same speed range, the conical disc flywheel has higher electromotive force than the thick rimmed flywheel, as shown in Figure 18.

From the analysis of the experimental results in terms of various factors obtained from the experiment, we can see differences in every factor. The most noticeable is the time of flywheel energy storage, speed of the flywheel, and electromotive force. It was also found that the conical disc flywheel had better results than the thick rim flywheel owing to the different mechanical properties of the conical disc flywheel.

When comparing the calculated values for the shape factor, kinetic energy, the moment of inertia, energy density, and stress obtained from theoretical calculations, it can be seen that the conical disc flywheel has a higher value than the thick rim flywheel. Therefore, the researcher believes that a conical disc flywheel can be further developed.

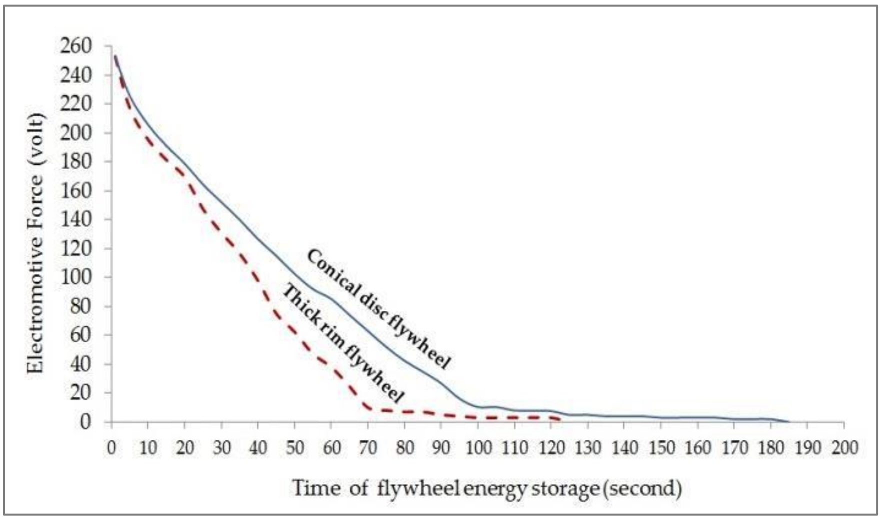

Fig. 17. Comparison of time of flywheel energy storage and electromotive force between thick rim flywheel with conical disc flywheel.

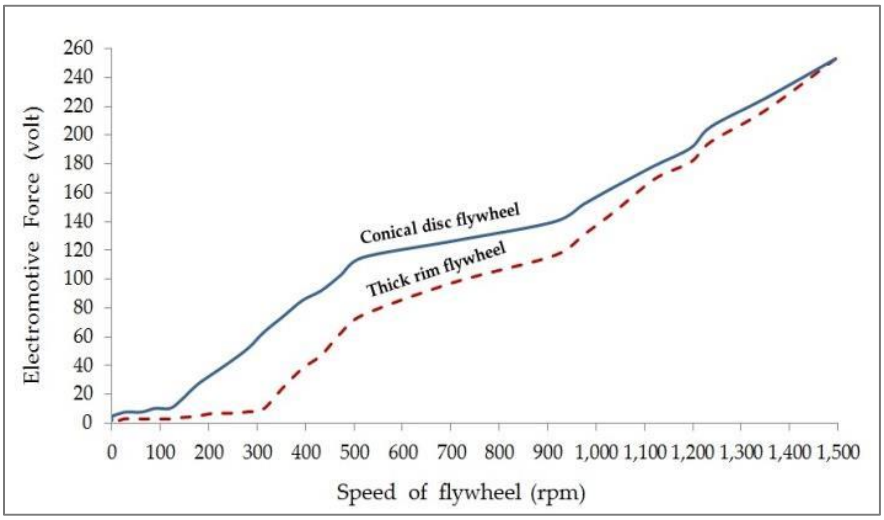

Fig. 18. Comparison of the speed of flywheel and electromotive force between thick rim flywheel with conical disc flywheel.

