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Model Predictive-based PID Controller Design and Order Reduction of Higher Order System

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Abstract – *This paper demonstrates the implementation of controllers that use control strategies like Proportional-Integral-Derivative (PID) and Model Predictive Control (MPC) for reducing the higher-order systems. Based on the Routh stability criterion, a model reduction technique is presented. The elements in the high-order denominator and numerator's Routh stability arrays are directly used to calculate the reduced order transfer function. This paper introduces a method for discovering MPC-based PID controllers for complex dynamic systems (higher-order systems). Higher-order systems significantly hamper controller design because of their complex dynamics and higher degree of differential equations. It is used to optimize a controller to match the required performance criteria. Here, the second-order system is initially tested, and then the higher-order system is assessed by the proposed PID controller. A general MPC-based second-order system is defined and compared with the observed closed-loop response for the stabilization process. Simulation results conducted on various higher-order systems consistently show that the MPC-based PID controller outperforms conventional tuning methods in terms of stability, response time, and overall performance. The findings underline the method's efficiency in achieving optimal control of higher-order systems while maintaining computational simplicity.*

Keywords – Higher Order Systems, Model Order Reduction, Routh Stability Criterion, PID Controller, Model Predictive Control.

1. INTRODUCTION

The mathematical technique known as "model order simplification" is used to approximate large-scale systems to lower-order systems while preserving key elements of the original systems. The last forty years have seen a significant increase in utilising model order simplification techniques in system modelling and design. The literature contains a plethora of methodologies, and ongoing research indicates how important it is to create a robust and dependable reduced-order system for the study of higher-order models and for the conception purpose [1]. When a system's dimensions are so great that even with acceptable computational effort, traditional, analysis, design, control, and modelling computation techniques are unable to produce accurate results, the system is considered large-scale [2]. Therefore, model reduction for a system of higher order is a crucial issue for both analysis and controller synthesis of a working system. There are numerous approaches within the scholarly works that use domains of time and frequency methods to reduce huge systems to their low order. An essential method for order reduction in a frequency domain is the Pade approximation. [2]. The available literature significantly addressed that the reduced-order model's stability is not guaranteed using the Pade approximation technique; as a result, numerous other approaches have occasionally been documented. These include the Hurwitz polynomial approximation, the Routh approximation and the Mihailov stability criterion. Lower-order linear systems are approximated from higher-order ones the subject of numerous investigations [3]. This correspondence proposes a straightforward approach of immediately decreasing the order of the system as determined by the Routh stability array [4].

PID controllers are essential to automation and are widely utilized in many sectors for a range of purposes. Because higher order, time delay, and nonlinearities are frequently present in industrial facilities, it has become increasingly important to appropriately tune the gains of the PID controller[5] [6]. Higher-order network models capture complicated relationships beyond bilateral interactions, offering new perspectives on the understanding of complex systems [7]. We reduced the higher order models into lower order as it is quite challenging to analyze the model behavior and characteristics of complex dynamic systems and reduced order modelling is a suitable fit for real-time applications like augmented reality where maintaining a high frame rate is essential because it significantly lowers computational expenses [8]. To ensure that the obtained lower order preserves the qualities of the original system, a lower order system must be obtained [9]. By doing this, the variances that occur during the creation and application of appropriate control system components that are connected to the original system are reduced [10].

Higher order systems are typically estimated to lower order systems, in these works using model reduction approaches such as Routh stability criterion techniques [11]. The Routh stability criterion is a useful technique for matching the initial Markov parameters of the system and its transient components to the simplified

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model. In the lower order system, it does not preserve the higher order model's dominant poles for non-minimum phase systems. The Routh stability approach occasionally yields the same lower-order model for several kinds of large-scale systems; the work presented by V. Singh identifies this non-uniqueness in 1979. The approach that is being given is really straightforward and ensures that the large-scale stable systems' reduced models remain stable [1].

Three terms are commonly used to refer to the PID controller: proportional (P), derivative (D) and integral (I). The ideal closed-loop system output can be accomplished by properly adjusting the controller's settings. Once the three settings in the PID controller algorithm are optimally tuned, the controller can offer reduced error performance and optimized control action [11]. For fine-tuning the PID controller, there are hundreds of tools, techniques, and theories accessible like the Ziegler- Nichols Method. Recent advances in computing techniques have led to the frequent proposal of optimization algorithms to adjust control settings and discover the best possible performance [7].

