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Power Flow Solution for Balanced Radial Distribution Networks: A New Approach

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Abstract – *This paper presents an efficient power flow solution method to analyze balanced radial distribution networks having high R/X ratio by using Tellegen's theorem (TT) and Kirchhoff's laws. A set of iterative power flow equations has been proposed at the computational stage to find more accurate value of injected current from the upstream at each node of a network. A computer program has been developed with the help of MATLAB Ver 7.0. The results obtained exhibit a better rate of convergence when compared to the existing backward-forward methods of power flow solution for various balanced radial distribution networks. The proposed method has a flexibility to extend for 3-phase networks also.*

Keywords – Backward-forward sweep, distribution network, injected power, and radial.

1. INTRODUCTION

Electrical power flow study, forms an important part of power system analysis. This study is necessary for planning, economic operation and control of existing system as well as planning its future expansion in transmission as well as in distribution system. Modern Computer Aided Distribution System (MCADS) analysis requires precise formulation and efficient algorithms to resolve power flow solution in a complex radial distribution network. Some of the basic power flow algorithms, which are already developed and applied, include methods of Newton Raphson (NR) and Gauss Seidal (GS).

The method developed as in [1] for transmission network, reveals that NR method of power flow algorithm for solving high R/X ratio of distributed network was not successful, as it failed to converge in several network studies. The decoupling assumptions necessary for simplifications used in the standard fast-decoupled NR method as documented in [2] are often not valid for distribution system. Implementation of fast decoupled power flow for unbalanced radial distribution systems as in [3] requires special process of ordering for the input data format as mentioned in [4]. The ladder network theory as in [5] considers dependency of the load demands on voltage changes, which is much similar to the forward-backward substitution method as reported in [6]. The power flow methods for both radial and weakly meshed network structure are presented in [7]. In case of radial network current, is taken as the variable, which proves to be to less efficient to converge power flow solution. Likewise, many other algorithms for radial distribution network have been developed as in [8]-[10].

The network topology based algorithm as in [11] has problems for large radial networks, like developing the special matrices by writing a special program. It also takes more computation time for convergence of solution, when compared with the backward-forward technique and ladder network theory methods as expressed in [12] for the given data as in [13].The power flow solution obtained using node current variable technique as in [14] was also found to be less efficient. In the proposed method, most recent upstream and power losses as variable are used as variables, to find efficient solution, when compared to [11] and [14] power flow methods.

The backward-forward-forward concept is effectively utilized to develop power flow equations for distribution system, and is presented in this work. Tellegen's theorem (TT) as in [15] states that the algebraic sum of complex powers meeting at a node is zero. Using TT, the backward sweep of load power and losses are performed from downstream to upstream of a network. The efficient technique developed during the forward sweep is helpful to compute downstream branch currents. Applying KVL in the forward sweep power flow solution can be obtained directly. The validation of algorithm performance is carried out by writing programs in MATLAB specifications as in [16] for the networks reported in [17] and [18].

2. SOLUTION METHODOLOGY

Numbering Technique

The node oriented numbering for a typical radial distribution network is shown in Figures 1 and 2, having 'n' nodes and b (= n-1) branches.

The network nodes are numbered level by level from left to right side of the network is reached till the end of network is reached. In case of network branch, which lies between k^{th} top node and $(k+1)^{th}$ bottom node, it is suggested that the branch number be the downstream node number itself, because network is radial in nature.

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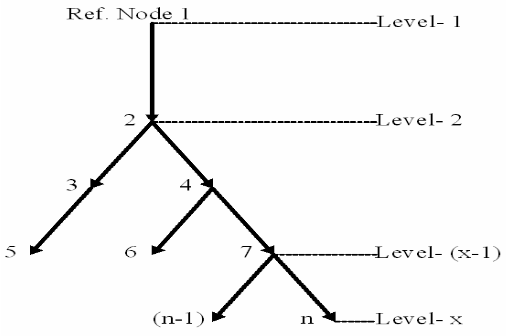

Fig. 1. Radial distribution network structure.

