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FDR PSO-Based Optimization for Non-smooth Functions

M. Anitha*[1], S. Subramanian* and R. Gnanadass[+]

Abstract – *In this paper, an improved structure of the standard Particle Swarm Optimization (PSO), called Fitness Distance Ratio PSO (FDR PSO) is proposed to solve non-smooth test functions. In the conventional PSO method, the particle's velocity is updated using cognition and social components. But it suffers from premature convergence. To overcome this drawback, in the proposed algorithm, in addition to cognitive and social component, each particle also learns from the experience of the neighboring particles that have a better fitness than itself. The demonstration of the FDR PSO algorithm was carried out on six bench mark test functions and a practical Optimal Power flow (OPF) problem. The results of the proposed algorithm outperformed the solution obtained through the standard PSO. The minimum solution of OPF problem is also compared with the results obtained through the other optimization methods.*

Keywords – Fitness distance ratio particle swarm optimization, non-convex fuel cost functions, optimal power flow, optimization.

1. INTRODUCTION

The objective of optimization is to seek values for a set of parameters that maximize or minimize the objective function subjected to various constraints. In practice, optimization problems become more and more complex. To search an optimum of a function with continuous variables is difficult, if there are peaks and valleys. In these cases, traditional optimization methods fail to provide global optimum solution. They will either be trapped to local minima or need much more search time. In recent years, many researchers have been trying to propose new algorithms to solve such complex optimization problems.

Teo [1] augmented the Generalised Generation Gap (G^3) algorithm with adaptive and mutation operations to improve its performance for solving multimodal optimization problems. The effectiveness of the proposed algorithm has been demonstrated on five benchmark test problems with highly deceptive fitness landscapes. Yao and Liu [2] proposed a Fast Evolutionary Programming (FEP) which uses a Cauchy instead of Gaussian mutation operator to overcome the drawback of EP such as slow convergence. The suitability and performance of the FEP algorithm is validated on different function optimization problems. Li and Jiang [3] introduced a new stochastic approach based on proper integration of Simulated Annealing algorithm (SAA), Genetic Algorithm (GA) and Chemotaxis Algorithm (CA) for solving complex optimization problems. The proposed approach has been applied to solve such problems as scheduling, training of artificial neural networks and the optimization of complex functions.

Laskari *et al.* [4] investigated the ability of PSO method to cope up with minmax problems through experiments on well-known test functions. The performance of PSO is compared with that of other optimization methods. Pongchairerks and Kachitvichyanukul [5] proposed two non-homogeneous PSO algorithms based on a structure that is built by combining previously published structures. The algorithms were tested using benchmark test functions.

Perem *et al.* [6] introduced the FDR PSO algorithm to combat the problem of premature convergence faced by the standard PSO. The algorithm is shown to outperform PSO on many benchmark problems. Liang and Suganthan [7] introduced a novel dynamic multiswarm PSO based on dividing the population into many small swarms and then regrouping the swarms. The effectiveness of the proposed method is validated on benchmark problems. Liang *et al.* [8] proposed Comprehensive Learning Particle Swarm Optimization (CLPSO) which uses a new learning strategy to make the particles have different learning exemplars for different dimensions. The authors conducted experiments on benchmark functions with and without coordinate rotations.

This paper proposes the solution techniques for different modal functions and a practical OPF problem to validate the FDR PSO technique. Optimal power flow problem is one of the important optimization problems in power system which is aimed to optimize steady state performance of the power system with respect to the objective of minimum operating cost while subjected to various operating constraints. Nowadays, power system planners and operators often use OPF as a powerful assistant tool in both planning and operating stage. Many traditional optimization methods, including non-linear programming, quadratic programming, linear programming, mixed integer programming and interior point method have been used to solve OPF problem. These methods rely on convexity to obtain the global optimum solution and as such are forced to simplify relationships in order to ensure convexity. Traditional methods offer good results but

[*] Department of Electrical Engineering, Annamalai University, Annamalai Nagar – 608 002, Tamil Nadu, India.

[+] Department of Electrical and Electronics Engineering, Pondicherry Engineering College, Pondicherry – 605 014, India.

[1] Corresponding author;
E-mail: vishanitha@yahoo.com

when the search space is non-linear and has discontinuities such as valve point loading effects, fuel switching and prohibited operating zones, these methods become difficult to solve with a slow convergence ratio and not always seeking to the optimal solution [11].

