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Optimal Control Strategy using Pseudo-Decentralization for Coordination of Power System Stabilizer and FACTS in a Multi-Machine System

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ABSTRACT

The power system stabilizers (PSS) are widely used for damping out the oscillations in a large power system. The interactions of multiple PSS may adversely affect the system stability, hence, they must be properly coordinated. An optimal control strategy with pseudo-decentralization is proposed for the coordination of multiple PSS. FACTS controllers such as static Var compensator (SVC) and thyristor controlled series compensator (TCSC) are used mostly for improving voltage/reactive power and power flow control. These along with supplementary controllers can also be used for damping out the oscillations in the system. The strategy for coordination of the multiple PSS has also been extended for the coordination of supplementary controllers of SVC and TCSC with PSS. The proposed technique has been tested on 39-bus New England system.

1. INTRODUCTION

Maintaining system stability has been one of the major concerns for integrated operation of power systems. For this purpose more efficient control strategies have been utilized. To improve power system transient stability, the fast excitation control has been popularly used. However, under certain operating conditions, it produces a detrimental impact on power system small signal stability. This problem can be solved or at least mitigated through the use of power system stabilizers (PSS). The flexible AC transmission system (FACTS) like static var compensators (SVC), thyristor controlled series compensator (TCSC) and unified power flow controller (UPFC) etc are used, in general, for the reactive power support, to control power flow and to improve voltage profile. These FACTS controllers can also be used for damping the oscillations and improve the system stability, when supplementary controllers are added to the main controllers. Different controllers like PSS and the FACTS supplementary controllers, when operating in a multi-machine system may have interactions adversely affecting the system stability, and hence these controllers need to be coordinated carefully.

Several design strategies for multi-machine power system having PSS are proposed in the literature during the last two decades. These methods are usually sequential, in the sense that the PSS parameters of each machine are adjusted at a time, as an extension of the classical single machine infinite bus system [7-9]. Since, one machine is taken at a time in the single machine infinite bus system, the dynamics of all the generators cannot be considered simultaneously and hence these will not be able to take control interactions into account.

In this context, integrated methods, as opposed to the sequential procedure, become very attractive due to their ability to properly take into account all dynamic interactions. Optimal control coordination methods using linear matrix inequality were proposed in ref [1, 2], which have considered the dynamics of all the generators simultaneously. The main drawback of these methods are to calculate the system parameters for a set of operating condition, which is formed by removing a line at a time.

The parameters are required to be averaged and then optimal control strategy with linear matrix inequalities is applied for obtaining the coordinated parameters of the controllers. This method becomes infeasible if the system is large as the number of operating points will be very high. Damping torque coefficients [6], which are quite difficult to calculate for large power systems have been used for coordination of controllers. References [1, 4, 10] described the selection of the feedback signal to FACTS controllers for damping the inter-area power oscillations. They used the local signals as feedback which may not increase the damping of the local modes of oscillations of the generators.

In this paper, an optimal control strategy with pseudo-decentralization has been proposed for the coordination of PSS with SVC and TCSC supplementary controllers. An approach has been suggested to find optimal location and selection of input signals to the SVC and TCSC controllers. The proposed method has been implemented on 39-bus New England system.

2. POWER SYSTEM AND PSS MODELING

A two-axis machine model of synchronous generator along with IEEE type-1 DC exciter has been considered in this work. The system dynamic equations are given in the appendix, which have been linearised around an operating point for a small perturbation (Δ). The linearised dynamic equations of the machines along with exciter can be represented in state space form as:

$$\Delta \dot{\mathbf{X}} = A \Delta \mathbf{X} + B \Delta \mathbf{U} \quad (1)$$

$$\Delta \mathbf{Y} = C \Delta \mathbf{X} \quad (2)$$

An N machine power system can be considered to be composed of N subsystems. The state vector ($\Delta \mathbf{X}$) can be partitioned into N state vectors of each sub system as:

$$\Delta \mathbf{X} = [\Delta \mathbf{X}_1^T, \Delta \mathbf{X}_2^T, \dots, \Delta \mathbf{X}_N^T]^T \quad (3)$$

where,

$$\Delta \mathbf{X}_i = [\Delta \delta_i, \Delta \omega_i, \Delta E'_{qi}, \Delta E'_{di}, \Delta E_{fdi}]^T \quad i = 1, \dots, N \quad (4)$$

Similarly, the input vector can also be partitioned as:

$$\Delta \mathbf{U} = [\Delta \mathbf{U}_1^T, \Delta \mathbf{U}_2^T, \dots, \Delta \mathbf{U}_N^T]^T \quad (5)$$

where, $\Delta \mathbf{U}$ = The vector of references for the voltage regulator of the machines.

