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Socially Structured Transferable Utility Game Applied to Electricity Markets

S. Balagopalan*[1], S. Ashok*, and K.P. Mohandas*

Abstract *– The procedure of sharing the Transmission Service Charge (TSC) among the Discos, the players in an electricity market, is modeled as a Socially Structured Transferable Utility (SSTU) game. Such an algorithm requires a hierarchical ordering of the Discos, in a given social structure and a method of ranking them in a permutation. These parameters have been designed to be endogenous in electricity markets, modeled in a Cooperative game theory (CGT) environment using multilateral trades. A suitably crafted TSC is used as the characteristic value of the game to identify trades with least loss on transmission lines. The development of a power vector indicating the strength of the players, ranking based on objective of the game and design of TSC are in tune with the requirements of an electricity market, the multilateral trade structure, a CGT environment and the socially stable core. The combinatorial game is applied to a five bus power system and results are analyzed. This work is important in the context of a still embryonic electricity market requiring a reliable, secure and harmonious transmission sector.*

Keywords **–** Cooperative game theory, core, electricity markets.

1. INTRODUCTION

Any joint venture, faces imminent failure if a constituent is dissatisfied, and wants to deviate. Economic or social discontent prompts the digression, whereas pay-off-vector acceptable to all partners shelters a coalition. In electricity markets, real time deviations in power transaction are tantamount to network anarchy. These markets evolved due to a general failure of vertically integrated power systems and the desire for efficiency and accountability via competition [1]. Of the many trade structures used to carry out functions of the power business [2] multilateral format with idealized market concepts is used here [3]-[5] since reliable cost-benefit data of trades are not available. But distributed information and decision structures and consequent information asymmetry and market games that lead to abuse of transmission lines, is an eventuality not to be dismissed in such trades.

CG theory (CGT) environment is suggested as a solution to coordinate trades. In coalitional [6]-[8] games action sets are available to coalitions of players. The Nash program claims that all games can be reduced to non-cooperative formulation, but Aumann program [9] argues that two agents may have a common interest in ganging up on a third. While non-cooperative games claim that agents' interests are opposed, CGT argues that with more than two players, their incentives are not wholly opposed to one another. In multilateral trades with several Discos and Gencos (load or generators agents), coalitions of at least two players at every stage have common motivations. TSC is advocated as an instrument of coordination and its reduced allocation acts as an incentive. Generally, TSC is designed to recover operation and sunk costs of the grid [10]-[13]. The projected design is power flow based; assesses impacts on lines and penalizes exceeded limits [14]. TSC allocation is a conscious, coalitional choice, motivating traders to contract optimal trades. When agents coalesce, ensuring counter-flows reduce total TSC. Pay-off refers to a lesser share of TSC, and is made possible via co-operation of participants. The concept of power vectors [15]-[17] is adapted to identify partners capable of causing counter-flows in lines.

In this paper electric power transaction is proposed as a CG coming under the class of SSTU game [18]-[19]. The solution to this game is a mapping which assigns a set of pay-off distributions over the players, construed as TSC allocated to each agent. The game and its features are expressed, properties established and methods to derive a solution are shown after some preliminaries are given, both in electricity markets and in a SSTU game.

2. PRELIMINARIES

In this work, two instruments that have been developed to coordinate multilateral trades in an electricity market are briefly explained first.

2.1 Transmission Service Charge

A line flow based differential price function is devised using current paradigms and market mechanisms. For a network with n nodes, L lines, flow through L lines z, and loss on L lines q, if weights for penalizing loss, sum of power flowing in all lines and flow in congested lines are a (Rs./MW^{2}h), b and d (Rs./MWh) respectively, and embedded cost is c in (Rs./hr.), then the price function P(q) in Rs./hr

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$$P(q) = aq^2 + b\sum_L z + \sum_{congested} z + c \quad (1)$$

A quadratic equation ensures adequate penalization and minimization of loss. The weights must in general reflect local priorities. TSC must act as a fiscal instrument and deter bad impacts on grid.

2.2 Power vector

For a social structure in a subset of players, a dominance or hierarchical ordering exists. A power vector defines this structure for every coalition, and shows the strengths of all its members. Originally proposed to measure positional power for application in tournaments and games [18] it is developed here to identify partners for amassing trades along the least loss route. Directed graphs are used to represent transactions and vertices denote buses on which players of the game are stationed. Arcs with directions embody lines and power flow through them. The method used to measure power of nodes (players) consists of a node, deriving power, from both the number as also power of its successors as given below. Let **A** be the collection of ir-reflexive digraphs on the vertex set N={1,2,…,n} with $(i,j)\epsilon NxN$ denoting the arc $\overrightarrow{ij}$, $((i,j)\epsilon A$ (meaning node i dominates j). The positional power function is the function $f^p: \mathbf{A} \rightarrow R^n$ which maps each $A \epsilon \mathbf{A}$ to

$$f^p(A) = \frac{1}{n}(I - \frac{1}{n}T^A)^{-1}s^A \quad (2)$$

Here T^A is the adjacency matrix of A, with the ij^{th} entry t^A_{ij}=1 if (i,j) is an arc of A and 0 otherwise; $s^A$is the score vector giving the number of successors of each node. In electricity markets nodes represent buses and Gencos or Discos located on them are players who can evaluate power vector based on system characteristics and directions of line flows. Players have maximum power if they can influence maximum outflows.

2.3 SSTU Game Preliminaries [18], [19]

A CG with TU describes a situation in which coalitions of players can obtain certain payoffs by cooperating. Some useful definitions and theorems are outlined to extend the concepts of SSTU Game to electricity market restructure.

2.3.1 Some Definitions of relevance

1. Structured Transferable Utility (STU) Game: A STU game is given by the triple Γ= (N, v, p), with N ={1,2,…,n} finite set of players, $v: 2^N \rightarrow \mathcal{R}$ a characteristic function, assigning to any coalition $S \subseteq N$ of players a real number as its worth v (S), and a social structure on every coalition represented by a power function $p: \mathcal{N} \rightarrow R^n$. It assigns to each coalition S of its players the power vector $p(S)$ within the underlying social structure of S, where $\mathcal{N} = 2^N \setminus \{\varphi\}$ is the collection of all non-empty subsets of N. A power vector of coalition S, is a non-negative vector in $\mathcal{R}^n$, p_i (S) =0 for any i not in S and p_i (S) >0 for at least one player i in S.