This research suggests a straightforward algebraic method for creating a PID controller for a linear time-invariant (LTI) system. Utilizing an adjunct polynomial scheme, The MPC is suggested to derive the fundamentals and create a refined second-order system that represents the system's initial characteristics by taking a second-order system from the first higher-order system. The best PID gain values are found by this technique. The suggested scheme's resilience is evaluated against a second-order model that is formulated using an MPC [11]. One of the more promising sophisticated control strategies is MPC. Conversely, one popular industrial controller is the PID controller, which is renowned for being reliable and straightforward [12]. It is difficult to modify the PID's settings while taking system limits into account. To increase the performance of industrial processes while taking operational limits into account, both MPC and PID controllers were combined in a hierarchical framework in this work[13].

2. PROBLEM DETERMINATION

Let us consider a high-order transfer function F(s) as:

$$G(s) = \frac{\sum_{i=0}^{p-1} m_i s^i}{\sum_{i=0}^{q} n_i s^i} \quad (1)$$

Equation (2) represents the lower-order model of the system found in equation (1). The primary necessity for the lower-order model is that it must have all the crucial parameters of the original system.

$$G_r(s) = \frac{\sum_{j=0}^{r-1} d_j s^j}{\sum_{j=0}^{r} c_j s^j} \quad (2)$$

3. ROUTH STABILITY CRITERION

Assume that the high-order system's transfer function is-

$$G(s) = \frac{p_{11}s^u + p_{21}s^{u-1} + p_{12}s^{u-2} + p_{22}s^{u-3}\ldots}{q_{11}s^v + q_{21}s^{v-1} + q_{12}s^{v-2} + q_{22}s^{v-3}\ldots} \quad (3)$$

Where u > v.

Tables 1 and 2 below display Routh's stability criterion for the numerator and denominator polynomials of equation (3).

The coefficients of the two polynomials are used separately to build the first two rows of Tables 1 and 2, which is noteworthy [13].

Table. 1. Stability array of numerator.

p_{11}	p_{12}	p_{13}	p_{14}	.
p_{21}	p_{22}	p_{23}	p_{24}	.
p_{31}	p_{32}	p_{33}	.	.
p_{41}	p_{42}	p_{43}	.	.
.	.	.	.	.
.	.	.	.	.
$p_{u,1}$				
$p_{u+1,1}$				

Table. 2. Stability array of denominator.

n_{11}	n_{12}	n_{13}	n_{14}	.
n_{21}	n_{22}	n_{23}	n_{24}	.
n_{31}	n_{32}	n_{33}	.	.
n_{41}	n_{42}	n_{43}	.	.
.	.	.	.	.
.	.	.	.	.
$n_{v-2,1}$	$n_{v-2,2}$	.	.	.
$n_{v-1,1}$	$n_{v-1,2}$	.	.	.
$n_{v,1}$	.	.	.	.
$n_{v+1,1}$	.	.	.	.

Each table has odd coefficients in the first row and even coefficients in the second row. The tables are finished in the traditional manner by using the algorithms to calculate the coefficients of subsequent rows [14].

4. MODEL PREDICTIVE CONTROL (MPC)

The controlled system's future behaviour is predicted using a process model by a collection of advanced control techniques called MPC. MPC implicitly determines the control law by resolving an optimization problem, which may be constrained. As a result, the focus of controller design now focuses on modelling the process that has to be governed. Given that these models can be found in numerous technical fields, MPC eliminates the first obstacle to implementing control [15]. The system parameters' physical understanding is preserved by its implicit formulation, which makes controller tuning easier. Therefore, by altering a process model, MPC offers more intuitive parameterization at the cost of greater computational effort than classical controllers [16]. The MPC solves a limited optimization problem in order to minimize the objective function of the system while obtaining the optimal output. This control mechanism is unique in that it takes restrictions into account immediately. The cost function is frequently written so that, over a given horizon N_2, a given reference r is tracked by the system output y. The system is only provided with the initial value derived from the optimal output trajectory. The same forecast and optimization are done each time. For this reason, MPC control is also known as "receding horizon" control [17]. Essentially, the notion is that optimality is attained over an extended period by short-term (predictive) optimization. Given that a proximal forecast's mistake is typically smaller than a distant prediction's, this is taken to be true. Prediction and optimization are integrated, which is the main difference from conventional control techniques that use precomputed control principles [12]. The forecast horizon N_2 ought to should be sufficiently lengthy to appropriately reflect the changes to the manipulated variable which significantly affects the variable under control. Delays could arise accounted for by either the system model or the lower prediction horizon N_1. The latter is often easier to understand and sets the lower prediction horizon to $N_1 = 1$ to allow for computation time. Consequently, one time step completes the calculation, and the result u is put into practice after the next time step [18]. MPC is a kind of control method that estimates the system's future behaviour over an extended time horizon using a mathematical description of the system. MPC is included following steps such as:

4.1 System Modeling: Transfer Function

MPC is designed with a SISO transfer function, given by equation (4).

$$G(s)=\frac{N(s)}{D(s)}=\frac{b_0 s^{k}+b_1 s^{k+1}+....+b_m}{a_0 s^{j}+a_1 s^{j-1}+....+a_j} \tag{4}$$

Here, z represents the discrete-time variable.

