The Optimal Power Flow Algorithm Considering Load Power Factor Limits

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ABSTRACT

Efficient reactive power operation enhances economic operation of the power system. To provide reliable and economic system operation, the acceptable load power factor must be analyzed according to the demand level of load bus. This paper presents two-bus equivalent method combined with stability index analysis to calculate acceptable load power factor range. The above method begins with searching the critical load bus with the line stability index, where the two-bus equivalent circuit is applied to determine the acceptable load power factor limit for the critical load bus. The acceptable range of load power factors is portrayed as bandwidths of load power factor expressed as a function of load level. After critical bus selection and load power factor calculation, an optimal power flow considering load power factor is carried out to optimize reactive power source. A case study using an IEEE 26 buses system is presented to demonstrate the suggested OPF method, and results show that system loss is reduced as well as load power factors are improved.

1. INTRODUCTION

Large power systems are exposed to many disturbances. Recent requirements for more intensive use of available generation and transmission have magnified the possible effects of these disturbances. Many of these disturbances directly affect voltage control and stability.

Inadequate reactive power support from generation units, transmission lines and load power factor correction leads to voltage instability or voltage collapse, which has resulted in several major system failures. The voltage control and instability phenomenon are not new to power system managers, operators, engineers and researchers.

Sufficient reactive resources must be located throughout the power systems, with a balance between static devices and dynamic devices that change in response to control signals. Static and dynamic reactive power resources are needed to supply the reactive power requirements of customer demands and the reactive power losses in the generation, transmission and distribution systems, and provide adequate system voltage including both support and control. They are also necessary to avoid voltage instability and widespread system collapse in case of certain contingencies. Transmission and distribution systems cannot perform their intended functions without an adequate reactive power supply.

Reactive power sources must be distributed throughout the power system among the generation, transmission, and distribution facilities, as well as at load (customer) locations. Because reactive demands and facility loadings are constantly changing, coordination of transmission, distribution and generation reactive power is required. Unlike active or real power (MW), reactive power (MVAR) cannot be transmitted over long distances and must be supplied locally.

The installation of a reactive source close to the load improves power factor, relieves reactive loading on lines and equipment, reduces transmission and distribution line voltage drop, and reduces active and reactive losses. Location of the reactive source closest to the load minimizes the amount of reactive capacity needed to support voltage. Therefore, consideration of the load power factor is required to control reactive power in large power systems.

This paper proposed an optimal power flow (OPF) considering the load power factor limits. In view of the results, OPF considering the load power factor limits reduces the generator's total fuel cost and enhances the efficient generator's operation in this paper.

2. LITERATURE REVIEW FOR LOAD POWER FACTOR

The NEPOOL Operating Procedure establishes ranges of acceptable load power factors for various NEPOOL areas to provide for reliable system reactive performance. The ranges of acceptable load power factors are portrayed as bandwidths of load power factor expressed as a function of load level. For a specific load level, the bandwidth between a pair of limiting curves represents the range of acceptable load power factors. The ranges are determined through load flow simulations at various load levels.

The methodology shall be used to establish minimum and maximum load power factor limits for each area at three discrete load levels: heavy (100%), medium (75%), and light load (40%).

A curve connects the three minimum points and another curve connects the three maximum points. The bandwidth between these two curves represents the range of load power factors that establish the standard for the area. The following figure shows an example of minimum and maximum power factors for an area, as a function of load level.

The ISO is charged with the task of evaluating the survey results. If all points for a Participant fall within the bandwidth of the curves, the standard has clearly been met. If there are points outside the bandwidth, the ranges of acceptable load power factors have not been met. The degree of noncompliance for each noncompliant point is quantified by identifying the VAR variance from the nearest curve (upper or lower). The largest variance from the lower or minimum allowable load power factor curve represents the amount of shunt capacitance that must be added to achieve compliance. Transmission Providers have the authority to develop and implement actions to achieve compliance. The largest variance from the upper or maximum allowable load power factor curve represents the amount of shunt capacitance that must be switched out, or reactors that must be added to become compliant.

On an annual basis, the ISO Staff will survey the NEPOOL Participants to measure compliance with the ranges of acceptable load power factors. For compliance purposes, the audit data will be then compared against curves, which are based on studies representative of the system at the time the audit data is collected. By surveying load power factors and comparing them against the ranges, the ISO can determine if load power factors are within reasonable tolerances for reliable operation. The survey data also provides the ISO with more accurate data for conducting system studies [2].