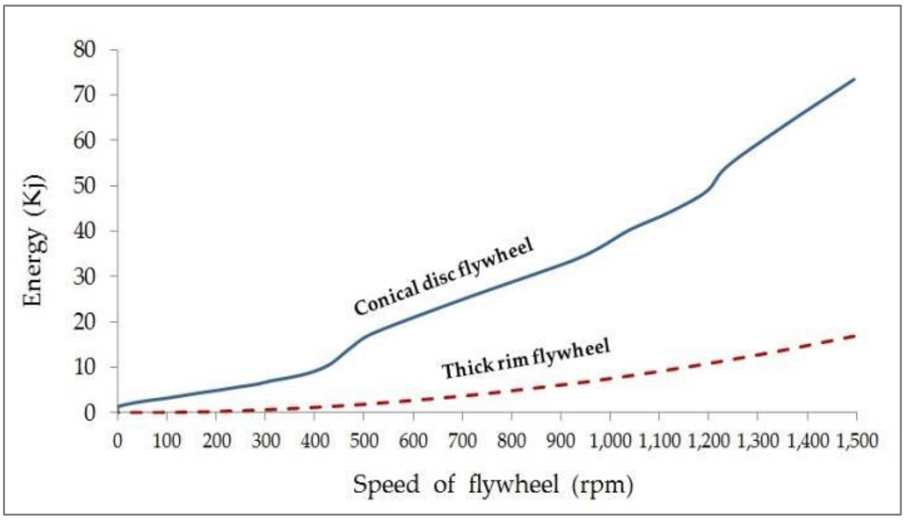

Fig. 19. Comparison of energy between thick rim flywheel with conical disc flywheel.

5.7 The Comparison of Kinetic Energy Between the Thick Rim Flywheel and the Conical Disc Flywheel

Equation 1 is used to calculate kinetic energy and Equation 5 is used to calculate angular velocity. The angular velocity depends on the rotational speed of the flywheel obtained from the test. When the result of the velocity obtained from the experiment is substituted in Equation 5, the angular velocity is gained. From Table 10, the speed of flywheel of the conical disc flywheel compared at the same time and at different times is higher than that of the thick rim flywheel. When calculating angular velocity and kinetic energy, he conical disc flywheel has higher and longer storage of kinetic energy than the thick rim flywheel.

From Figure 19 shows the comparison of the kinetic energy between the thick rimmed flywheel and the conical disc flywheel. From the graph, it can be seen that the kinetic power of the conical disc flywheel is higher than the thick rim flywheel at all speeds.

This ensures that when the kinetic energy is high, it will result in good energy retention as well. Furthermore, it is found that the conical disc flywheel has better energy storage than the thick rimmed flywheel. Important factors for conical disc flywheel performance are the shape factor affecting energy density, the weight of the flywheel affecting inertia, and angular velocity affecting energy. These factors make the conical disc flywheel perform better than the thick rimmed flywheel.

6. CONCLUSION

By designing and building a flywheel energy storage kit, the researcher starts by considering the shape factor of the flywheel. As a result, the thick rim flywheel and conical disc flywheel are chosen for study in the design and to be tested. The researcher then uses the shape of the flywheel to analyze the mathematical profile to calculate the kinetic energy, moment of inertia, energy density, hoop and radial stress according to the shape of the flywheel. After that, an experimental design in multiple-time series design is carried out and a test is performed to evaluate its performance. Summary of the results from the factors studied in all three areas:

Factor 1, Time of flywheel energy storage: It was found that the conical disc flywheel had a time value of 180 s and a thick rim flywheel with a time value of 120 s. For the time of flywheel energy storage during which the electromotive force can be generated for use, it is found that the conical disc flywheel has a run time of 0-15 s, which is longer than the thick rim flywheel, which is 0-10

Factor 2, Speed of flywheel: It is found that the conical disc flywheel has a higher rotational speed than the thick rim flywheel at every test time .The suitable rotational speed for the production of the applied electromotive force was 1,219-1,495 rpm at time value of 0-15 s.