The system output vector is formed by at least one measurable output for each subsystem.

$$\Delta \mathbf{Y} = [\Delta \mathbf{Y}_1^T, \Delta \mathbf{Y}_2^T, \dots, \Delta \mathbf{Y}_N^T]^T \quad (6)$$

With the above partition of vectors, $\Delta \mathbf{X}$ and $\Delta \mathbf{U}$ matrix B will exhibit a block diagonal structure that is

$$B = \text{block_diag}\{B_1, B_2, \dots, B_N\} \quad (7)$$

where,

$$B_i = [0, 0, 0, 0, K_{ai}/T_{ai}]^T \quad i = 1, \dots, N \quad (8)$$

K_{ai} and T_{ai} are the gain and time constant of the IEEE Type-1 DC exciter reduced to a first order representation. Speed deviation has been taken as the output in this work and the output matrix C is given as:

$$C_i = [0, 1, 0, 0, 0]^T \quad i = 1, \dots, N \quad (9)$$

2.1 Power System Stabilizer Representation

Many signals, such as the electric power, rotor speed, and AC bus frequency etc has been used as input signal to the PSS [6]. In this paper, only rotor speed deviation is used as the input signal. The signal must be processed through an appropriate lead-lag block in order to compensate the lag introduced by the exciter of the generator. The PSS transfer function is usually composed of two lead-lag stages and a washout filter to reject high-frequency noise. The PSS transfer function can be represented as given in Fig. 1.

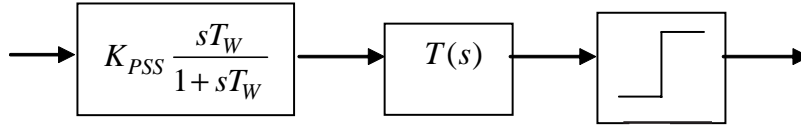


Fig. 1 Basic block diagram of a PSS

The transfer function of the PSS can be written as:

$$G_{PSS} = K_{PSS} \left(\frac{sT_W}{1 + sT_W} \right) \left(\frac{1 + sT_1}{1 + sT_2} \right) \left(\frac{1 + sT_3}{1 + sT_4} \right) \quad (10)$$

where, T_W and T_1, T_2, T_3, T_4 = the wash out and lead-lag time constants, respectively.

This transfer function can be rewritten as: [1, 2].

$$G_{PSS} = d + \left(\frac{\beta_1 s + \beta_0}{s^2 + \alpha_1 s + \alpha_0} \right) \quad (11)$$

where, $d = -k_{PSS} \frac{T_1 T_3}{T_2 T_4}$, $\alpha_0 = \frac{1}{T_2 T_4}$

$$\alpha_1 = \frac{T_2 + T_4}{T_2 T_4}, \quad \beta_0 = \left(\frac{K_{PSS}}{T_2 T_4} \right) \left(1 - \frac{T_1 T_3}{T_2 T_4} \right)$$

$$\beta_1 = \left(\frac{K_{PSS}}{T_2 T_4} \right) \left(T_1 + T_3 - \left(\frac{T_1 T_3}{T_2 T_4} \right) (T_2 + T_4) \right)$$

The linearised state space representation of the PSS can be expressed as:

$$\Delta \dot{\mathbf{X}} = A_c \Delta \mathbf{X}_c + B_c \Delta \mathbf{U}_c \quad (12)$$

$$\Delta \mathbf{Y}_c = C_c \Delta \mathbf{X}_c + D_c \Delta \mathbf{U}_c \quad (13)$$

$$\text{where, } A_c = \begin{bmatrix} 0 & -\alpha_0 \\ 1 & -\alpha_1 \end{bmatrix}, \quad B_c = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$$

$$C_c = [0 \quad 1], \quad D_c = d$$

2.2 The PSS Design Procedure

The PSS design problem consists of determining the parameters of the transfer function, $G_{pss}(s)$ so that the compensated system is sufficiently damped. We select T_p , T_2 , T_3 , and T_4 of a generator by making other generator buses as infinite buses. Then the phase lag introduced by the transfer function of the exciter from voltage reference to the torque output is found out. The time constants of the PSS are so chosen that the phase-lag introduced by the exciter is exactly compensated by the PSS. The wash out filter time constant is taken as 10 sec.