It is therefore becomes necessary to develop new, more general and reliable algorithms, which are capable of incorporating new constraints arising from non-smooth solution surfaces. Many heuristic search algorithms, such as Genetic Algorithms, Evolutionary Programming [9], [10], Tabu Search, Simulated Annealing [11] have been proposed by many authors to solve OPF problem. These techniques searched for the global optimum for any type of objective function subjected to various types of constraints. Moreover, many hybrid algorithms have been introduced to enhance the search efficiency. For instance, a hybrid PSO–Sequential Quadratic Programming algorithm was used to solve power dispatch problem with units having valve point loading effect [12].

In this paper, a new variation of PSO method, namely Fitness Distance Ratio PSO is proposed to solve the problems with non-convex, non-continuous and highly non-linear solution space. Initially, the proposed FDR PSO method is validated with benchmark problems. Then, it deals with the implementation of the proposed algorithm to solve OPF problem considering three different fuel cost functions and the results are compared with other optimization methods.

2. PROBLEM FORMULATION

Generally, in the modal functions, the number of local minima increases with respect to the nature/ dimension of the problem. The functions of the bench mark problem are unimodal, continuous, discontinuous, non-differentiable and multimodal in nature. In this paper, to prove the effectiveness of the proposed algorithm, different types of benchmark equations and a practical OPF problem are considered and given below.

A. Benchmark Problems

1. De Jong's Function 1

$$\text{Min } f(x) = \sum_{i=1}^{n} x_i^2 \tag{1}$$

where $-5.12 \leq x_i \leq 5.12$.

This simple test function is continuous, convex and unimodal.

2. Axis parallel hyper-ellipsoid

$$\text{Min } f(x) = \sum_{i=1}^{n} i * x_i^2 \tag{2}$$

where $-5.12 \leq x_i \leq 5.12$.

This function, also known as weighted sphere model, is continuous, convex and unimodal.

3. Sum of different powers

$$\text{Min } f(x) = \sum_{i=1}^{n} | x_i |^{i+1} \tag{3}$$

where $-1 \leq x_i \leq 1$.

This is a commonly used unimodal test function.

4. Rotated hyper-ellipsoid

$$\text{Min } f(x) = \sum_{i=1}^{n} \left(\sum_{j=1}^{i} x_j \right)^2 \tag{4}$$

where $-65.536 \leq x_i \leq 65.356$.

5. Rosenbrock's Valley (Banana function)

$$\text{Min } f(x) = \sum_{i=1}^{n-1} 100 \times (x_{i+1} - x_i^2)^2 + (1 - x_i)^2 \tag{5}$$

where $-2.048 \leq x_i \leq 2.048$.

The above equation has global optimum in a long, narrow, parabolic shaped valley. To find the convergence to the global optimum is difficult. Hence this is often used in assessing the performance of the optimization algorithm.

6. Griewangk's function

$$\text{Min } f(x) = \sum_{i=1}^{n} \frac{x_i^2}{4000} - \prod_{i=1}^{n} \text{cox}\left(\frac{x_i}{\sqrt{i}} \right) + 1. \tag{6}$$

where $-600 \leq x_i \leq 600$.

Griewangk's function is a multimodal problem and the location of minima is regularly distributed.

The following subsection describes about the formulation of practical optimal power flow problem.

B. Power System Problem

OPF problem seeks to optimize steady state performance with respect to a non-linear/ non-smooth objective function. In the OPF problem, it is required to minimize the total operating cost of the generating units while satisfying the system constraints.

The objective function is mathematically stated as:

$$F = \text{Min } \sum_{i=1}^{N} f_i(P_i) \qquad \$ / \text{hr} \tag{7}$$

where, F is the total optimal cost of generation, $f_i(P_i)$ is the fuel cost of the i^{th} generator, P_i is the real power generation of the i^{th} generator, N is the total number of generators connected in the system subjected to the equality constraint in real power balance.

$$\sum_{i=1}^{N} P_i - P_L - P_D = 0 \tag{8}$$

where, P_D is the total load of the system and P_L is the transmission losses of the system.

In this paper, three different fuel cost functions of the generators are considered and they are given below.