2. Qualities of a TU game: A TU-game v is super-additive (SA) for any pair of subsets $S,T \subseteq N$ if

$$v(S \cup T) \geq v(S) + v(T) \text{ such that } S \cap T = \varphi \quad (3)$$

Convex if

$$v(S \cup T) + v(S \cap T) \geq v(S) + v(T) \text{ for all } S,T \subseteq N \quad (4)$$

and permutationally convex if there exists a permutation π such that for all 1< j< k< n and defining rank number π(i) of any player $i \in N$, $\pi^i = \{j \in N | \pi(j) < \pi(i)\}$ set of all players with rank number at most equal to rank of i including i himself it holds that Max

$$[v(S), v(\pi^j \cup S) - v(\pi^j)] \leq v(\pi^k \cup S) - v(\pi^k) \text{ for all } S \subset N \setminus \pi^k \quad (5)$$

3. Socially Stable (SS) Pay-off: For a STU game, a payoff vector $x \in \mathcal{R}^n$ is SS if Equation 6 has a non-negative solution

$$\sum_{\{S|x(S)\leq v(S)\}} \Lambda_s\, p(S) = e^N \quad (6)$$

If for some $x \in \mathcal{R}^n$ and coalition $S \in N$ it holds that $x(S) \leq v(S)$ then S is said to sustain x since S can obtain value $x(S)$ without cooperating with players outside. If within S an individual at x sustained by S, has more power than others in S and x cannot be sustained by any other coalition, then he is able to increase his payoff at the expense of others and x is not SS. If nonnegative real numbers can be assigned to coalitions sustaining x such that weighted total strength of all agents is equal to 1, then x is SS. A POV x is feasible when total payoff is attained by cooperating as per some partition of grand coalition.

4. Economically Stable Payoff and Socially Stable Core: For a STU-game, a POV $x \in \mathcal{R}^n$ is economically stable if $x(N)=\overline{v(N)}$ and $x(S) \geq v(S)$ for all $S \in \mathcal{N}$(un-dominated) i.e. a POV x is economically stable if and only if x is in the SA cover core of v. The SS core of the TU game is denoted SC(v ,p) and consists of set of all economically and SS POV of Γ. If a POV x can be sustained only by the grand coalition {N} and if $p^N \neq e^N$, the collection {N} is not stable and therefore x is not an element of SC (v, p). A core element need not be Socially Stable.

5. Socially Stable Game: A STU game is SS if every SS POV of Γ is feasible.

6. Marginal Value Vector (MVV): For a permutation $p: \mathcal{N} \rightarrow R^n$ on the player set N where Π denotes set of all permutations, giving rank number $\pi(i) \in N$ to any player $i \in N$, then MVV $m^\pi(v) \in \mathcal{R}^n$ of game v and permutation π assigning to player i his marginal contribution to worth of coalition consisting of all his predecessors in π is

$$m^\pi_i(v) = v(\pi^i) - v(\pi^i \setminus \{i\}), i \in N) \quad (7)$$

7. π-consistent power function: Power function $p: \mathcal{N} \rightarrow R^n$ is π-consistent for a permutation π of N, when for all coalitions S and for all players i and j in S it holds that $\pi(i) < \pi(j)$ implies $p_i(S) < p_j(S)$.

8. π-compatible power function: Power function $p: \mathcal{N} \rightarrow R^n$ is π-compatible for a permutation π of N, when for all players $i \in N$ and for all coalitions S containing i such that $S \setminus \pi^j \neq 0$ it holds that $p_i < \sum_{j=1}^{n} p_i(S)/n$.

2.3.2 Some Relevant Theorems

1. A SSTU game Γ= (N, v, p) has a non-empty socially stable core if Γ is SS.

2. If v is a convex game, then for every p the STU-game has a non-empty SS core.

3. For a STU game, for some permutation π of N, v is permutationally convex and p is π-consistent, then SC (v, p) of Γ contains marginal vector $m^{\pi}(v)$ as an element.

4. If a STU game v is convex and p is π-compatible for some permutation π of N then SC (v, p) of Γ contains the marginal vector $m^{\pi}(v)$ as its unique element.

Based on above assertions TSC sharing is modeled as a SS game with a stable set of coalitions. Then solution is a SS POV, realized by all elements of a stable group of coalitions. A unique solution resolves ambiguity.

3. THE GAME IN ELECTRICITY MARKETS

Concepts from CG with TU are extended to restructured electricity sector such that worth and social structure developed indigenously are valid to transmission sector.

3.1 Participants in the Game

In power transaction parlance Discos are considered to be the players of the game who form coalitions to reduce TSC. Scheduling of Gencos is based on trades contracted by Discos. So, onus of decision of choice of Gencos and trade volumes is on Discos. So coalition $S \subseteq N$ has only Discos discussing power purchases for best values.

3.2 Characteristic Value of the Game (v)

Value of coalitions is a set of possible configuration conveying benefit of cooperating, especially since gain is not known when joining a coalition. A game is super-additive, convex or permutationally convex based on v. A solution assigns a set of POV to every TU game. A set valued solution is the core assigning to every game $\boldsymbol{v}$.

$$C(v) = \{x \in R^n | x(N) = v(N) \& x(S) \geq v(S), \text{all } S \subseteq N \quad (8)$$

Functionalities like an efficient POV and an un-dominated game are made the basis for choice of $\boldsymbol{v}$. Negative TSC (Equation 1) is chosen as $\boldsymbol{v}$. Thus coalition membership and values are linked with Discos only. Some factors that influenced these choices are discussed next.

1. Though Gencos pump energy into systems, Kirchhoff's laws determine power flow directions. On demand power reach Discos from a mix of sources. So intent is that Discos guide course of events towards least loss, the ultimate aim, by demand manipulation or switching benefactors via coalitions.

2. An implicit reason for choosing Discos as players and TSC as coalitional value is to keep decision makers of energy charges, the Gencos, isolated and powerless to control design and division of TSC in any manner to avoid undue influences or market power.

3. Discos wrangle a power purchase deal from available options. Concurrently, they can ensure themselves least possible TSC share, through coalitional agreements, only if value of coalition indicates gains of cooperating.

4. It must be possible to assess convexity, superadditivity, and permutational convexity of v. TSC based on Equation 1 is considered an apt choice because it penalizes loss by squaring it and ascertains convexity.