Compensating the system represented by Eqs. (1) and (2) with the controller given by the state Eqs.(12) and (13) in feedback connection implies that $\ddot{\mathbf{A}}\mathbf{U}_c = \ddot{\mathbf{A}}\mathbf{Y}$ and $\ddot{\mathbf{A}}\mathbf{U} = \ddot{\mathbf{A}}\mathbf{Y}_c$. The resulting composite system is then represented as:

$$\begin{aligned} \Delta \dot{\mathbf{X}} &= (A + BD_c C) \Delta \mathbf{X} + BC_c \Delta \mathbf{X}_c \\ \Delta \dot{\mathbf{X}}_c &= A_c \Delta \mathbf{X}_c + B_c C \Delta \mathbf{X} \end{aligned} \quad (14)$$

If a feedback loop is introduced with a gain K to the system represented by the Eq. (1) such that $\ddot{\mathbf{A}}\mathbf{U} = -K \ddot{\mathbf{A}}\mathbf{X}$ then the system with feed back is given by

$$\Delta \dot{\mathbf{X}} = (A - BK) \Delta \mathbf{X} \quad (15)$$

Comparing Eqs. (14) and (15), we get

$$K = -D_c C \quad (16)$$

Thus, the problem reduces to finding a static feedback gain K so that D_c can be calculated, thereby the coordinated gains of the PSS. To find out the K , an optimal control strategy with pseudo-decentralization has been used [2] as:

$$J(\Delta \mathbf{X}, \Delta \mathbf{U}) = \frac{1}{2} \int_0^{\infty} (\Delta \mathbf{X}^T Q \Delta \mathbf{X} + \Delta \mathbf{U}^T R \Delta \mathbf{U}) dt \quad (17)$$

where, the semi positive definite matrix Q and the positive definite matrix R are weighting matrices. These are selected by trial and error process [2, 11]. This is the so-called linear quadratic regulator problem and its well-known solution is given by the state feedback strategy

$$\Delta \mathbf{U} = -K \Delta \mathbf{X} \quad (18)$$

where, $K = R^{-1} B^T P$ and P is the solution of the algebraic Riccati equation.

$$A^T P + PA - PBR^{-1} B^T P + Q = 0 \quad (19)$$

2.3 Pseudo-Decentralization

To implement a centralized control in a multimachine system, large numbers of measurements are required. As number of machines and controllers increase, computing the full state feedback gains become complicated. In order to deal with this difficulty, decentralized approach has been used by many researchers. If the feedback gain is decentralized in such a way that only gains pertaining to the local states of the generator are retained and the cross gains are neglected, then the complexity of feedback computation will decrease. This is called as pseudo-decentralization.

The system with pseudo-decentralization has similar performance as the centralization control. However, the computational burden is reduced [5]. If K is the centralized feedback gain of a multimachine system, its diagonal elements correspond to the gains of local states of each generator. According to the pseudo-decentralization, only the diagonal elements are considered and the cross gain terms are neglected. Thus,

$$K = \begin{bmatrix} K_{11}, 0, 0, \dots, 0 \\ 0, K_{22}, 0, \dots, 0 \\ \vdots \\ 0, 0, 0, \dots, K_{mm} \end{bmatrix} \quad (20)$$

With the decentralized gain, the coordinated gains of power system stabilizers can be obtained from D_c values which can be computed as:

$$D_c = KC^T(CC^T)^{-1} \quad (21)$$

3. STATIC VAR COMPENSATOR

Fig. 2 shows a block diagram representation of SVC [12]. The measurement module for sensing the voltage and converting it into dc feedback signal has been ignored as it has small time constant. The gain, K_{SVC} , is the inverse of slope of voltage regulation characteristics (usually 1-5%), lies in the range 20-100. The regulator time constant, T_r , usually lies between 20 and 150 milliseconds. Susceptance B_{SVC} is the controlled output of the SVC controller. The supplementary controller of the SVC has the same structure as that of the power system stabilizer. It has a washout filter followed by lead-lag block. The output of the supplementary controller is given to the summing junction of the SVC block. U_{SVC} is the input to the supplementary controller. K_{PSVC} is the gain of the SVC supplementary controller. $FDS1, FDS2$ are the intermediate states and SVC_dsig is the output of the supplementary controller. T_w and T_p, T_2, T_3, T_4 are the wash out and lead-lag time constants, respectively.