Proposed Power Flow Approach

Backward sweep

Appling TT and using backward sweep in all leaf nodes of a network, till reference node is reached the power injections are computed. The distribution network shown in Figures 2 and 3 are so configured that more than one laterals and sub laterals can emanate from the same node.

(a) Power at node: In Figure 2 leaf node the power referred from the upstream Si is equal to load Sd and this can be expressed as:

$$\mathrm{Si}(n) = \mathrm{Sd}(n) \tag{1}$$

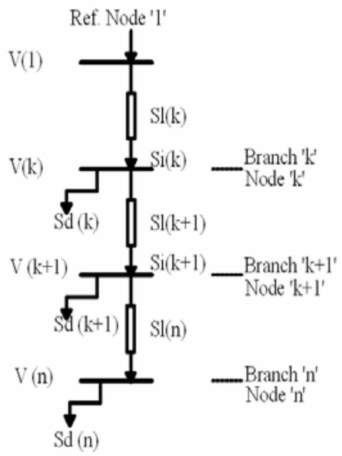

Fig. 2. Representation of main feeder in the radial distribution network.

Further power injection calculations are performed by backward sweeping from the sub-lateral or lateral end to the main lateral using

$$\mathrm{Si}(k) = \mathrm{Si}(k+1) + \mathrm{Sl}(k+1) + \mathrm{Sd}(k) \tag{2}$$

where,

$\mathrm{Si}(k)$ = upstream injected power

$\mathrm{Si}(k+1)$ = downstream injected power

$\mathrm{Sl}(k+1)$ = line loss along the branch

$$\mathrm{Sl}(k+1) = \mathrm{abs}\left(\frac{\mathrm{Si}(k+1)}{\mathrm{V}(k+1)}\right)^2 \mathrm{Z}(k+1) \tag{3}$$

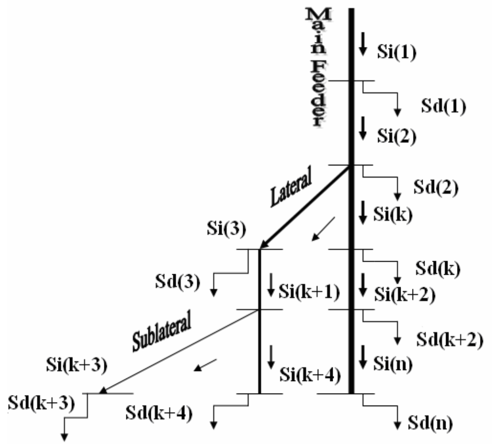

Fig. 3. Lateral and sub-lateral and main lateral connections.

(b) Reduction of terms in Equation 2: Figure 4 represents injected power at a node in the upstream is equal to sum of loads and losses in the downstream. Hence three terms on right hand side of Equation 2 can be considered into two terms as:

$$\mathrm{Si}(k) = \sum_{k=\text{downstream nodes}}^{n-\text{upstream nodes}} \text{Loads} + \sum_{k=\text{downstream branches}}^{n-\text{upstream branches}} \text{Losses} \tag{4}$$

$$\mathrm{Si}(k) = \sum_{k=\text{downstream nodes}}^{n-\text{upstream nodes}} \mathrm{Sd}(k) + \sum_{k=\text{downstream nodes}+1}^{n-\text{upstream nodes}} \mathrm{Sl}(k) \tag{5}$$

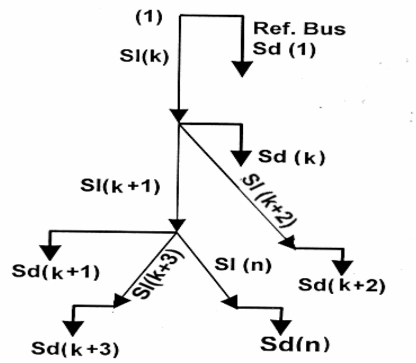

Fig. 4. Representation of loads at nodes and losses in the branches.