3.3 Power Function

The choice of a power function is crucial to get a marginal vector and for it to be the unique element of the core. Two qualifications for power vectors - π consistency and π compatibility are imbibed and Equation 2 is used with some changes like excluding generator buses, reducing the order of the vector whenever membership is reduced *etc*. Thus Discos excluded from a coalition have zero strength.

The game and some concepts for extending to electricity markets have been presented. A few relevant features like solution space or core, ranking *etc*. are next explained.

4. CORE OF THE GAME AND ITS DERIVATION

Assuming superadditivity and collective rationality, grand coalition will form at the end of the game. Solution to the game is the division of the joint payoff among the players.

4.1 Imputation and Core

The division of joint payoff $v(N)$ represented by POV $\tilde{x} = (x_1, x_2, \ldots, x_n)^T$ if it satisfies group ($v(N) = \sum_i^n x_i$) and individual ($x_i \geq v(i)$) rationality is a logical candidate for solution and is called an imputation. These notions of rationality when includes coalitional rationality to a solution with all possible coalitions of players too, a new concept, the core is defined. A set of imputations fulfilling individual, coalitional and group rationality is a core.

4.2 Derivation of the Core

Considering a game of three players, the set of pareto-optimum equations such that no member increase his share without reducing that of another member are given below which can be extended to any number of players.

$$x(1) \geq v(1), x(1) \geq v(1), x(1) \geq v(1)$$
$$x(1,2) \geq v(1,2), x(1,3) \geq v(1,3), x(2,3) \geq v(2,3)$$
$$x(1) + x(2) + x(3) = v(1,2,3) \text{ (Efficient allocation)} \quad (8)$$

Giving numerical values to these inequalities, the core restricted to a grand coalition of all 3 players, is a triangle on a 3 dimensional space bounded by vertices in payoff space. This solution concept is simple, persuasive and consists of a set of imputations leaving no coalition in a position to improve the payoff to all its members.

4.3 *Socially Stable Core* (SC(v,p))

A socially stable core (SC(v,p)) is the outcome of a convex, super-additive and permutationally convex game. A POV lies in the SC(v,p) if and only if x is economically feasible and un-dominated, fulfills individual, coalitional and group rationality and is sustained by a SS set of coalitions. The STU game has a non-empty SC(v,p) if Γ is SS. But to establish feasibility for any SS POV, is a tough task. Social stability is sufficient but not a necessary condition for non-empty core. If v is a convex game, then for every power function, p the STU game has a non-empty SC(v,p). But a game is convex if and only if core contains all the marginal vectors. Convexity implies super-additivity, so $x(N)=v(N)$ for any un-dominated POV. Then even if a POV can be found which is SS and sustained by a collection of stable coalitions, if POV is not feasible, then game Γ is not SS. Thus joint assumptions on power vector p and worth v, weaken assumptions made on v. Notion of π consistency is useful to tackle this problem.

4.4 *Notion of π Consistency*

When power function p is π consistent, rank of players assigned by π is consistent with the power of players in any coalition. MVV belongs to SC(v,p) of Γ for a STU game, where for some π of N, v is permutationally convex and p is π-consistent. Then, core may contain multiple elements and the best solution is to be extracted. Clearly when $p = e$ the power function is π consistent for any π and the core SC(v,p) = $C(v)$. However when p is such that for any π a player i has little power in any coalition involving players $N \setminus \pi^i$, SC(v,p) can be shown to consist of a unique element, the MVV, which is precisely the π compatibility condition referred next.

4.5 *Notion of π compatibility*

When power function is π compatible, power of a player i in any coalition that involves another player who is ranked higher according to π, is less than the average power $\sum_{j=1}^{n} p_i(S)/n$. Power function is π compatible implies that any player has so much power compared to his lower ranked players that he is able to extract all pay-offs from them up to a point when the lower ranked players could form a deviating coalition. However, neither π compatibility nor π-consistency implies the other.

At this stage players must be ranked to appraise the position of a player in any coalition. This rank is different from the strength of a player, as in a power vector. Some investigations made to assign a rank or hierarchical permutation is given, in the subsequent section.

5. INDIGENOUS HIERARCHY OR RANKING

By interpreting the definition of a permutationally convex game, it is inferred that a higher ranked agent brings more value to a coalition and so is much sought, though not uniformly by all parties. Thus in a cooperative game, based on the nature of the game being played, the route coalitions take, is generally dependant on 'who needs whom' more. Some of the facets considered in assessing hierarchy in this work are technical aspects, social features, and commercial considerations.

5.1 *Technical Aspects*

In the 'TSC Distribution game', main aim is to minimize impact on the grid. Thus prospects of reduction of loss, line loading, congestion *etc*. are taken as technical benefits that coalition members seek. Another decisive factor is position of an agent in the network configuration in which the game is played. In short, some alliances are ranked higher because they can be forged easily based on who the neighbors are and the impedance of line linking them. The reasons in such cases are both technical and social.

5.2 *Social Features*

Another aspect that affects rank of players is an evaluation of how weak an agent is. Reference is to the incapacity of an agent due to which threat looms of a coalition formation which excludes him. This is weighed against how powerful another agent is, whereby a promising coalition can be clinched, by wresting the chance from a contender. Here, social proximity of players and position in the network topology are important decisive factors. More than the electrical distance, network configuration or inter-Disco and Genco-Disco proximity influences coalition decisions here. The compensation package at an agent's disposal, which he can offer to lure partners are commercial aspects that are used to rank players.

5.3 *Commercial Considerations*

A major indicator used to rank players is minimum TSC per unit of energy the player has to pay for his transaction from his vantage point. Such commercial aspects are going to be crucial issues that decide the hierarchy.

A game with nonempty core is sociologically neutral since every cooperative demand by any coalition can be granted and there is no need to resolve conflicts. But in a coreless game, coalitions are too strong for any mechanism to satisfy every coalition demand. Simultaneously, too many elements in the core, makes it difficult to arrive at a solution, because of little predictive power for the core.

6. SOLUTION TECHNIQUES

The stable set is said to have a more general solution concept and is based on dominance. If every player in the subset S prefers payoff offered by say $\boldsymbol{y}$ and if members of subset *S* have the power to form a coalition and position to enforce their preferences then solution has both internal and external stability. The bargaining set is another technique based upon objections and counter-objections of players through imputations. Solution based on excess theory has the Kernel as a solution concept and is based on a pair-wise comparison of all players in a game in terms of excess payoff that one of the players could have by forming a coalition that excludes the other.