The linearised dynamic equation of the SVC controller is given as:

$$T_r \frac{d\Delta B_{SVC}}{dt} = -\Delta B_{SVC} + K_{SVC}(\Delta V_{ref} + \Delta V_{sup}) \quad (22)$$

$$\Delta M_J(\lambda_h) = \frac{\partial M_J(\lambda_h)}{\partial K_J} \Delta K_J + \frac{\partial M_J(\lambda_h)}{\partial \lambda_h} \Delta \lambda_h \quad (26)$$

Substituting Eq. (26) in Eq. (25) and solving for $\Delta \lambda_h$, we get

$$\Delta \lambda_h = \frac{r_{mq}^h}{1 - r_{mq}^h \frac{\partial M_J(\lambda_h)}{\partial \lambda_h}} Q_J(\lambda_h) \Delta K_J \quad (27)$$

If K_J is so chosen that $\left\| r_{mq}^h \frac{\partial M_J(\lambda_h)}{\partial \lambda_h} \right\| \ll 1$, then Eq. (27) reduces to

$$\Delta \lambda_h \approx r_{mq}^h Q_J(\lambda_h) \Delta K_J \quad (28)$$

Based on the Eq. (28) the following procedure has been adopted for the design of the stabilizer. To cause a left shift in the eigen-value of interest, choose T_p, T_z , such that

$$\arg\{r_{mq}^h Q_J(\lambda_h)\} = \pm 180 \quad (29)$$

Residues for critical eigen mode with SVC bus reference as input and the angles of the generator buses as output are found out. The residue gives the controllability of the output and observability of the input. The generator bus angle residue with maximum magnitude is taken as the input signal to the SVC supplementary controller.

4. THYRISTOR CONTROLLED SERIES COMPENSATOR (TCSC)

Thyristor controlled series compensator is used for enhancing the power transfer capability of a line between two buses. The capacitor compensation can be between 10 to 30 % of the line inductance. TCSC is usually represented by a first order transfer function with a time constant T_r , which represents the time lag in firing pulses of the thyristors. Time constant is between 20 to 50 milliseconds. The block diagram of TCSC is shown in Fig. 3. X_{ref} can be calculated according to the requirement of the compensation of the line. X_{SUP} is the signal from the supplementary controller.

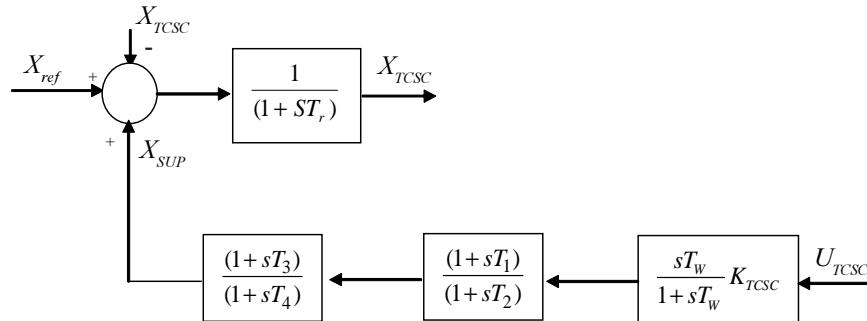


Fig. 3 Block diagram of the TCSC with supplementary controller

The differential equation representing dynamics of the TCSC is

$$T_r \frac{dX_{TCSC}}{dt} = -X_{TCSC} + (X_{ref} + X_{SUP}) \quad (30)$$

The supplementary controller is similar to that of the PSS. The differential equations for supplementary controller are similar to that of the SVC controller as explained in section 3. K_{TCSC} is the gain of the supplementary controller of the TCSC. T_w , and T_p , T_2 , T_3 , T_4 are the wash out and lead-lag time constants respectively. U_{TCSC} is the input to the supplementary controller.

Criteria for Placement and Selection of Control Input Signal of TCSC

Optimal placement of TCSC has been decided based on a method given in reference [11]. The severity of the system loading under normal and contingency cases can be described by a real power line flow performance index, as given below.

$$PI = \sum_{m=1}^{N_l} \frac{w_m}{2n} \left(\frac{P_{lm}}{P_{lm}^{\max}} \right)^{2n} \quad (31)$$

where, P_{lm} is the real power flow and P_{lm}^{\max} is the rated capacity of line- m , n is an exponent and w_m a real non-negative weighting coefficient which may be used to reflect the importance of lines.