Table 10. Experimental results comparison of kinetic energy, angular velocity, speed of flywheel between thick rim flywheel with conical disc flywheel.

Time of flywheel energy storage (second)	Speed of flywheel (rpm)		Angular velocity (rad/s)		Kinetic energy (Kj)	
	Thick rim flywheel	Conical disc flywheel	Thick rim flywheel	Conical disc flywheel	Thick rim flywheel	Conical disc flywheel
1	1,495	1,495	156.47	156.47	16.895	73.457
5	1,350	1,385	141.3	144.96	13.776	63.040
10	1,237	1,285	129.47	134.5	11.566	54.270
15	1,196	1,219	125.18	127.59	10.812	48.837
20	1,121	1,160	117.33	121.41	9.498	44.221
25	1,040	1,108	108.85	115.97	8.175	40.347
30	975	1,049	102.05	109.8	7.185	36.168
35	915	1,005	95.77	105.19	6.328	33.194
40	706	959	73.89	100.38	3.767	30.228
45	517	914	54.11	95.67	2.020	27.458
50	472	876	49.4	91.69	1.683	25.221
55	432	670	45.22	70.13	1.410	14.754
60	393	521	41.13	54.53	1.167	8.920
65	353	489	36.95	51.18	0.942	7.858
70	313	464	32.76	48.57	0.740	7.077
75	281	437	29.41	45.74	0.596	6.276
80	243	415	25.43	43.44	0.446	5.661
85	210	393	21.98	41.13	0.333	5.075
90	176	372	18.42	38.94	0.234	4.548
95	143	348	14.97	36.42	0.154	3.979
100	120	330	12.56	34.54	0.108	3.579

105	89	306	9.32	32.03	0.059	3.077
110	64	288	6.7	30.14	0.030	2.725
115	40	266	4.19	27.84	0.012	2.325
120	24	247	2.51	25.85	0.0434	2.004
125	0	205	0	21.46	0	1.381
130	0	192	0	20.1	0	1.212
135	0	166	0	17.37	0	0.905
140	0	148	0	15.49	0	0.719
145	0	130	0	13.61	0	0.555
150	0	113	0	11.83	0	0.419
155	0	95	0	9.94	0	0.296
160	0	80	0	8.37	0	0.210
165	0	61	0	6.38	0	0.122
170	0	45	0	4.71	0	0.066
175	0	37	0	3.87	0	0.044
180	0	22	0	2.3	0	0.015
185	0	0	0	0	0	0

Factor 3, The electromotive force aspect, at the same time, it was found that the electromotive force produced by the conical disc flywheel generator is higher than that of the thick rim flywheel. The usable ranges of the electromotive force produced by the conical disc flywheel generator were 191.2-253 V.

The speed is suitable for practical application at speeds from 1,000-1,495 rpm.at a time value of 0-15 s. The key variables that affect the experiment are: kinetic energy, angular velocity, shape factor and energy density. From the experimental data, it is discovered that the conical disc flywheel performs better than the thick rim flywheel when considering the results obtained from the test. The researcher studied the efficiency of energy storage systems by storage methods. The methods are optimizing energy storage systems by designing permanent magnet bearings with magnetic clutch devices for cutting the power and reducing the friction of the motor control system and emphasizing the use of electricity in the energy storage system without electrical interference.

7. COST ANALYSIS

This comparison in Table 11, shows that from 2018 until now, a flywheel rotor with a flywheel energy storage system has low cost but high efficiency. The disc flywheel performed better than the thick rim flywheel in all factors studied because of being created at the real size and shape of flywheels, was tested with real working conditions. (without-load condition), be an application multiple-time series design for use in experimental design, be able to work at low revs well, reduced vibration while working, be high security, make the flywheel high strength, and be low price.

Table 11. Comparative of cost analysis.