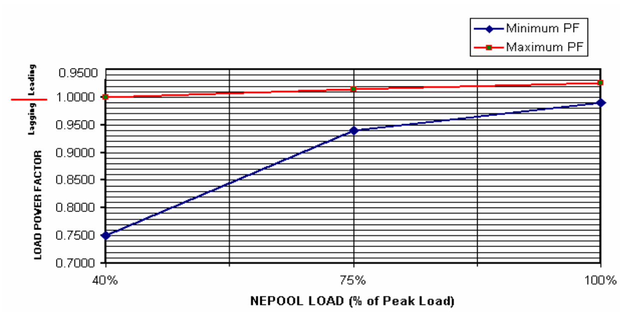


Fig. 1 Load power factor curve for a load area

3. ESTABLISHMENT OF THE LOAD POWER FACTOR

3.1 Determining The Load Power Factor Constraints

Fig. 2 shows a procedure of determining the load power factor constraints. A procedure as follows: Firstly, critical bus is determined using line stability index. Secondly, the load power factor limits are established by two-bus equivalent. IEEE 26 bus test system was chosen to carry out the test of the OPF considering the load power factor limits.

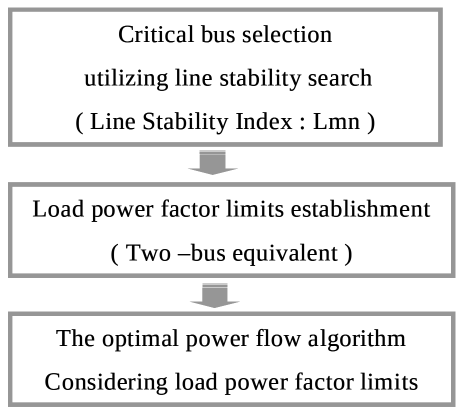


Fig. 2 Procedure of OPF considering load power factor limits

3.2 Critical Bus Selection Utilizing Line Stability Index

In conventional research of critical bus selection method, Moghavvemi [3] proposed an overall system stability index based on the concept of power flow through a single line. A technique is used to reduce the system into a single line and then applying the method, an overall system stability is obtained. The index reflects the severity of loading as well as the stability of the system.

Moghavvemi derived a voltage stability criterion based on power transmission concept in a single line. An interconnected system is reduced to a single line network and then applied to assess the overall system stability. Utilizing the same concept but using it for each line of the network a stability criterion is developed which will be used to assess the system security.

Let us consider a single line of an interconnected network, where the lines are connected through a grid network. Any of the lines from that network can be considered to have the following parameters.

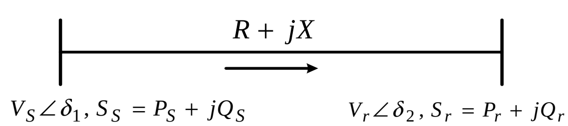


Fig. 3 One line diagram of a typical transmission line

Utilizing the concept of power flow in the line and analyzing with ‘π’ model representation, the power flow at the sending and receiving end can be represented as:

$$S_s = \frac{|V_s|^2}{Z}\angle\theta - \frac{|V_s||V_r|}{Z}\angle(\theta + \delta_1 - \delta_2) \tag{1}$$

$$S_r = \frac{|V_s||V_r|}{Z}\angle(\theta - \delta_1 + \delta_2) - \frac{|V_r|^2}{Z}\angle\theta \tag{2}$$

From the above equation of power we can separate the real and the reactive power

$$P_r = \frac{V_s V_r}{Z} cos(\theta - \delta_1 + \delta_2) - \frac{V_r^2}{Z} cos\,\theta \tag{3}$$

$$Q_r = \frac{V_s V_r}{Z} sin(\theta - \delta_1 + \delta_2) - \frac{V_r^2}{Z} sin\,\theta \tag{4}$$

Now putting $\delta_1 - \delta_2 = \delta$ into (4), and solving it for V_r .

$$V_r = \frac{V_s \sin(\theta - \delta) \pm \left\{[V_s \sin(\theta - \delta)]^2 - 4ZQ_r \sin\theta\right\}^{0.5}}{2\sin\theta} \tag{5}$$

Now for $Z\, sin\, 6 = x$, we are obtained as (6).

$$V_r = \frac{V_s \sin(\theta - \delta) \pm \left\{[V_s \sin(\theta - \delta)]^2 - 4xQ_r\right\}^{0.5}}{2\sin\theta} \tag{6}$$

In order to obtain real values of V_r in terms Q_r the equation derived above must have real roots. Thus the following conditions, which can be used as a stability criterion, need to be satisfied.