PI will be small when all the lines are within their limits and reach a high value when there are overloads. Thus, it provides a good measure of severity of the line overloads for a given state of the power system. In this study, the value of exponent has been taken as 2 and $w_m = 1.0$ for all m . The real power flow PI sensitivity factors with respect to the parameters of TCSC can be defined as:

$$c_k^c = \left. \frac{\partial PI}{\partial x_{ck}} \right|_{x_{ck}=0} = \text{PI sensitivity with respect to TCSC placed in line-}k \ (k=1, \dots, N_l)$$

Using Eq. (30), the sensitivity of PI with respect to parameter x_{ck} of TCSC connected between bus- i and bus- j for $n=2$, can be written as:

$$\frac{\partial PI}{\partial X_k} = \sum_{m=1}^{N_l} w_m P_{lm}^3 \left(\frac{1}{P_{lm}^{\max}} \right)^4 \frac{\partial P_{lm}}{\partial X_k} \quad (32)$$

The TCSC has been placed in a line (k) having most negative sensitivity index.

There are two ways in which one can coordinate SVC and TCSC controllers. The first way is to use SVC for damping one mode of oscillation and TCSC for different mode. The second method is to use both SVC and TCSC controllers for damping the same mode of oscillations and coordinate them. Both these methods have been used in the literatures [4, 6]. There are different input signals to the TCSC supplementary controller like real power in the line, bus voltage and the angle difference between two generators [4, 10]. In this paper, the angle difference between the generators which are participating maximum in a critical mode is taken as the input signal to the supplementary controller. The linearised differential equations for the supplementary controllers of TCSC are similar to that of SVC given in section 3.

5. NUMERICAL RESULTS

The proposed optimal control strategy with pseudo-decentralization for coordination of the controllers in the system has been implemented on New England 39-bus system [14] which contains 10 machines. Machine 10 has been taken as infinite bus. There are fourteen under damped modes in the system without any controllers (modes with damping ratio less than 0.05 are taken as under-damped modes). The under-damped modes of the system are given in Fig. 4. For this system, the voltage participation factors for critical mode were calculated and the bus, which was participating maximum, was found out. It is found that the bus 8 is participating maximum to the critical mode and hence the SVC is placed at bus 8. The bus voltage participation factors are given in the Table 1. Only voltages with participation factor greater than 0.2 are listed in this table.

Table 1 Participation factor of bus voltage for critical mode

Bus	PF	Bus	PF
5	0.216	6	0.220
1	0.299	7	0.231
9	0.235	8	0.257

The optimal placement of TCSC according to the method given in reference [11] was found between bus number 14 and 15. The input signal to the SVC was selected by computing the residues of the generator bus angle. The residues of the generator bus angles for critical mode are given in Table 2. It is observed that the generator 5 bus angle participates maximum and hence, it is taken as the input signal to the SVC supplementary controllers.

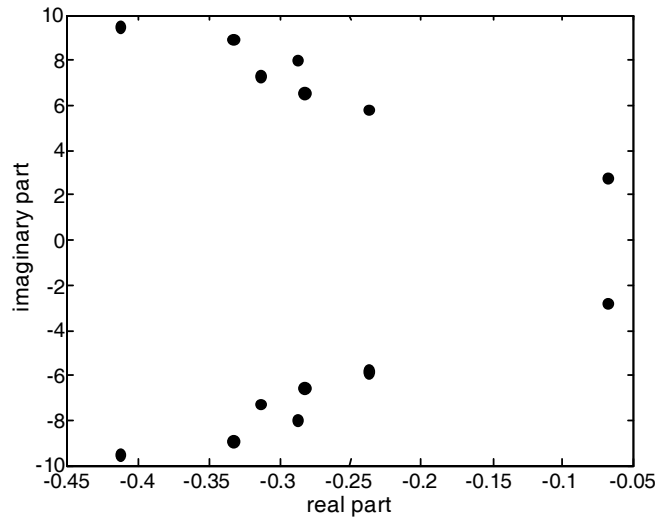


Fig. 4 Under-damped modes without any controllers

Table 2 Residue for critical eigen mode

1	2	3	4	5	6	7	8	9	10
2.4731	2.4374	2.6414	3.3708	3.5066	3.3291	3.4291	2.6105	3.4949	0.3472

Generators 9 and 5 are participating maximum in the critical mode and hence the angle difference between these generators is taken as the input signal to the TCSC supplementary controller. The coordinated controller parameters obtained using the pseudo-decentralization method is given in Table 3.