Quadratic cost function:

$$f_i(P_i) = a_i + b_iP_i + c_i P_i^2 \quad (9)$$

where, a_i, b_i and c_i are the fuel cost coefficients and P_i be the real power generation of i^{th} unit. This function is non-linear in nature.

Sine component function:

$$f_i(P_i) = a_i + b_iP_i + c_i P_i^2 + \left|d_i \sin(e_i (P_i^{min} - P_i))\right| \quad (10)$$

where, d_i and e_i are fuel cost coefficients of the i^{th} unit with valve point effects. This function is non-differentiable one.

Piecewise quadratic cost function:

$$\begin{aligned} f_i(P_i) &= a_{i1} + b_{i1}P_i + c_{i1} P_i^2; && \text{if } \underline{P_i} \le P_i < P_{i1} \\ &= a_{i2} + b_{i2}P_i + c_i P_i^2; && \text{if } P_{i1} \le P_i < P_{i2} \\ &= a_{im} + b_{im}P_i + c_{im} P_i^2; && \text{if } P_{im-1} \le P_i < \overline{P_i} \end{aligned} \quad (11)$$

where, a_{im}, b_{im}, c_{im} are fuel cost coefficients of i^{th} generator with m^{th} fuel and $\underline{P_i}$, $\overline{P_i}$ are minimum and maximum level power generation of the i^{th} generator. The above function is discontinuous in nature.

The generator inequality constraints include its real power outputs (P_i), reactive power outputs (Q_i), voltage magnitudes (V_i) and voltage angles (θi). They are restricted by their lower and upper limits as follows:

$$P_{i\,min} \le P_i \le P_{i\,max}, \quad i = 1, 2..... n \quad (12)$$

$$Q_{i\,min} \le Q_i \le Q_{i\,max}, \quad i = 1, 2..... n \quad (13)$$

$$V_{i\,min} \le V_i \le V_{i\,max}, \quad i = 1, 2..... n \quad (14)$$

$$\theta_{i\,min} \le \theta_i \le \theta_{i\,max}, \quad i = 1, 2..... n \quad (15)$$

Line flow inequality constraint is given as:

$$L_{fi} \le L_{fi\,max} \quad (16)$$

where, $L_{fi\,max}$ is the maximum line flow limit (MVA) of the i^{th} transmission line.

3. OVERVIEW OF PARTICLE SWARM OPTIMIZATION

In 1995, Kennedy and Eberhart first introduced the PSO method which is motivated by social behavior of organisms such as fish schooling and birds flocking [13]. In a PSO system, particles fly around a 'd' dimensional problem space. During flight, each particle adjusts its position according to its own experience as well as by the best experiences of other neighboring particles. Let us consider $X_i = (X_{i1}, X_{i2}, \ldots X_{id})$ and $V_i = (V_{i1}, V_{i2}, \ldots V_{id})$ be the position and velocity of the i^{th} particle. Velocity V_{id} is bounded between its lower and upper limits. The best previous position of the i^{th} particle is recorded and is given by $P_{besti} = (P_{i1}, P_{i2}, \ldots P_{id})$. Let $g_{besti} = (P_{g1}, P_{g2}, \ldots P_{gid})$ be the best position among all individual best positions achieved so far. Each particle's velocity and position is updated using the following two equations.

$$V_{id}^{k+1} = W * V_{id}^{k} + C_1 * rand1 * (P_{id} - X_{id}) + C_2 * rand2 * (P_{gid} - X_{id}) \quad (17)$$

$$X_{id}^{k+1} = X_{id}^{k} + V_{id}^{k+1} \quad (18)$$

where, C_1 and C_2 are the acceleration constants, which represent the weighting of stochastic acceleration terms that pull each particle towards P_{best} and g_{best} positions, while k represents the current iteration and rand1 and rand2 are two random numbers in the range [0,1]. Inertia weight (W) is a control parameter that is used to control the impact of the previous velocities on the current one. Hence, it influences the trade-off between the global and local exploration abilities of the particles. The search process will terminate if the number of iterations reaches the maximum allowable number.

4. FDR PSO ALGORITHM

In the literature, it has been proved that the particle positions in PSO oscillate in damped sinusoidal waves until they converge to points in between their previous P_{best} and g_{best} positions [14], [15]. During this oscillation, if a particle reaches a point which has better fitness than its previous best position, then the particle continues to move towards the convergence of the global best position discovered so far. All the particles follow the same behavior to converge quickly to a good local optimum. Suppose, if the global optimum of the problem does not lie on a path between original particle positions and such a local optimum, then the particle is prevented from effective search for the global value. In such cases, many of the particles are wasting their computational effort in seeking to move towards the local optimum already discovered. Better results may be obtained if various particles explore other possible search directions.