The first term (loads) of Equation 5 is independent of assumed voltage, whereas second term (losses) depends on square of absolute value of voltage. It is noted that the losses are about 8% of the system demand and therefore the initial error anticipation will be small.

Forward sweep

Improved forward sweep technique computes the currents injected at downstream node k using the accurate upstream currents and downstream emanating currents.

(a) The relation between power and current at the reference node can be expressed as:

$$\mathrm{Ii}(k) = \left(\mathrm{Si}(k)/\mathrm{V}(k)\right)^* \tag{6}$$

(b) Calculation of branch currents: The downstream branch current Ii(k+1) is equal to most recent upstream current entering at k^{th} node minus emanating currents in the downstream at the same node.

$$I_i^{(p)}(k+1) = I_i^{(p-1)}(k) - \sum_{\substack{k \neq k+1 \\ k = k+2}}^{\text{emanating branches}} \left(\text{Lateral} + \text{Node}\right) \text{currents} \tag{7}$$

where, p= Number of iteration i.e. p = 1, 2...

Similarly for subsequent currents of the network expressed in Equation 7 can be later written as:

$$I_i^{dow}(k+1) = I_i^{up}(k) - \sum_{k \neq k+1}^{\text{emanating branches}} I_i^{dow}(k+2) - I_d^{junc}(k) \tag{8}$$

The injected current shown in Figure 5 can be expressed as

$$Ii(k+1) = [Si(k)/V(k+1)]^* - \sum_{k \neq k+1}^{\text{emanating node}} [Si(k+2)/V(k)]^* - Sd(k)/V(k) \tag{9}$$

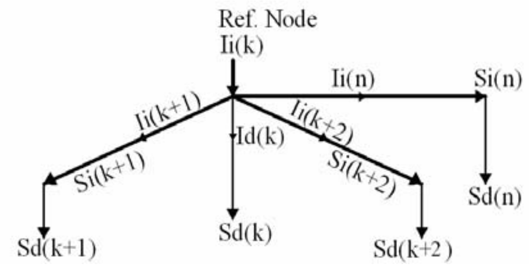

Fig. 5. Current entering and leaving at node 'k' of a distribution network.

Forward sweep

Appling KVL directly to update the node voltage for the network shown in Figure 6 the voltage at (k+1) node is equal to

$$Vi(k+1) = Vi(k) - Z(k+1)Ii(k+1) \tag{10}$$

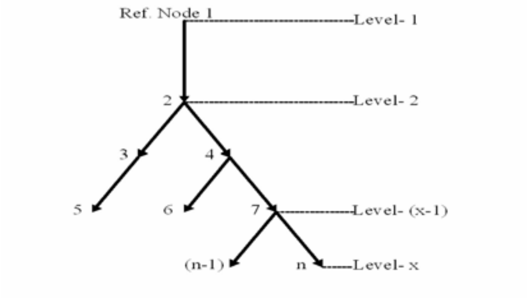

Fig. 6. A typical voltage distribution radial network.

Equations 6, 9, and 10 are to be executed repeatedly until convergence is reached. The voltage mismatch at node can be expressed as

$$\Delta V(k)^{(p+1)} = V(k)^{(p+1)} - V(k)^{(p)} \tag{11}$$

3. COMPARISION OF PROPOSED METHOD

The basic forward–backward techniques are analyzed and compared in Tables 1 and 2. The complete calculation flow diagram of the proposed method is as shown in Figure 7.

Table 1. Comparison between method [11] and [14].

Sl	Method [11]	Method [14]
A	Nodal currents: At iteration 'p' the nodal current injection 'Ii (k)' is $Ii(k) = \left(\frac{Si(k)}{V(k)}\right)^*$	Nodal currents: At iteration 'p' the nodal current injection 'Ii (k)' is $Ii(k) = \left(\frac{Si(k)}{V(k)}\right)^*$
B	Bus-Injection to Branch-Current (BIBC):Branch currents in matrix form as [Branch currents] = [BIBC] [Nodal currents] where [BIBC] having 1's and 0's conevrts given nodal currents information into branch currents	Backward sweep: Expression for branch currents $I_i^{(p)}(k+1) = \sum_{k=1}^{\text{emanating nodes}} In_i^{(P)}(k)$ where 'In' is nodal currents
C	Branch-Current to Bus-Voltage (BCBV): Final voltage at bus using [BCBV] [BIBC] and [Ii] is $[V(k)] = [V(1)] - [BCBV][BIBC][Ii(k)]$	Forward sweep: Nodal voltages are computed in forward sweep as $Vi(k+1) = Vi(k) - Z(k+1)Ii(k+1)$