6.1 The Nucleolus

One advantage of nucleolus is that every game has one and only one nucleolus, and is in the non-empty core. It combines ideas of excess, stability of core and equity of value. It is an imputation with maximal excess minimized i.e. max e_{min} (e_{min} = min {Σxi - v(S)}), over all coalitions *S*. The basic idea is to find an imputation which makes the unhappy member of a potential coalition happier than under any other imputation.

6.2 The Shapley Value (SV)

A CGT solution concept of a fair division of a common utility among *n* coalition members and is the weighted average of marginal contributions of a member to all possible coalitions in which it may participate. The game is super-additive and assigns a-priori worth to each player of any coalition structure *S*. Shapley proved that a unique solution results and the POV or reward is given by

$$x_i = \sum_{\forall S|i\in S} P_n(S)\,[v(S\cup\{i\} - v(S)],\ P_n(S) = \frac{|S|!(n-|S|-1)}{n!}.$$

It means that player *i*'s reward is the expected amount he adds to the coalitions of players who are present when he arrives. Two inclusions are probability that players in *S* are already present when *i* arrives, and combinatorial nature of playing. One player starts the game and each player is added one at a time till a grand coalition is formed. Order of arrival is a pure chance mechanism and all permutations equi-probable. A striking feature of SV solution is that it is unique for every game. A defect is that the strength of players is not reflected as in a core and so is not suitable for decentralized games.

6.3 Bilateral Shapley Value (BSV)

To avoid the exponential complexity of Shapley Value calculation in a multi agent system, with probabilities for any kind of coalition formation, the Bilateral Shapley Value for some coalition S_i in a bilateral coalition *S* is *bsv* S_i,S_j (S_i)= 0.5 v (S_i) + 0.5 [v (S) - v (S_j)].

It is inferred that a synchronized algorithm is needed to adopt the Shapley value solution. The nucleolus may be used wherever minimization of excesses is possible. If equilibrium can be established kernel is a good solution concept. The bargaining set depends a lot on persuasive powers of players and there is no end to such a process. For applying BSV, coalition formation is considered. If game is super-additive a grand coalition forms and a backward process for redistribution of worth only is done.

6.4 Marginal Contribution Vector

The Marginal Value Vector of game $\boldsymbol{v}$ and permutation π is $m_i^{\pi}(v) = v(\pi^i) - v(\pi^i \setminus \{i\}),\ i \in N$). The SV is the average of MVV over all permutations and is an element of the non-empty Weber set. The core and Weber coincide if and only if and when the game is convex.

7. APPLICATION TO ELECTRICITY MARKETS

In multilateral format, trades are contracted by local agents in electricity markets. In the first phase, Discos derive local information like power vector and possible TSC levied for different trades in order to compare with energy charges. Transmission Providers (TP) determine TSC using load flow analysis, conduct least loss iteration and broadcast corresponding TSC and power vectors in central computation phase. In common information derivation and negotiation phase Discos negotiate and merge based on TSC reduction as influenced by cooperation. Socially stable coalitions result if TSC distribution game is played effectively. Procedure and illustration are as follows.

7.1 Application Procedure (Figure 1)

The following steps are used to illustrate the route.

1. The game Γ= (N, v, p), players, coalitional values and power vectors are identified
2. Convexity and super-additivity are checked and if positive, permutational convexity is verified after ranking the players, judiciously in a permutation.
3. Based on the rank of the players π consistency and π compatibility are examined.
4. Core is obtained and if possible marginal vector is derived. Else POV is obtained and social stability checked

7.2 Application to a 5-bus System

The objectives are cited before expressing an SSTU game. A 5-bus power system [20] is taken for illustration (Figure 2). Pay-off solutions are obtained and compared with other methods. At first, initial trades are derived in *local information phase* (Table1). *Central computation phase* as conducted by the TP is also given. Here, the outcome of initial trades contracted by Discos for a total transaction of 165 MW is a loss of 4.74 MW whereas optimal trades incur 1.6 MW. It provides guidelines for targeting optimal trades using power vectors. The lowest TSC to be shared (26.6 % of initial trades) as the SSTU game is also shown.

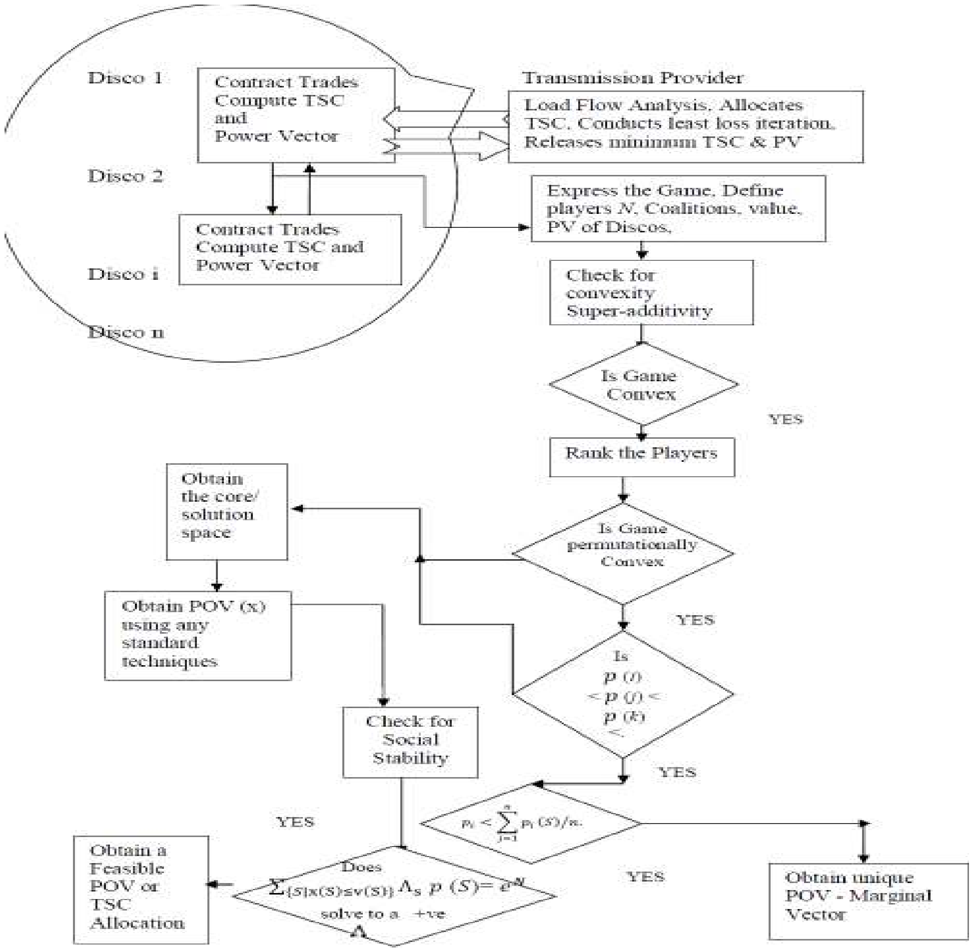

Fig. 1. Algorithm of application of SSTU game to electricity markets.