In the FDR PSO algorithm, in addition to the Socio-cognitive learning processes, each particle also learns from the experience of neighboring particles that have a better fitness than itself [6]. This approach results in change in the velocity update equation, although the position update equation remains unchanged. It selects only one other particle at a time when updating each velocity dimension and that particle is chosen to satisfy the following two criteria.

1. It must be near the current particle.
2. It should have visited a position of higher fitness.

The simplest way to select a nearby particle which satisfies the above mentioned two criteria is that maximizes the ratio of the fitness difference to the one-dimensional distance. In other words, the d^{th} dimension of the i^{th} particle's velocity is updated using a particle called the n_{best}, with prior best position P_j. It is necessary to maximize the following Fitness Distance Ratio which is given by:

$$\frac{Cost(P_j) - Cost(X_i)}{\left|P_{jd} - X_{id}\right|} \quad (19)$$

In the FDR PSO algorithm, the particle's velocity update is influenced by the following three factors:

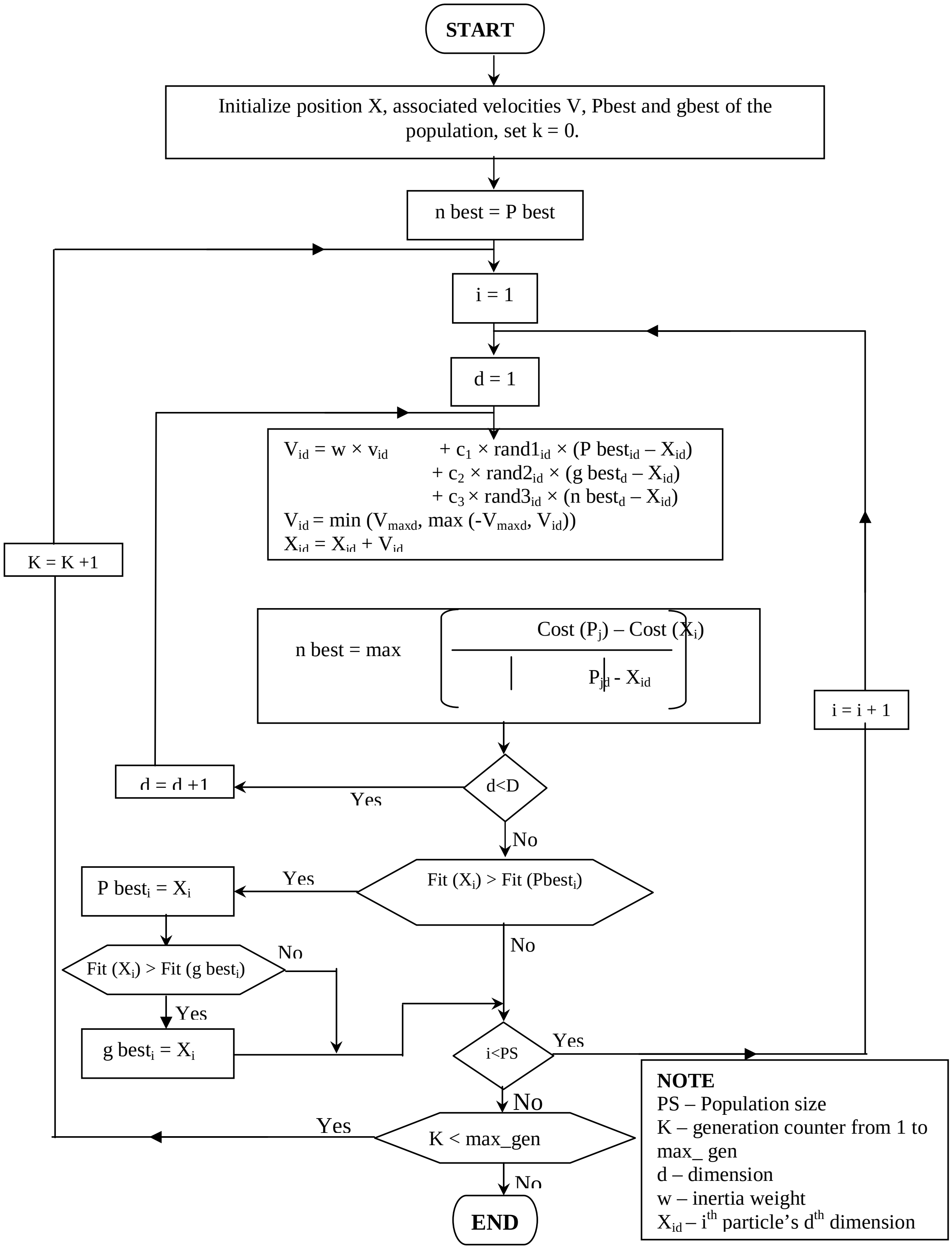

Fig. 1. Flow diagram – FDR PSO algorithm.

1. Previous best experience i.e. P_{best} of the particle.
2. Best global experience i.e. g_{best} , considering the best P_{best} of all particles.
3. Previous best experience of the "best nearest" neighbor i.e. n_{best}.

Hence, the new velocity update equation becomes:

$$V_{id}^{k+1} = W*V_{id}^{k} + C_1*rand1*(P_{id}-X_{id})$$
$$+ C_2*rand2*(P_{gid} - X_{id})$$
$$+ C_3*rand3*(P_{nd} - X_{id}) \quad (20)$$

where, P_{nd} is the nearby particle that have better fitness.