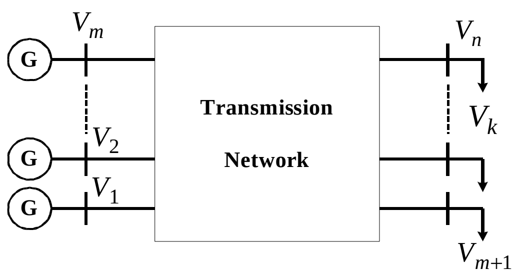
$$\begin{gathered} \left\{[V_s\ sin(\theta - \delta)]^2 - 4xQ_r\right\}^{0.5} \geq 0 \\ or \\ \frac{4xQ_r}{[V_s\ sin(\theta - \delta)]^2} = L_{mn} \leq 1.00 \end{gathered} \tag{7}$$

where; $L_{mn} = 21$ the stability index of that line.

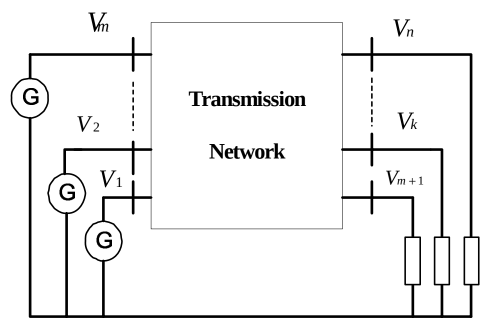
The above stability criterion is used to find out the stability index for each line connected between two buses in an interconnected network. Considering each line of the network as a single line we can use the above stability formula to calculate the line stability index. Based on the stability indices of lines, voltage collapse can be accurately predicted. As long as the stability index L_{mn} remains less than 1, the system is stable and when this index exceeds the value 1, the whole system loses its stability and voltage collapse occurs. Using this technique of calculating line stability index, we can monitor the status of all the connected lines of the network i.e. we can identify the lines which are in stressed condition and also we would be able to locate the exact location of voltage collapse.

3.3 Two-bus Equivalent

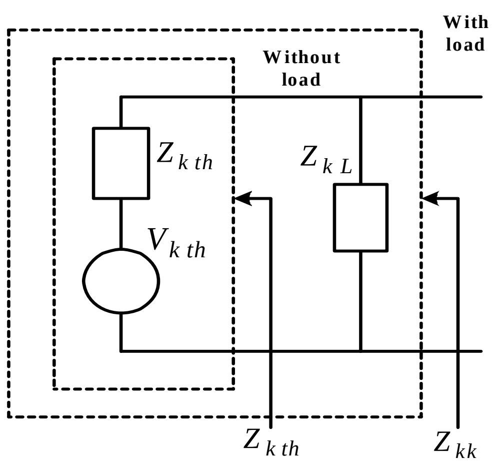
Consider a general power system consisting of n buses as shown in Fig. 4(a). It is assumed that the first m buses (1 to m) are the generator or voltage controlled buses where the voltage magnitudes are kept constant and the rest buses (m+1 to n) are the voltage uncontrolled buses. In general, a generator in a power system is modeled by an internal voltage source behind a series reactance. However, in LF simulations, the generator adjusts its reactive power output to maintain a constant terminal voltage. In this respect, a generator can be modeled by a constant voltage source with no series reactance. This model can preserve the terminal characteristic of the generator even for the change in operating conditions and thus can faithfully be used to determine the voltage stability limit by repetitive LF simulations. The load of a voltage uncontrolled bus can be modeled by constant shunt impedance. The above generator and load models are shown in Fig. 4(b). It may be mentioned here that when a generator cannot maintain the constant terminal voltage because of finite reactive power limit, it can be modeled as a negated load and the corresponding bus is to be considered as a voltage uncontrolled type.