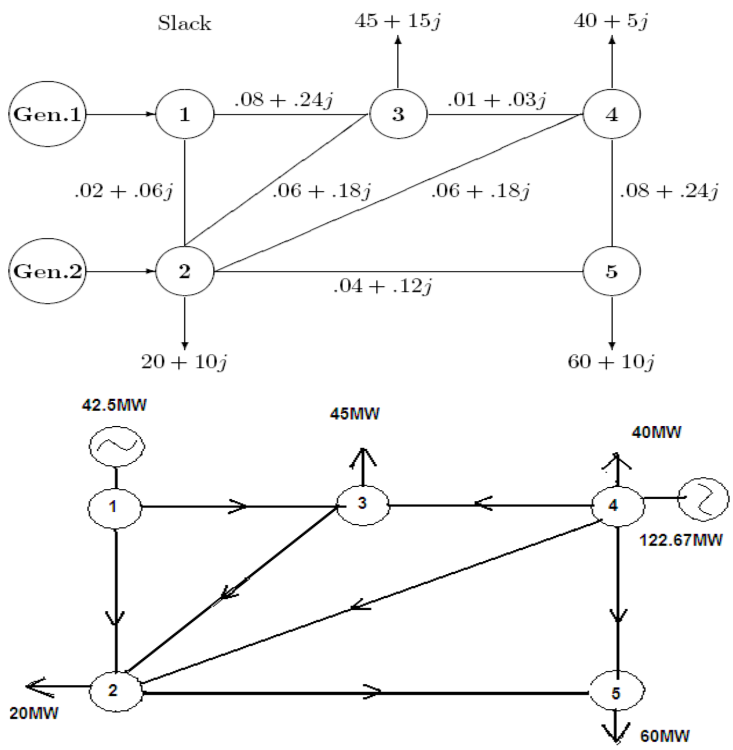

Fig. 2. Five bus power system with original and desired trade configurations.

Next the game is expressed and qualities of the game are checked for searching out a solution.

***Players in the Game*:**

The players are Discos on buses 2, 3 and 5. They form $N = 2^N - 1 = 7$,[(2),(3),(5),{2,3}, {2,5}, {3,5},{2,3,5}] coalitions trading in several ways with the two optional Gencos. (6 solo, 4 duos and a grand coalition of Discos with optimal trades shown in Tables 1 and 2).

Table 1. TSC and trade data for original problem and optimal trades for the 5 bus problem.

	Original Problem		Optimal Trades	
Disco on buses No: (Demand)	Genco bus 1 Load in MW	Genco bus 2 Load in MW	Genco bus 1 Load in MW	Genco bus 4 Load in MW
2 (20MW)	13.623	6.376	13.846	6.154
3 (45MW)	39.544	5.456	5.002	39.996
4 (40MW)	30.463	9.536	0	40
5 (60MW)	41.374	18.628	23.654	36.546
Total loading	129.74	40	44.102	122.696
Line loss	4.77MW		1.6 MW	
Sum of power flow in all lines	262.6 MW		163.7 MW	
TSC in Rs /hr.	3,11,400		82,515	

Table 2. Trade details locally derived and commonly derived after negotiation.

Local computation Phase (Trade in MW)			Loss (MW)	TSC (Rs./hr)	Ave. Rs./ kWhr	Common Information Derivation of coalitions (2,3), (2,5) and (3,5)				TSC Rs./hr. (MW loss)	Ave. Rs/kWh
Disco	Genco					Members		Gen 1	Gen 4		
Bus 2	1	(20)	0.0674	8260	0.413	(2,3)	2	13.32	6.68	21400	.329 or
Bus 2	4		0.0869	12650	0.632		3	5.89	39.11	(.2312)	.204
Bus 3	1	(45)	0.6017	38015	0.845	(2,5)	2	12.73	7.27	62915	.786 or
Bus 3	4		0.1794	15595	0.346		5	21.91	38.09	(1.3622)	.524
Bus 5	1	(60)	1.571	75140	1.252	(3,5)	3	4.995	40	68305	.650 or
Bus 5	4		1.307	63610	1.06		5	25.76	34.24	(1.3512)	.471

Table 3. Preliminary allocation of TSC.

merger	Sum self values	v(S)	TSC	profit vide (1)	profit vide (2)	POV of 2	POV of 3	POV of 5
(2,3)	8260+15,595 =23,855	9726+18,530 = 28,256	21,400	2455	6856	7032	14368	(1)
						6298	15102	(2)
(2,5)	8260+63610 = 71,870	9856+67,820 = 77,676	62,915	8955	14,761	3783	←(1)→	59132
						2475	← (2)→	60440
(3,5)	15,595+63,610 = 79,205	18,082+68,560 = 86,642	68,305	10,900	18,337	(1)→	10145	58160
						(2)→	8914	59391
(2,3,5)	87,465	9610+18,085+ 68,120=95,815	82,515	4950	13,300	6610	13945	61960
						5177	13652	63687

Table 4. Power vector for the 7 coalitions considered in the 5 bus problem.

Coalition→	(2)	(3)	(5)	(2,3)	(2,5)	(3,5)	(2,3,5)
Disco 2	0.2067	0	0	0.4620	0.4619	0	0.4565
Disco 3	0	0.2123	0	0.2532	0	0.4541	0.4911
Disco 5	0	0	0.2122	0	0.2531	0.2527	0.2484

Table 5. Checking convexity and superadditivity of coalitions in the 5 bus game.