The position update equation remains the same as in Equation 18. The flow diagram of the FDR PSO algorithm is given in Figure 1.

5. SIMULATION RESULTS AND DISCUSSION

The optimum solutions for six different types of modal functions and a power flow problem are obtained in this section using swarm intelligence techniques. A comparative performance of the conventional PSO and FDR PSO methods are also illustrated. All the simulation studies were carried for 30 numbers of trials. They were carried out on P-IV, 3 GHz system in MATLAB environment.

A. Benchmark problems

The simulation parameters and constants for PSO and FDR PSO methods are given in Appendix I and II. The obtained minimum solution of the complex modal functions using both by PSO and FDR PSO methods is given in the Table 1. From this table, it is inferred that the results obtained by both of these methods are close with each other. To illustrate the convergence, the simulation characteristics of Rosenbrock's function (a narrow solution surface) obtained through these methods is given in Figures 2 and 3. It is also observed that the PSO algorithm suits well in the initial iterations but fails later. The average and best fitness characteristics reveal that the proposed FDR PSO method is less susceptible to premature convergence and less likely to get into the local minimum of the function being optimized. Thus it outperforms the standard PSO.

Table 1. Minimum solution - bench mark problems.

Optimization Function	Minima achieved	
	PSO	FDR PSO
De Jong	0.0134	2.047e-6
Axis parallel hyper ellipsoid	3.42e-5	7.105e-12
Rotated ellipsoid	5.846e-4	7.902e-8
Rosenbrock	1.942e-6	1.409e-12
Griewangk	0.4993	7.178e-11
Sum of powers	1.337e-11	2.755e-33

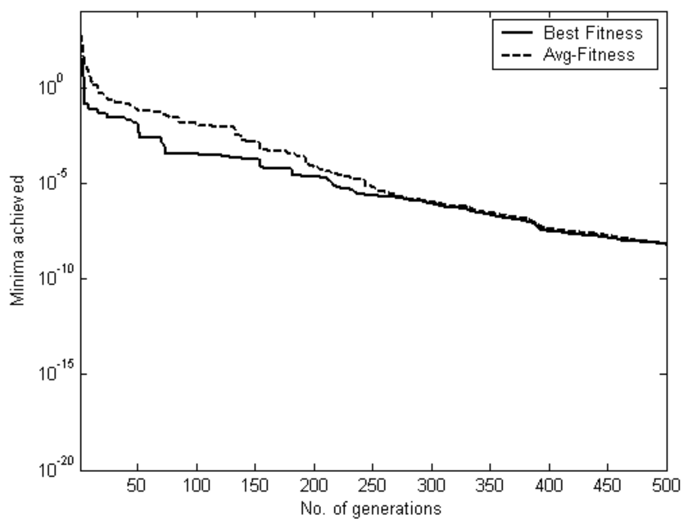

Fig. 2. Best and average fitness characteristics - PSO technique.