Research topic	Description	Conclusions	Advantages	Disadvantages
Xiaojun L., and Alan P., 2022. A review of flywheel energy storage systems state of the art and opportunities [54].	To review the latest developments in FESS technology that explores different designs, approaches, subsystem options, performances, cost, applications, and possible directions for future development of FESS technology.	Utility using of FESS development, low-cost flywheel materials, and new materials with compact designs can increase the power, energy density, and the application of energy harvesting systems in the form of a hybrid power system.	– More compact size – Use new materials to make flywheels for increasing more energy and density.	– Need new materials to make flywheel and need more experimental research.
Xiaojun L., Alan P and Zhiyang W., 2021. A Combination 5-DOF Active Magnetic Bearing for Energy Storage Flywheels	To study a combination -5 DOF active magnetic bearing for energy storage flywheels. An active magnetic bearing was an integrated design of -5DOF (C5AMB); high-	An active magnetic bearing in the form of -5DOF can provide good buoyancy stability for the flywheel weighing 5, 540kg	Be used and efficient to work with large flywheels.	– Limitations in design. – non-silicon solid steel for AMB. – High cost.

[55].	strength steel with hub-less, shaft-less, and hub-less for energy storage flywheel systems (SHFES). Its energy density increases twice, enables radially, axial, and tilting synchronously, and is ready for use at a low cost.	and has low power consumption. Integrated magnetic bearing performance can be applied to larger flywheels, too.		
Xiaojun L., Bahareh A., Alan P., Zhiyang W. and Hamid T., 2018. A Utility Scale Flywheel Energy Storage System with a Shaft-less, Hub-less, High Strength Steel Rotor [56].	To study the utility scale flywheel energy storage system with a shaft-less, hub-less, high strength steel rotor. Flywheel style was made of high-strength and low-cost steel, the system is equipped with active magnetic bearings for controlling the buoyancy of the flywheel	This flywheel can be used more commercially because of its innovative design with less shaft. CAMB can provides high power and energy capacity that seems to be a modern advancement in flywheel technology.	– Compacted shape – Less axles – Not use wheel hubs. – Active magnetic ball bearings that reduce friction well. – High strength material	– Tested in real situation. – Require deeply study about the shaftless flywheel. – Require a lot of CPU in controlling. – Quite expensive
Aphichit S., 2022. Experimental Design of Flywheel Rotor with a Flywheel Energy Storage System for Residential uses.	To study the design and construction of a conical disc flywheel compared with a thick rim flywheel and to compare the performance of the flywheel. The material for this flywheel is ss 400 steel. The experimental design is based on the multiple-time series design technique. The experiment considers the factors in terms of the time of flywheel energy storage, speed of the flywheel, electromotive force, and kinetic energy.	The conical disc flywheel performed is better than the thick rim flywheel in all factors studied.	– Real size and shape of flywheels. – No-load condition – Need multiple-time series design in experimental design. – Work at low revs well – Reducing vibration while working – High security and high strength. – Low price.	– Loss of energy to friction. – Permanent magnet bearings for reducing friction even though untested with actual load conditions.

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NOMENCLATURE

E	Kinetic energy	Joule
I	Moment of inertia	$Kg.m^2$
t	Thickness of flywheel	M
r_o	Outer radius of flywheel	M
r_i	Inside radius of flywheel	m
M	Mass	Kg
k	Shape factor	
e_v	Energy density per unit volume	MJ
e_m	Energy density per unit mass	MJ
c_1, c_2	Constants of integration	
h	Constant of disk profile	
m_1, m_2	Constants	
r	Radius of flywheel	m
r_a	Inside radius	m
r_b	Outer radius	m
s	Exponential constant of disk profile	
$t(r)$	Thickness at radius	m
t_a	Thickness at r_a	m
t_b	Thickness at r_b	m
v	Poisson's ratio	
ρ	Density of material	kg/m^3
ω	Angular velocity	rad/s
σ_r	Radial stress	MPa

σ_t	Hoop stress	MPa
FESS	Flywheel energy storage system	

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