(a)



(b)



(c)

Fig. 4 (a), (b), and (c) Process of finding the two-bus equivalent of a general power system

The *k*-th diagonal element Z_{kk} of bus impedance matrix of Fig. 4(b) represents the Thevenin impedance of bus *k*. The Thevenin voltage V_K of the bus can be obtained from the base-case LF solution. Note that the above equivalent includes the load impedance Z_K^L of the candidate bus. However, in voltage stability studies, it is necessary to consider the load as an external element of the Thevenin equivalent circuit. The parameters of such an equivalent can easily be obtained from Z_{KK} and V_K with the help of Fig. 4(c).

$$Z_{KTh} = \frac{Z_{KK} Z_K^L}{Z_K^L - Z_{KK}} \tag{8}$$

$$V_{KTh} = \left(1 + \frac{Z_{KTh}}{Z_K^L}\right) V_K \tag{9}$$

3.4 Load Power Factor Limits Establishment

Fig. 5 shows a simple single-line diagram of the considered two-bus system in this paper. The generator at bus 1 transfers power through a transmission line having an impedance of $Z = R_{KTh} + jX_{KTh}$ to a load center at bus 2. For a given load $(S_K = P_K + jQ_K)$ at the selecting critical bus, the load current (I_{KL}) and voltage (V_K) can be expressed as (10).

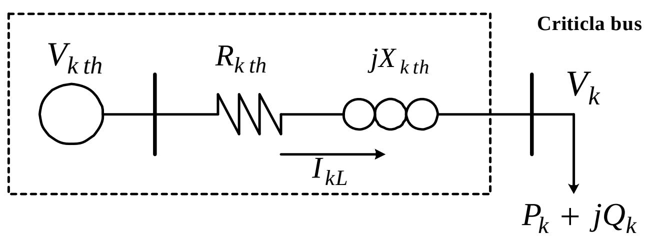


Fig. 5 Two-bus equivalent of power system

$$I_{KL} = \frac{P_K - jQ_K}{V_K^*} \tag{10}$$

$$V_K = V_{KTh} - Z_{KTh} I_{KL} \tag{11}$$

After some mathematical manipulations of the above two equations, the load voltage magnitude V_K of the critical bus can be expressed as (12).

$$V_K^4 + 2(R_{KTh}P_K + X_{KTh}Q_K)V_K^2 - V_{KTh}^2 V_K^2 + (R_{KTh}^2 + X_{KTh}^2)(P_K^2 + Q_K^2) = 0 \tag{12}$$

3.5 Problem Formulation

General OPF has been quickly developed and widely used for the purposes of economical and secure operation and planning of power systems. The OPF has been applied to regulating the generator active power outputs and voltages, shunt capacitors and reactors, transformer tap-settings and other controllable variables to minimize the fuel cost while keeping the load bus voltages, generator reactive power outputs, network power flows and all other state variables in the power system in their operational and secure limits.

The objective function of OPF is expressed as follows:

$$\text{Min} \qquad f\,cos\,t = \sum_{i \in Ni} \left(a_i + b_i P_{gi} + c_i P_{gi}^2 \right) \tag{13}$$

Subject to:

$$\begin{gathered} 0 = P_i - V_i \sum_{j \in Ni} V_j \left(G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij} \right) \\ 0 = Q_i - V_i \sum_{j \in Ni} V_j \left(G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij} \right) \\ P_{gi}^{min} \le P_{gi} \le P_{gi}^{max}, Q_{gi}^{min} \le Q_{gi} \le Q_{gi}^{max} \\ T_k^{min} \le T_k \le T_k^{max}, V_i^{min} \le V_i \le V_i^{max} \\ Q_{ci}^{min} \le Q_{ci} \le Q_{ci}^{max} \end{gathered} \tag{14}$$

where, P_g = active power generation,

Q_g = reactive power generation,

T_k = Tap-setting,

V_i = load bus voltages, and

Q_c = reactive power of shunt capacitor/ reactor.