Table 3 The coordinated parameters of controllers

Gen. No.	gain	T_w (sec)	T_1 (sec)	T_2 (sec)	T_3 (sec)	T_4 (sec)
PSS 1	55.5	10	0.9	0.04	0.9	0.04
PSS 2	58.2	10	0.7	0.03	0.7	0.03
PSS 3	35.6	10	0.8	0.06	0.8	0.06
PSS 5	125.9	10	0.4	0.02	0.4	0.02
PSS 6	25.37	10	0.7	0.035	0.7	0.035
PSS 7	47.7	10	0.2	0.025	0.2	0.025
PSS 8	35.35	10	0.8	0.03	0.8	0.03
PSS 9	9.5	10	0.6	0.02	0.6	0.03
SVC_sup	0.52	10	0.3	0.2	0.3	0.2
TCSC_sup	0.27	10	0.2	0.02	0.2	0.02

The under-damped eigen values which are present in the system without any controllers are sufficiently damped with the controllers introduced in the system. The damped eigen values are given in Fig. 5.

The robustness of this method was verified by disturbing the system with a step input of 0.1 pu magnitude and the speed response of all the generators were plotted. The step responses without any controller, with only PSS and with PSS, SVC, and TCSC supplementary controllers are plotted in Figs. 6-8. Fig. 6 is plot of oscillations without any controllers. It is observed that the oscillations are persistent and peak of the oscillation of generator 9 without controllers is 4 times that of the peak of the speed oscillations with all controllers. Fig. 7 is a plot of speed deviation oscillations with only PSS and without SVC, TCSC supplementary controllers. Even though, in this case, the peak is not reduced, the system becomes stable after 40 sec. However, settling time is quite high. Fig. 8 is a plot of speed deviation oscillations with PSS and SVC, TCSC supplementary controllers. It is observed that the peak is $\frac{1}{4}$ that of the oscillations without any controller and with only PSS. The settling time is very less as compared to that without the controllers.