S	*T*	*v*(*S*)-(Ph.2)	*v(T)* (Ph.2)	*v(S)* + *v(T)* *v*(*S*U*T**)(Ph.2)	*v*(*S*U*T*) Ph. 1	*v(S* ∩ *T*) (Ph.2)	Phase 1 and 2
2	3	-8260 (-9726)	-15,595 (-18530)	-23,855 (-28256)	-21,400	0	Convex,S A
2	5	-8260 (-9856)	-63,610 (-67820)	-71,870 (-77676)	-62,915	0	,, ,,
3	5	-15,595 (-18082)	-63,610 (-68560)	-79,205 (-86642)	-68,305	0	,, ,,
(2,3)	5	- 23,855 (-28256)	-63,610 (-63610)	-87,465 (-91866)	-82,515	0	,, ,,
(2,5)	3	-71,870 (-77676)	-15,595(,,)	-87,465 (-93271)	-82,515	0	,, ,,
(3,5)	2	-79,205 (-86642)	-8260 (,,)	-87,465 (-94902)	-82,515	0	,, ,,
(2,3)	(2,5)	- 23,855 (-28256)	-71,870 (-77676)	-95,725(-105932)	-82,515	-8260 (-9610)	,, ,,
(2,3)	(3,5)	- 23,855 (-28256)	-79,205 (-86642)	-103,060(-114898)	-82,515	-15,595 (-18085)	,, ,,
(2,5)	(3,5)	-71,870 (-77676)	-79,205 (-86642)	-151,025(-164318)	-82,515	-63,610 (-68120)	,, ,,

***Characteristic Value*:**

TSC allocated is to be minimized and is taken as the objective function (negative). Central Electricity Regulatory Commission (CERC) guidelines were used to choose the weights (a = Rs.500/MW^{2}h, b = Rs.250/MWh and d = Rs.10,000/MWh)TSC computed for each coalition is given in Table 2. Tentative allocation in Table 3 uses a simple sharing of coalitional benefits from trades causing least impacts and their actual allocation.

***Power Vectors*:**

Power vectors are given in Table 4 for trades linked to least TSC of seven relevant coalitions.

***Convexity and Superadditivity of the Game*:**

To study the game from perspectives of TP and Disco interactions (Phase 1) and intra-Disco dialogues (Phase 2), two modes are examined for above coalitions and given in Table 5. The actual minimum TSC for transactions is used in Phase 1 (for e.g. bus 2 incurs least TSC if buying from bus 1) to check convexity and superadditivity. Actual lower TSC are compared to sum of locally computed TSC of Discos. In Phase 2 sum of individual benefits are compared with divulged coalition information for a verdict.

***Checking Permutationally Convexity of the Game*:**

To assess this property which guarantees a workable solution for a game, a permutation is to be identified such that all n players can be arrayed as $1 \leq j \leq k \leq n$. Because only limited number of Discos (on bus 2, 3 and 5) play the TSC sharing game in this example, hierarchy structure is short. Scope for choosing various permutations and subsequent mapping of convexity is also limited. So, all possible permutations are attempted here. Also, if the information of least TSC of sub-coalitions is released in advance, collusion hinders further coalition formation and hence least loss formulation. Hence only common information values of coalitions are used in Table 6. Lowest TSC of each Disco is used in column (1) and sum of minimum TSC of both agents in column (2) (i.e. Disco i weighs his benefit in the coalition with Disco j). Column (3) picks the more beneficial value. But at the final stage the sentiment of TP is echoed, who releases overall minimum TSC the grand coalition pays (column (4)) and this agent thus loops in more people into the coalition. Hence, this game has permutational convexity. So, grand coalition alone is given the full benefit of coalitional value. The core is non empty because condition for permutational convexity is satisfied for all permutations.

Next ranking issue is investigated.

***Ranking the Players*:**

a. Technical considerations: For working out a ranking scheme, benefits of coalition are divided in the ratio of power demands of Discos who are in contention. For e.g. when Discos at bus 2 and 3 combine their transactions, loss is only 0.2312 MW due to counter-flows, in the least loss formulation. Similarly bus 2, buying from bus 1 incurs least loss of 0.0674 MW. For bus 3 when contracting with bus 4, loss is 0.1794 MW, totaling to 0.2468 MW. This means a lessening of losses by 0.0156 MW which when shared in the 20:45 MW demand ratio amounts to a share of 4.8 kW and 10.8 kW for bus 2 and 3 respectively. A similar computation of lower loss in kW for all coalitions is given in Table 7. The same situation is seen when roles of Discos in reduction of total power shuttling over lines and congestion of vulnerable lines are examined. Disco at bus 3 has top rank, then 2 and at the bottom rung is Disco on bus 5. This survey is extended to other considerations.

Table 6. Checking permutational convexity for all permutations in the 5 bus problem.

v(5) **(1)**	$v(5\cup 3)-v(3)$ **(2)**	$\max[v(5), v(5\cup 3)-v(3)$ **(3)**	$V(5\cup 2\cup 3)-V(2\cup 3)$ **(4)**	Is **(4)**> **(3)**
-63610	-(79,205-18,082) = -61,123	-61,123	-(82,515-23,855) = -58,660	Yes PC
v(5) -63610	$v(5\cup 2)-v(2)$ -(71,870-9,856) = -62014	$\max[v(5), v(5\cup 2)-v(2)\}$ -62,014	$V(5\cup 3\cup 2)-V(3\cup 2)$ -(82,515-23,855) = -58,660	Yes PC
v(2) -8260	$v(2\cup 3)-v(3)$ -(23,855-18,530) = -5,325	$\max[v(2), v(2\cup 3)-v(3)\}$ -5,325	$V(2\cup 3\cup 5)-V(3\cup 5)$ -(82,515-79,205) = -3,310	Yes PC
v(2) -8260	$v(2\cup 5)-v(5)$ -(71,870-67,820) = -4,050	$\max[v(2), v(2\cup 5)-v(5)\}$ -4,050	$V(2\cup 5\cup 3)-V(5\cup 3)$ -(82,515-79,205) =-3,310	Yes PC
v(3) -15595	$v(3\cup 2)-v(2)$ -(23,855-9,726) = -14,129	$\max[v(3), v(3\cup 2)-v(2)\}$ -14,129	$V(3\cup 2\cup 5)-V(2\cup 5)$ -(82,515-71,870) =-10,645	Yes PC
v(3) -15595	$v(3\cup 5)-v(5)$ -(79,205-68,560) = -10,645	$\max[v(3), v(3\cup 5)-v(5)\}$ -10,645	$V(3\cup 5\cup 2)-V(5\cup 2)$ -(82515-71,870) = -10,645	Equal PC

Table 7. Ranking players based on line loss reduction.