In (14), the power flow equations are used as equality constraints, the active and reactive power generation restrictions, shunt capacitor/reactor reactive power restrictions, apparent power flow restrictions in branches, transformer tap-setting restrictions and bus voltage restrictions are used as inequality constraints. The active power generation at the slack bus, load bus voltages, reactive power generations and branch apparent power flows are state variables, which are restricted by adding them as the quadratic penalty terms to the objective function to form a penalty function. Therefore, (13) is changed to the following generalized objective function.

$$\begin{aligned} \text{Min} \quad f\,cos\,t = & \sum_{i \in Ni} \left(a_i + b_i P_{gi} + c_i P_{gi}^2 \right) + \sum_{i \in N_B} \lambda_{Vi} \left(V_i - V_i^{lim} \right)^2 + \sum_{i \in N_h} \lambda_{Qg} \left(Q_{gi} - Q_{gi}^{lim} \right)^2 \\ & + \lambda_{P_{gs}} \left(P_{gs} - P_{gi}^{lim} \right)^2 + \sum_{i \in N_{sc}} \lambda_{PF} \left(PF_i - PF_i^{lim} \right)^2 \end{aligned} \tag{15}$$

Subject to:

$$
\begin{aligned}
0 &= P_i - V_i \sum_{j \in Ni} V_j \left(G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij} \right) \\
0 &= Q_i - V_i \sum_{j \in Ni} V_j \left(G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij} \right) \\
& P_{gi}^{min} \le P_{gi} \le P_{gi}^{max}, Q_{ci}^{min} \le Q_{ci} \le Q_{ci}^{max}, T_k^{min} \le T_k \le T_k^{max}
\end{aligned} \tag{16}
$$

where, $\lambda_{Vi}, \lambda_{Qg}, \lambda_{P_{gs}}, \lambda_{PF_i} =$ each penalty factor.

The inequality constraints are the only control variable, so they are self-restricted. $V_i^{lim}, Q_{gi}^{lim}, P_{gs}^{lim}$ and PF_i^{lim} are defined in the following (17).

$$
\begin{aligned}
V_i^{lim} &= \begin{cases} V_i^{min} & if \quad V_i \langle V_i^{min} \\ V_i^{max} & if \quad V_i \rangle V_i^{max} \end{cases} \\
Q_{gi}^{lim} &= \begin{cases} Q_{gi}^{min} & if \quad Q_{gi} \langle Q_{gi}^{min} \\ Q_{gi}^{max} & if \quad Q_{gi} \rangle Q_{gi}^{max} \end{cases} \\
P_{gs}^{lim} &= \begin{cases} P_{gs}^{min} & if \quad P_{gs} \langle P_{gs}^{min} \\ P_{gs}^{max} & if \quad P_{gs} \rangle P_{gs}^{max} \end{cases} \\
PF_i^{lim} &= \begin{cases} PF_i^{min} & if \quad PF_i \langle PF_i^{min} \\ PF_i^{max} & if \quad PF_i \rangle PF_i^{max} \end{cases}
\end{aligned} \tag{17}
$$

4. CASE STUDY

4.1 Critical Bus Selection Utilizing Line Stability Index

Table 1 presents the results of selecting critical bus by the line stability index. As can be seen Table 1, bus 17 and 21 are founded the value of line stability indices, which are very high. Therefore, bus 17 and 21 is selected as critical buses.

Table 1 Line stability index of IEEE 26

	From	To	Index		From	To	Index		From	To	Index		From	To	Index
1	1	2	0.0042	12	6	7	0.0000	23	10	22	0.0466	34	16	20	0.1248
2	1	18	0.0454	13	6	11	0.6559	24	11	25	0.2751	35	17	18	0.2632
3	2	3	0.1504	14	6	18	0.0952	25	11	26	0.1272	36	17	21	0.8805
4	2	7	0.0000	15	6	19	0.0639	26	12	14	0.0718	37	19	23	0.1158
5	2	8	0.0000	16	6	21	0.1330	27	12	15	0.0126	38	19	24	0.2509
6	2	13	0.0854	17	7	8	0.0000	28	13	14	0.0207	39	19	25	0.2635
7	2	26	0.2389	18	7	9	0.1350	29	13	15	0.0118	40	20	21	0.2906
8	3	13	0.0051	19	8	12	0.0548	30	13	16	0.1564	41	20	22	0.0653
9	4	8	0.0000	20	9	10	0.0000	31	14	15	0.0075	42	21	24	0.3197
10	4	12	0.0602	21	10	12	0.0433	32	15	16	0.1044	43	22	23	0.0915
11	5	6	0.0540	22	10	19	0.2521	33	16	17	0.5181	44	22	24	0.1909

4.2 Determining Load Power Factor Limits

In this section, the ranges of load power factor are portrayed as bandwidths of load power factor, and the load power factor limits are expressed as a function of load level with selected critical buses. The load power factor limits can be obtained from (12), when the load voltage (V_K) should not drop below a specified value (0.95 [PU]).