4.2 Discretization

For MPC, the continuous time model is needed for discretization with sampling period Ts. Using methods of zero-order Hold (ZOH), a discrete transfer function is represented by equation (5).

$$G(z)=\frac{N(z)}{D(z)}=\frac{b_0 s^{-k}+b_1 s^{-k-1}+....+b_m}{a_0 s^{-j}+a_1 s^{-j-1}+....+a_j} \tag{5}$$

4.3 State-Space Representation

Convert the discrete transfer function to a state-space by equation (6).

$$\begin{aligned} x_d(k+1) &= A_d x(k) + B_d u_d(k) \\ y_d(k) &= C_d x(k) + D_d u_d(k) \end{aligned} \tag{6}$$

Where, xd (k) is the state at step k, and $u_d(k)$ is the control input. $y_d(k)$ is the system output. A_d, B_d, C_d, D_d are matrices derived from the transfer function.

4.3 MPC Problem Formulation

The number of future time steps over which the model behavior is anticipated is known as the prediction horizon, or Np. The number of time steps is the control horizon Nc over which the control inputs are minimized or maximized. The cost function typically has the following equation (7).

$$J=\sum_{n=0}^{N_p}\left[\left\|y(n)-y_{ref}(n)\right\|^2 * Q+\left\|\Delta u(n)\right\|^2 * R\right] \tag{7}$$

Where $y_{ref}(n)$ is the reference output at time step 'n' and $\Delta u(n) = u(n) - u(n-1)$ is the change in the control input. Q and R are weighting matrices that determines the relative importance of the tracking error and control effort.

4.4 Constraints

The optimization problem must satisfy system constraints are given by equation (8)

$$\begin{aligned} x_{min} &\le x(n) \le x_{max} \\ u_{min} &\le u(n) \le u_{max} \end{aligned} \tag{8}$$

These constraints ensure that the states and control inputs remain within feasible bounds.

4.5 Optimization Problem

The control action is acquired by resolving the following optimization problem at each time step with equation (9).

$$O_p = \underset{u(0),......,u(N_c-1)}{\mathrm{Max}} J \tag{9}$$

There all the above-mentioned equations are useful for the implementation of MPC.

5. RESULTS AND DISCUSSIONS

Consider the LTI system as a transfer function given in equation (10).

Below is the whole table with the numerator and denominator-

Numerator table-

35.0	13285.0	278376.0	482964.0
1086.0	82402.0	511812.0	194480.0
10629.	261881.1	476696.1	
55645.5	463107.8	194480.0	
173419.1	439546.9		
322069.0	194480.0		
334828.5			
194480.0			

Denominator Table-

1.0	437	11870	37492	9600
33.0	3017.0	27470	28880	
345.6	11037.6	36616.8	9600.0	
1963.0	23973.4	27963.9		
6817.2	31694.0	9600		
14847.1	25199.0			
20123.7	9600.0			
18116.2				
9600.0				

$$G(s)=\frac{35.0s^7+1086.0s^6+13285.0s^5+82402.0s^4+278376.0s^3+511812.0s^2+482964.0s+194480.0}{s^8+33.0s^7+437.0s^6+3017.0s^5+11870.0s^4+27470.0s^3+37492.0s^2+28880.0s+9600.0} \quad (10)$$

Equation (11) was obtained by applying the Routh Stability criterion method in equation (10).

$$G_r=\frac{334828.50s+194480.0}{20123.70s^2+18116.20s+9600.0} \quad (11)$$

Table 3: Gain parameters for Original System (OS) and Reduced System (RS).

PID Gain Parameters	Original System	Reduced System
K_p	0.480	0.900
K_i	17.450	32.895
K_d	0.003	0.005

The gain parameters are listed in Table 3. In this table, Kp, Ki, and Kd values are 0.480, 17.450, and 0.003 for the original system. The reduced system contains gain parameters are 0.900, 32.895, and 0.005.

5.1 Step response of OS and RS by PID controller

Figure 1 displays the step response of OS and RS with a PID controller. The OS and RS with PID controller are both identical, as shown in the figure1.