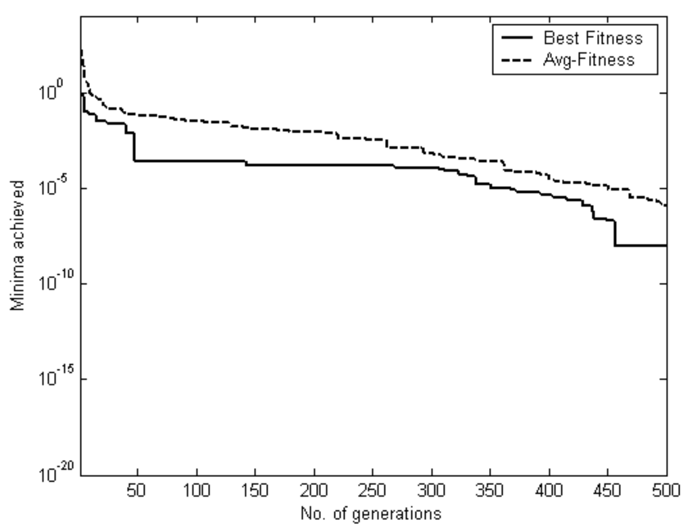

Fig. 3. Best and average fitness characteristics – FDR PSO technique.

B. Power system problem

IEEE – 30 bus power system is considered to describe the optimal solution obtained by swarm intelligence methods. It consists of six generators, 41 transmission lines, 4 tap changing transformers and 2 Var sources. The base load of the system is 283.4 MW. The details of simulation parameters, population size and maximum number of generations which decide the execution time of the swarm intelligence methods are given in Appendix III. Bus data, line data of the test system were taken from [9]. The generator real power limits and cost coefficients corresponding to three fuel cost functions are given in Tables 2 and 3. The PSO methods were used to solve optimal power flow problem of this practical power system problem. While solving this problem, the solution must satisfy the transmission and generator constraints. The equality and inequality constraints are described in the problem formulation section. In the proposed FDR PSO algorithm, the dimension of the particles is taken as six and a linearly decreasing inertia weight (w) from 0.9 to 0.2 is used to obtain the convergence characteristics [16].

Three different fuel cost objective functions were considered while solving the optimal power flow problem using the swarm techniques. One is quadratic, another one posses sine function and the later is piecewise in nature. The structure of these non-smooth functions is given in the II section of this paper. The optimal solution obtained through PSO and FDR PSO methods and their corresponding minimum costs are given in Table 4. The results of these techniques are also compared with other optimization technique called as Evolutionary Programming (EP), to prove its validity. In addition to the minimum cost of generation, settings of the generators and the corresponding convergence characteristics of the techniques are also obtained for all the three functions which are given in Figures 4 and 5 respectively. From the Figure 5, it is observed that after the considerable number of generations, the solution of the algorithm becomes constant that ensures the algorithm's convergence. Power system losses are determined through the power flow solution. It is obtained through Newton Raphson method. The losses obtained for the three fuel cost functions were 7.358 MW, 7.627 MW and 8.218 MW respectively. The computation time for the PSO and FDR PSO were found to be 2.313 sec and 4.344 sec respectively in P IV, 3 GHz system.

Table 2. Generator data and cost coefficients.

Gen No.	P_{max} (MW)	P_{min} (MW)	Quadratic a ($/hr)	Quadratic b ($/MWhr)	Quadratic c ($/MW^{2}hr)	Valve point loading a ($/hr)	Valve point loading b ($/MWhr)	Valve point loading c ($/MW^{2}hr)	Valve point loading d ($/hr)	Valve point loading e (rad/MW)
1	50	200	0.00	2.00	0.00375	150.0	2.0	0.0016	50.0	0.063
2	20	80	0.00	1.75	0.01750	25.0	2.5	0.0100	40.0	0.0980
3	15	50	0.00	1.00	0.006250	0.00	1.00	0.006250	0.0	0.0
4	10	35	0.00	3.25	0.00834	0.00	3.25	0.00834	0.0	0.0
5	10	30	0.00	3.00	0.02500	0.00	3.00	0.02500	0.0	0.0
6	12	40	0.00	3.00	0.02500	0.00	3.00	0.02500	0.0	0.0

Table 3. Generator cost coefficients for multiple fuel cost function.