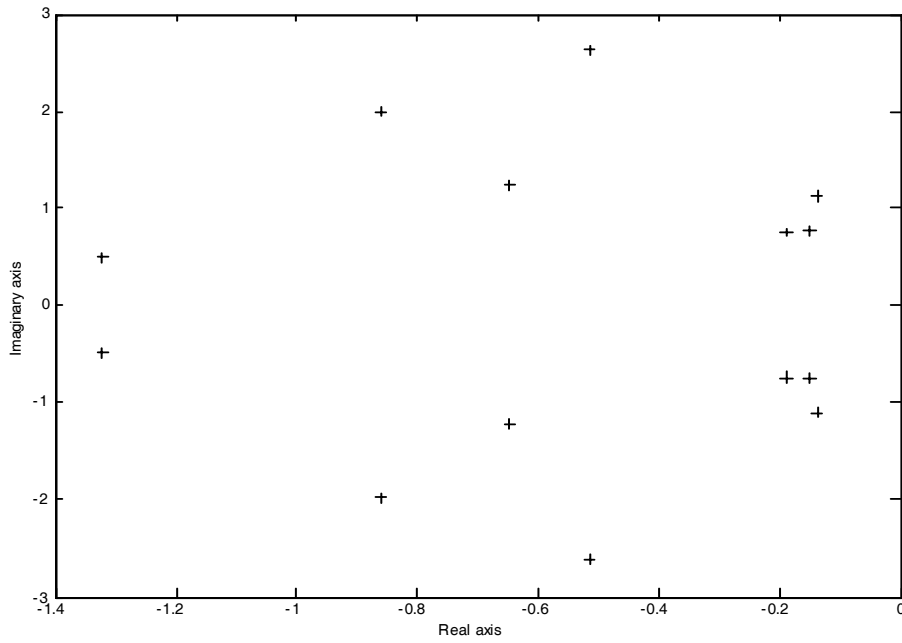


Fig. 5 Damped modes with all the controllers in the system

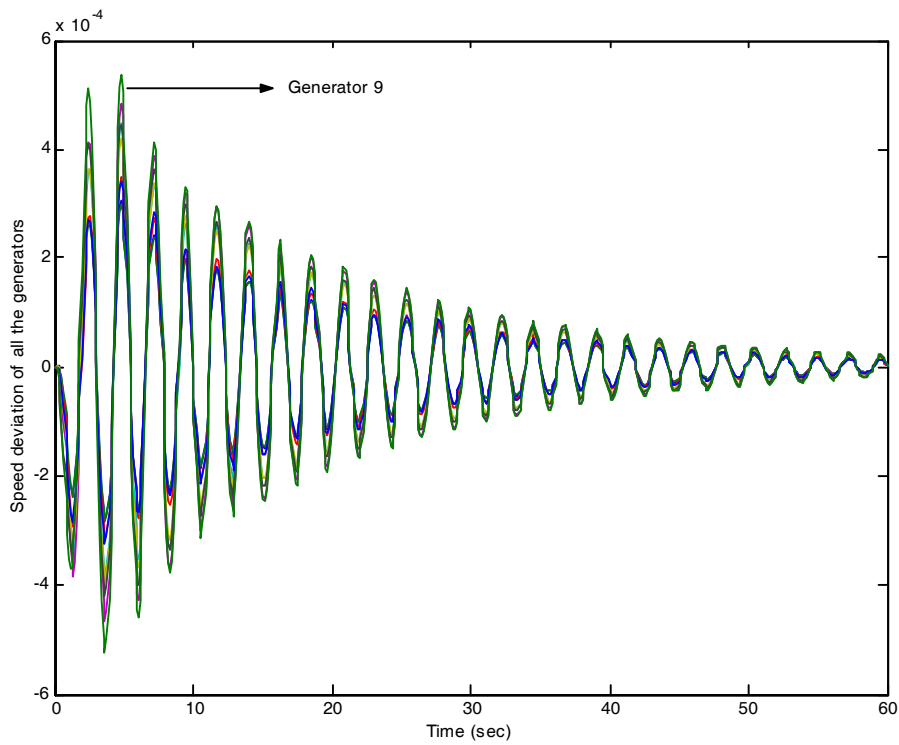


Fig. 6 Step response of generators speed deviation without any controllers

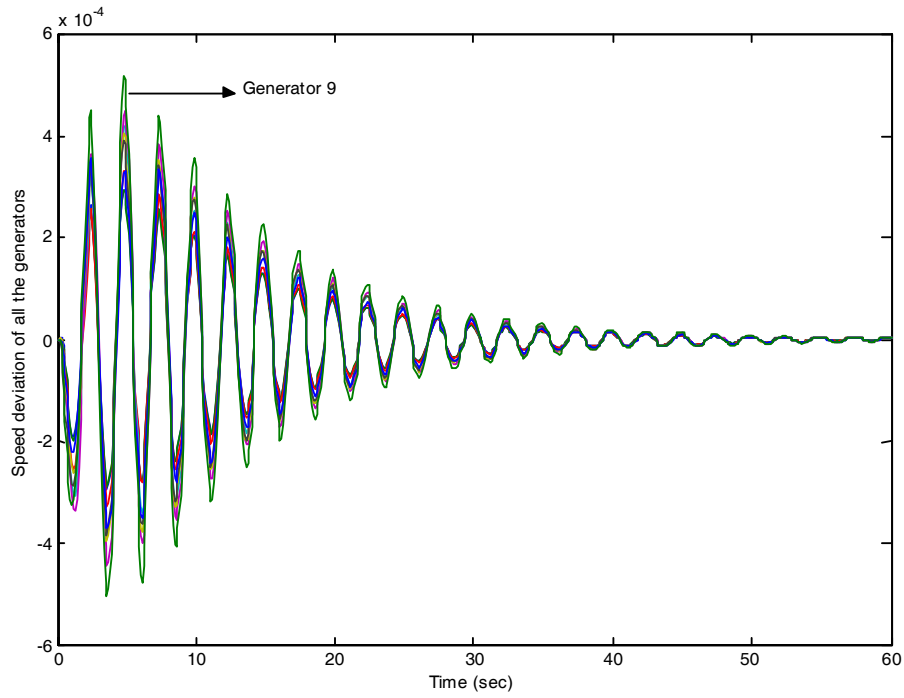


Fig. 7 Step response of the generator speed deviation with only PSS

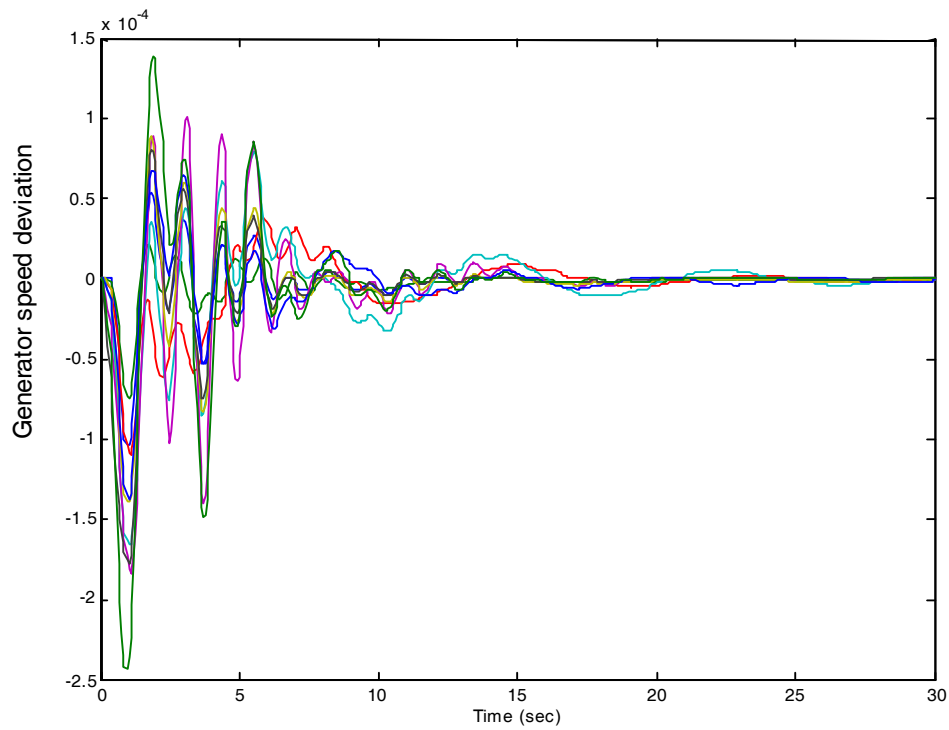


Fig. 8 Step response of the generators speed deviation with PSS, SVC & TCSC controllers

6. CONCLUSIONS

In this work, an optimal control strategy with pseudo-decentralization has been proposed for coordination of power system stabilizers. This proposed method coordinated all the power system stabilizers simultaneously and hence the negative interactions between the controllers are taken in to consideration. The FACTS controllers can also be used for the damping of the oscillations in the system. Two FACTS controllers viz, SVC and TCSC are considered along with supplementary controllers to damp out oscillations in the system. The supplementary controllers of the FACTS have been coordinated extending the proposed optimal control strategy with pseudo-decentralization. By analyzing the small signal stability of the system with and without controllers, it was found that there are many under-damped modes without any controller. After coordinating the power system stabilizer and the FACTS supplementary controllers, under-damped modes were eliminated. The small signal stability is verified by a step response, which clearly shows the effectiveness of the coordinated controllers in improving the system stability and also reducing the settling time.