Agent at bus	Benefit in reducing	Coalition (2,3)	Coalition (2,5)	Coalition (3,5)	Coalition (2,3,5)	Rank order	Reasoning
2	Loss	4.8	3.05	-	8	5	3 is sought by both 2 and 5 due
3	Loss	10.8	-	57.9	18.1	2	to more reduction of loss; 5
5	Loss	-	9.15	77.3	24.1	3	needs the coalition most

b. Social considerations: The network has a line of lowest impedance between generator bus 4 and Disco 3. Hence its demands are largely met by bus 4 whatever be its contracts because other paths offer higher impedances. Hence in the grid, bus 3 has a significantly superior position. Next lowest impedance line is between bus 2 and bus 1 and so bus 1 is the main supplier to power needs of bus 2. So it is ranked between bus 3 and 5. Bus 5 is situated strategically in a vulnerable position, almost equidistant electrically from both generators. Thus socially too, ranking echoes what was assigned based on impact of trades. Disco at bus 5 is last, 2 in the middle and 3 tops with rank numbers 1, 2, and 3, respectively.

c. Commercial considerations: Average TSC per unit of energy drawn is a good figure of merit to compare who needs coalitions most and the order of preference. Bus 3 transacts at a minimum TSC of Rs. 0.346/kWh. Next lowest TSC is achieved by bus 2 (Rs. 0.413/kWh.) The worst figures are for bus 5 which can, at best, aspire only to Rs. 1.06/ kWh. Definitely ranking will reflect this capability by according top rank to bus 3, next to bus 2 and the last to bus 5. It can be seen from Tables 3 that all preliminary POV allocation schemes associate a TSC of more than Rs. 60,000/- to bus 5, which is 7 to 8 times of allocation to bus 2 but demand is only three times that of bus 2. Whereas load ratio of bus 3 and 5 is 3:4, TSC payable or pay-off vector ratio is about 1:4. This implies a low stature for bus 5 commercially and a low rank too. Bus 3 is again superior to bus 2 on account of its pay-off data too. It transacts a load of 2.25 times that on bus 2 but is allotted only double the TSC apportioned to bus 2.

In this very short hierarchy structure, ranking is done as follows. Disco on bus 3 gets a rank number 3 as the 'top man' and joins the set having utmost rank equal to 3. Next ranked player is bus 2 accorded a rank number 2, with only two players- bus 2 and bus 5- in the set having utmost rank equal to 2. And at the bottom we have bus 5 as a sole member in this set. With the ranking published, the job of checking permutational consistency of power vectors of the game is taken up.

***Permutationally Consistent Power Vector*:**

Power vectors given in Table 4 are used to check permutational consistency and detailed in Table 8 for all coalitions. The permutation being examined has a ranking order of bus 3 > bus 2 > bus 5 in each of the coalitions. Coalition (2,3) alone is not permutationally consistent because bus 2 has many more successors than bus 3 in the power system configuration investigated. This shows the difference between a power vector and ranking.

***Permutationally Compatible Power Vector*:**

The property is studied via Table 9 given below. The lower ranked players are bus 5 and bus 2 when both play with bus 3 and bus 5 alone when playing with either bus

2 or bus 3.

Neither π-consistency nor π compatibility could be fully established for ***p***. However, though marginal vector is not the unique solution, existence of other solutions are explored to generalize the solution technique.

***Deriving the Core for the 5 Bus Power System*:**

Using Equation 8, solution is confined to a triangle as per the nucleolus concept and given in Figure 3. Separate values, available at two information phases are used to compare the nucleolus. Discos on bus 2, 3 and 5 are designated players 1, 2 and 3, respectively while considering TSC allocation in the local information phase. The same players are denoted as A, B and C respectively while TSC allocation, based on common information derived as a part of negotiation, is considered. The existence of a solution space admits experimentation with pay-off vectors derived using other methods to extract socially stable solutions.

Table 8. Checking permutational consistency of the power vectors.

Coalition	(2)	(3)	(5)	(2,3)	(2,5)	(3,5)	(2,3,5)
Power of 2	0.2067	0	0	0.4620	0.4619	0	0.4565
Power of 3	0	0.2123	0	0.2532	0	0.4541	0.4911
Power of 5	0	0	0.2122	0	0.2531	0.2527	0.2484
Comparison of PV	p(2)>p(3) and p(5)	p(3)>p(2) and p(5)	p(5)>p(2) and p(3)	p(2)>p(3) >p(5)	p(2)>p(5) >p(3)	p(3)>p(5) >p(2)	p(3)>p(2) >p(5)
π consistency	NA	NA	NA	No	Yes	Yes	Yes

Table 9. Checking permutational compatibility of the power vectors.

Coalition	(2)	(3)	(5)	(2,3)	(2,5)	(3,5)	(2,3,5)
Power of 2	0.2067	0	0	0.4620	0.4619	0	0.4565
Power of 3	0	0.2123	0	0.2532	0	0.4541	0.4911
Power of 5	0	0	0.2122	0	0.2531	0.2527	0.2484
p(5) : N \ (2,3)	Check if p(5) < $\sum_{j=1}^{n} p_i(S)/n$			NA	.2531<.357 5	.2527<.353 2	.2484<.398 7
$p(5, 2)^T$: N \ (3)	Check if p(5) and p(2) < $\sum_{j=1}^{n} p_i(S)/n$			5 not in S p(2)> ave.	NA	p(5) <ave. 2 not in S	,, p(2) > ave.
π compatibility	NA	NA	NA	No	Yes	Yes	No