Gen No.	P_{min} (MW)	P_{max} (MW)	a ($/hr)	b ($/MWhr)	c ($/MW^{2}hr)
1	50	140	55.0	0.70	0.0050
	140	200	82.5	1.05	0.0075
2	20	55	40.0	0.30	0.0100
	55	80	80.0	0.60	0.0200

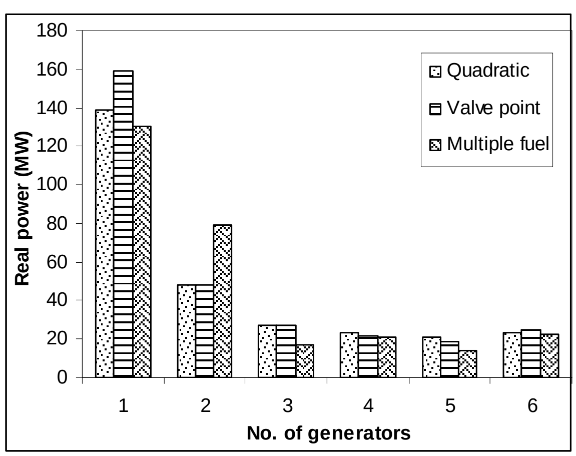

Fig. 4. Optimal generator settings obtained by FDR PSO technique.

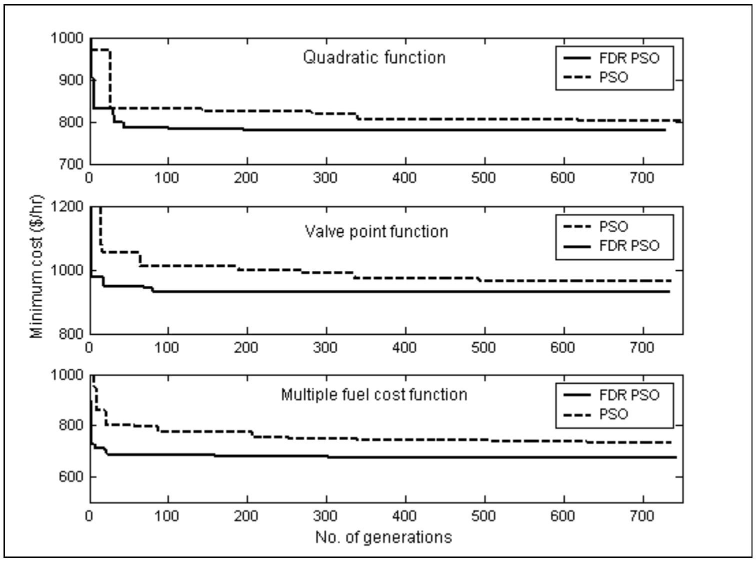

Fig. 5. Convergence characteristics for various fuel cost functions.

6. CONCLUSION

The effectiveness of the convergence and minimum solution of the proposed FDR PSO algorithm are validated using standard benchmark and power system problems. The solution capability of the proposed algorithm is demonstrated on the above functions which are non-linear, non-convex and non-smooth in nature. The implementation of this algorithm is also simple by computing and maximizing the ratio of fitness difference to one dimensional distance. Avoiding premature convergence, the proposed algorithm allows to be continuing in the search space for the global optima. The convergence of the proposed algorithm is less sensible with the nature of objective function.

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APPENDIX

Appendix I: Population sizes and problem dimensionality in various experiments for PSO and FDR PSO.

Function	Population size	Generations	Dimensions
De Jong's	10	1000	20
Axis Parallel hyper-ellipsoid	10	1000	10
Sum of Powers	10	1000	10
Rotated hyper-ellipsoid	10	1000	10
Rosenbrock's	10	1000	2
Griewangk's	10	1000	10

Appendix II: Simulation parameters used in PSO and FDR PSO.

Parameters / Algorithm	C_1	C_2	C_3
PSO	1.0	1.0	---
FDR PSO	1.0	1.0	2.0

Appendix III: Simulation parameters of PSO and FDR PSO methods.

Parameters/ Algorithm	C_1	C_2	C_3	Population size	Maximum number of generations
PSO	1.0	1.0	---	20	750
FDR PSO	1.0	1.0	2.0	20	750