Table 2. A, B, and C points of Table 1 are compared with the proposed method.

Sl	Proposed Method
1	Backward sweep: The power Si(k) is computed using Equation 5 by separating assumed voltage dependent term as losses and independent term as load. Usually the loss in a system is about 8% of the loads, which leads to minimization of errors.
2	Forward sweep: The recent values of upstream currents from Equation 6 are used to compute upstream currents from Equation 9, which leads to faster convergence.
3	Forward sweep: Nodal voltages are computed in forward sweep as $Vi(k+1) = Vi(k) - Z(k+1)Ii(k+1)$

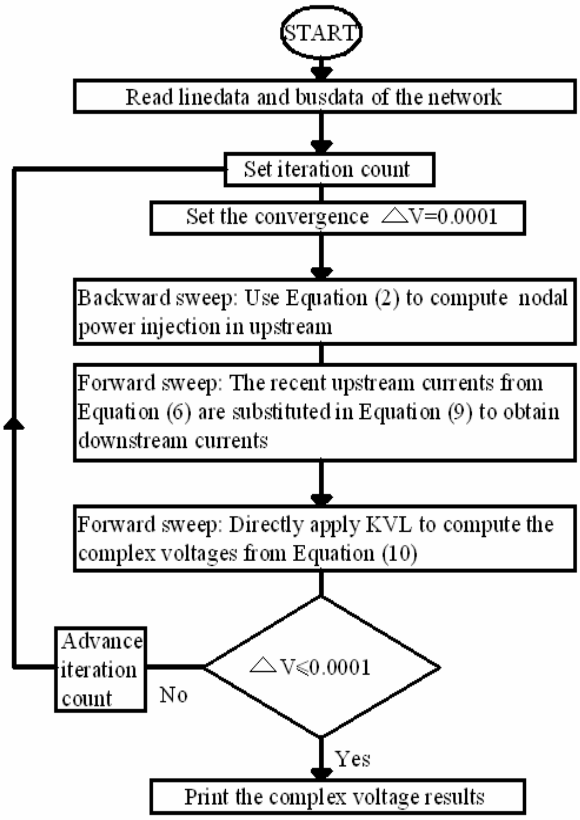

Fig. 7. Calculation flow diagram of the proposed method.

Table 3. Performance results for the IEEE 15 node distribution network [17].

Methods	Iteration	Accuracy
Method [11]	3	0.0001
Method [14]	3	0.0001
Proposed Method	2	0.0001

Table 4. Performance results for the IEE 28 node distribution network [18].

Methods	Iteration	Accuracy
Method [11]	4	0.0001
Method [14]	3	0.0001
Proposed Method	2	0.0001

4. RESULTS AND DISCUSSIONS

Using the network data in [16]-[17] the performance of the proposed algorithm is compared with the methods in [11] and [14]. However, for these data the NR and GS method do not converge. Tables 3 and 4 show the accuracy and number of iterations of all three methods. The results obtained in Tables 3 and 4 are validated by programming in MATLAB Ver 7.0 software package installed in the PC having specification as: 512MB-RAM, Intel Pentium IV-Processor, 1.73GHz-Speed. The strength of the algorithm has been demonstrated by considering losses associated with branches in Equation 2 and updating most recent current in Equation 9. The method is recommended based on the nodal voltage obtained from Equation 11. Table 5 and 6 represents the load flow results of a 15 and 28 bus distribution network. Thus the proposed method has been found to be superior in accuracy, number of iterations and robustness as per the comparison and the results given in Tables 3 to 6.