7. ACKNOWLEDGEMENTS

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9. APPENDIX

The differential equations of an i^{th} generator including the exciter are given below:

$$\begin{aligned}
 T'_{doi} \frac{dE'_{qi}}{dt} &= -E'_{qi} - (X_{di} - X'_{di})I_{di} + E_{fdi} & i = 1, \dots, m \\
 T'_{qoi} \frac{dE'_{di}}{dt} &= -E'_{di} - (X_{qi} - X'_{qi})I_{qi} & i = 1, \dots, m \\
 \frac{d\delta}{dt} &= \omega_i - \omega_s & i = 1, \dots, m \\
 M_i \frac{d\omega_i}{dt} &= T_{mi} - [E'_{qi} - X'_{di}I_{di}]I_{qi} - [E'_{di} - X'_{qi}I_{qi}]I_{di} - D_i(\omega_i - \omega_s) & i = 1, \dots, m \\
 T_{Ai} \frac{dE_{fdi}}{dt} &= -E_{fdi} + K_{Ai}(V_{refi} - V_i) & i = 1, \dots, m
 \end{aligned} \tag{33}$$

where,

- δ = Machine rotor angle with respect to a synchronously rotating frame,
- $\dot{\delta}$ = Rotor speed,
- M = Machine inertia,
- T_m = Mechanical input torque,
- E'_m = Quadrature axis induced voltage behind the transient reactance,
- E'_d = Direct axis induced voltage behind the transient reactance,
- I'_q = Quadrature axis stator current,
- I'_d = Direct axis stator current,
- X'_q = Quadrature axis stator steady state reactance,
- X'_d = Direct axis stator steady state reactance,
- X''_q = Quadrature axis stator transient reactance,
- X''_d = Direct axis stator transient reactance,
- E_{fd} = Voltage induced due to rotor field excitation,
- K_A = Regulator amplifier gain,
- T_A = Regulator time constant,
- V_{ref} = Reference voltage setting, and
- V = Generator terminal voltage.