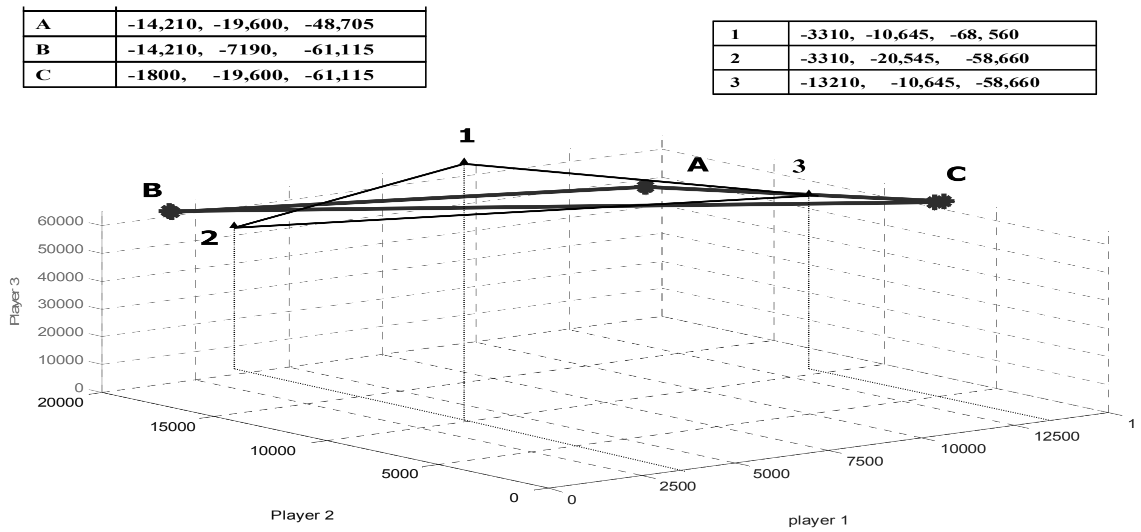

Fig. 3. Solution space for the 5 bus example using allocated values of TSC in local and common information phase.

Table 10. Checking for social stability for all feasible payoff vectors for the 5 bus game.

Method	Payoff vector	Coalitions	$\sum_{\{S\mid x(S)\leq v(S)\}}\Lambda_S\, p(S)$	Is $[\Lambda]^T$ +ve	SSC
nucleolus	-[6610,13945,61960]	(2,5)and (3,5)and (2,3,5)	Λ_1p(2,5)+Λ_2p(3,5)+Λ_3p(2,3,5)	[1.79, .79,.38]Y	Yes
demand	-[7468,13813,61234]	(2,5) and (3,5) and (2,3,5)	Λ_1 p(2,5) + Λ_2 p(3,5)+ Λ_3 p(2,3,5)	[1.79, .79, .38]Y	Yes
,,	-[5177,13652,63687]	(3)(2,5) and (3,5) and (2,3,5)	Λ_1p(2,5)+Λ_2p(3,5)+Λ_3p(2,3,5) +Λ_4p(3)	[1.8,.79,.38, 0]Y	Yes
loss	-[7485,13300,61740]	(2,5) and (3,5) and (2,3,5)	Λ_1 p(2,5)+Λ_2 p(3,5)+ Λ_3 p(2,3,5)	[1.79, .79,.38]Y	Yes
BSV	-[6908,13244,62363]	(2,5) and (3,5) and (2,3,5)	Λ_1 p(2,5) + Λ_2 p(3,5)+ Λ_3 p(2,3,5)	[1.79,.79,.38]Y	Yes
BSV(2)	-[7022,14358,61135]	(2,5) and (3,5)and (2,3,5)	Λ_1 p(2,5) + Λ_2 p(3,5)+ Λ_3 p(2,3,5)	[1.79,.79,.38]Y	Yes

Solution as Applicable to 5 Bus System:

1. The nucleolus is computed for coalitional values in Phase 1: x (2,3,5) = [- 10075, -15460, -56980]^T Rs/ hr. Phase 2: x (2,3,5) = [- 6610, -13,945, -61,960]^T Rs/ hr. A problem with this solution is that e_{min} is not less than or even equal to any individual payoff and must be resolved for a generalized and hence accepted as a unique solution.

2. The Shapley values computed show that both the payoff vectors sum to a non-feasible total coalitional value. Thus the remainder is to be shared via a suitable procedure. A more acceptable solution with more relevance to the core i.e. the bilateral Shapley value is examined next.

3. In the bilateral Shapely value derivation, a reallocation is made of the total coalition worth of -82,515, assuming that based on power vector information, agents at bus 2 and bus 3 form the first meta-agent. In Phase 1

$$[x(2)\ \ x(3)\ \ x(5)\ ]^T = [-6908, -20{,}152, -62{,}363]^T$$

At the final stage, founder member of the coalition gets a better worth. But till then recursive calculation is used. Similarly payoff vector for Phase 2 coalitional values are $[x(2)\ \ x(3)\ \ x(5)\ ]^T = [\ -7022\ \ -14{,}358,\ -61{,}135]^T$

4. Marginal contribution is also assessed before the social stable core is derived to identify the payoff vector if a unique solution is likely. This is so if a permutationally convex game is permutationally consistent and compatible too. Here marginal vector is shown to fail, as expected. It is based on bus 5 starting the coalition and bus 3 with the top rank being the last entrant and is as follows. In Phase 1: MC(3) = - 19600, MC (2) fails and MC (5) = - 63610.

With Phase 2 coalitional values, MVV(2,3,5)= [-8260, - 10645, - 63610] Rs./hr. The payoff vector must be designed to be at most equal to these values.

***Socially Stable Core*:**

A core or solution space is derived for both phases with vertices as illustrated in Figure 3. Next a socially stable core is examined using feasible payoff vectors so far derived and shown in Table 10. This exercise aims to identify coalitions that can sustain the corresponding payoff vector. So far every feasible POV derived is socially stable. The only problem is that the core does not contain a unique solution.

The results are still interesting and significant since the solution space offers scope for further negotiation and bargaining power, based on players' persuasive skills.

8. CONCLUSION

A method for TSC allocation is introduced, intuitionally more applicable to electricity markets than conventional methods because the procedural components take advantage of many traits of market agents. One of them is the inherent choice factor and its capacity to promote competition among both Discos and Gencos. Simultaneously, the sense of responsibility that comes with conscious choices reduces incidences of real time retraction, especially since the spirit is of TSC sharing and not of allocation. Moreover, scope for negotiation and extraction of coalitional information in this method, resolves the uncertainty factor in an information asymmetric market. The biggest engineering advantage is that security of the grid becomes a common agenda and a unifying force in a profit motivated milieu where commercial considerations overrule engineering requirements. There are inbuilt mechanisms for checking efficiency of the pay of vector. Another attractive feature is the social stability associated with it and consequently the method, which is crucial in real time operation of an electricity market.

A unique solution could not be obtained in the example considered because Disco at bus 2 has more influence on power flowing in the network but a lower rank as per the hierarchy considerations used here. The indigenously constructed power vector, ranking and TSC need to be investigated further for obtaining unique payoff vector as the solution. The choice of weights in TSC evaluation also needs a rethink or expert procedures.

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