Table 5. Comparative results for 15 bus network.

Voltage	Proposed Method (P)	Method [14] (S)	Method [11] (T)	% Difference (P-S)/P	% Difference (P-T)/P
V_2	0.9699	0.9696	0.9734	0.0003	0.0036
V_3	0.9690	0.9687	0.9717	0.0003	0.0028
V_4	0.9541	0.9541	0.9591	0.0000	0.0052
V_5	0.9535	0.9537	0.9587	0.0002	0.0055
V_6	0.9528	0.9535	0.9585	0.0007	0.0060
V_7	0.9388	0.9389	0.9461	0.0001	0.0078
V_8	0.9229	0.9231	0.9325	0.0002	0.0104
V_9	0.9112	0.9114	0.9223	0.0002	0.0122
V_{10}	0.9061	0.9061	0.9172	0.0000	0.0123
V_{11}	0.8939	0.8937	0.9049	0.0002	0.0123
V_{12}	0.9658	0.9667	0.9697	0.0009	0.004
V_{13}	0.9632	0.9654	0.9683	0.0023	0.0053
V_{14}	0.9600	0.9638	0.9668	0.0040	0.0071
V_{15}	0.9590	0.9635	0.9666	0.0047	0.0079

Table 6. Power flow solution for 28 bus network.

Voltage	Method [14]	Proposed Method	Method [11]
V_1	1.0000	1.0000	1.0000
V_2	0.9544	0.9568	0.9569
V_3	0.9070	0.9120	0.9126
V_4	0.8817	0.8882	0.8891
V_5	0.8656	0.8731	0.8744
V_6	0.8061	0.8160	0.8187
V_7	0.7682	0.7795	0.7838
V_8	0.7496	0.7615	0.7668
V_9	0.7179	0.7303	0.7380
V_{10}	0.6798	0.6921	0.7038
V_{11}	0.6560	0.6677	0.6828
V_{12}	0.6457	0.6571	0.6739
V_{13}	0.6194	0.6305	0.6519
V_{14}	0.5993	0.6099	0.6360
V_{15}	0.5873	0.5975	0.6274
V_{16}	0.5787	0.5887	0.6223
V_{17}	0.5714	0.5812	0.6200
V_{18}	0.5689	0.5787	0.6210
V_{19}	0.9470	0.9495	0.9516
V_{20}	0.9408	0.9476	0.9505
V_{21}	0.9298	0.9450	0.9496
V_{22}	0.9126	0.9431	0.9506
V_{23}	0.9010	0.9060	0.9072
V_{24}	0.8933	0.9026	0.9044
V_{25}	0.8822	0.8991	0.9019
V_{26}	0.8023	0.8122	0.8161
V_{27}	0.8010	0.8109	0.8154
V_{28}	0.8003	0.8103	0.8154

5. CONCLUSION

A simple and powerful algorithm has been proposed for balanced radial distribution network to obtain power flow solution. It has been found from the cases presented that the proposed method has fast convergence characteristics when compared to existing methods. The algorithm is found to be robust in nature. The method can be easily extended to solve three phase networks also.

ACKNOWLEDGEMENT

This research work is sponsored and supported by SNIST, Hyderbad. The first author would like to thank Dr. P.S.R Murthy, Director, School of Electrical Engineering, SNIST, Hyderabad and Dr. S.V. Kulkarni, IIT, Mumbai. for their useful comments on this paper.

NOMENCLATURE

k	Node or branch number (Branch number is assigned with reference to receiving end node number)
Si(k)	Power injected at node
Sd(k)	Power specified at node
Sl(k)	Complex power loss in k^{th} branch
V(1)	Voltage at reference node number '1'
V(k)	Unknown voltage
Ii(1)	Current injected at node number '1'
Id(k)	Current drawn at k^{th} node, due to load demand
Z(k)	Impedance of the branch referred to k^{th} node of receiving end
n	Number of nodes in the network
b	Number of branches in the network (= n-1)
p	Number of iteration
x	Network termination level

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