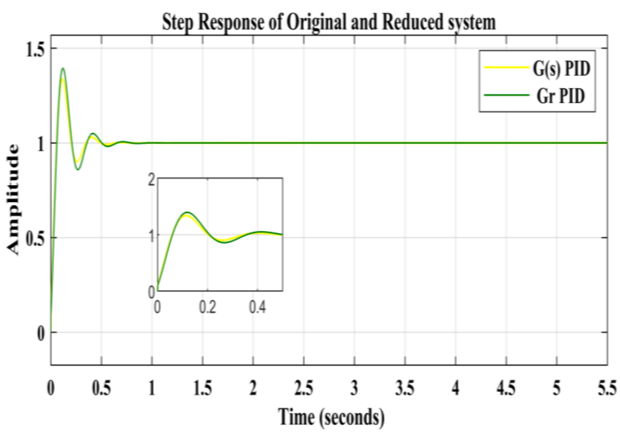

Fig. 1. Step response of OS and RS by PID controller

5.2 Step response of OS and RS by MPC controller

Figure 2 shows the step response of the OS and RS by the MPC controller. The OS and RS with the MPC controller are identical as shown in Figure 2.

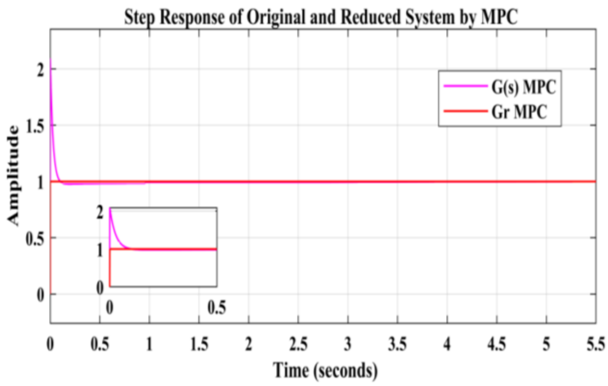

Fig. 2. Step response of OS and RS by MPC controller.

5.3 Comparison between Step response of OS and RS by PID and MPC controller

The contrast is displayed in Figure 3 between the step response of the OS and RS by PID and MPC controller. Analysis reveals that the suggested controller's design provides improved robustness and excellent performance. The results of the simulation demonstrate an improvement in the step response's time domain specifications. The significance of settling down is high in the case of the original and reduced system with a PID controller. The significance of settling down is minimal in the case of the OS and RS with an MPC controller. Using the MPC approach could be simultaneously found for better tuning of the controller parameter.

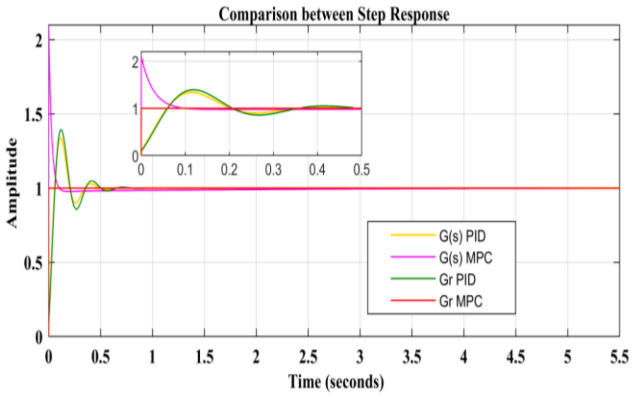

Fig. 3. Comparison between Step response of OS and RS by PID and MPC controller.

Table 4: Characteristics parameters of G(s) and Gr by PID

Characteristics	OS	RS
Rise Time	0.0606	0.1165
Settling Time	0.1088	0.7747
Settling Min	0.1088	0.7747
Settling Max	0.8614	0.8629
Overshoot	0.1052	0.9757
Peak Time	0.1974	0.4186

Table 4 presents a tabulation of the original system and a decreased system performance comparison. The purpose of the response is to demonstrate the efficacy and precise representation of the suggested strategy.

6. CONCLUSION

This work proposes a method for large-scale model approximation into reliable lower-order equivalents. It is clear from the study conducted for this paper that the suggested Routh stability criteria approach approximates a higher-order model at a lower level. This demonstrates unequivocally that the proposed approach creates a stable system of decreased order that shares the increased (higher) order system's properties in the frequency domain. The strategy that has been provided ensures that the fundamental traits of the lower-order system, including stability, passivity, and steady-state value, will not change from the original model. The derived simplified model shows virtually equivalent time-domain performance parameters including settling time, rising time, peak overshoot, and steady-state error.

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