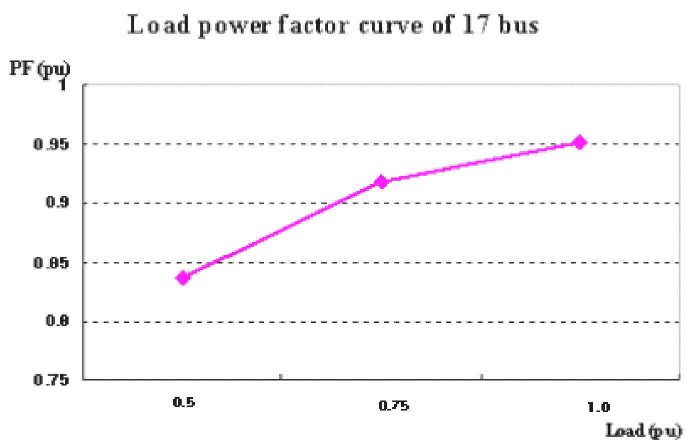


Fig. 6 Load power factor curve of bus 17

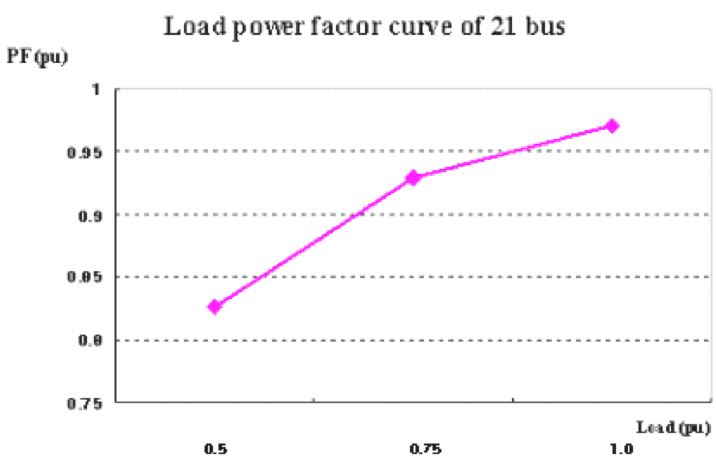


Fig. 7 Load power factor curve of bus 21

4.3 Simulation Results

Table 2 shows the results that OPF considering the load power factor limits compared with case of power flow and the general OPF. The fuels cost savings are given using the proposed OPF with load power factor limits as follows.

Case 1: Power flow compared with OPF without PF limits

$$\text{Saving (\%)} = \frac{19710.9 - 17599.44}{19710.9} \times 100 = 10.7\,[\%]$$

Case 2: OPF without PF limits compared with OPF with PF limits

$$\text{Saving (\%)} = \frac{17599.44 - 17581.90}{17599.44} \times 100 = 0.1\,[\%]$$

Table 2 Total Fuel Cost Comparison

Case	P_g	Q_g	S_g	Cost ($/h)
Power Flow	1440.562	686.477	1595.7661	19710.9
OPF Without PF	1435.053	607.647	1588.4005	17599.44
OPF With PF	1433.025	592.023	1550.5005	17581.90

The load power factor limits are considered the reduction in total generator fuel cost from 17599.44 $/h to 17581.90 $/h (0.1 [%]). A yearly saving cost is 153650 $. Also, generator's apparent power is reduced from 1588.40 MVA to 1550.50 MVA.

5. CONCLUSIONS

For efficient and reliable operation of power system, the acceptable range load power factor limits is required. Because of the adequate reactive power operation enhances the economic and stable operation of the power system.

This paper presents a numerical formula expression for load power factor limits. The Two-bus equivalent method is used for an acceptable load power factor range.

As a result, OPF considering the load power factor limits reduces the generator's total fuel cost and enhances the efficient generator's operation.

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7. APPENDIX

Table 3 Generator Data

Generator power limits and operating costs						
Bus	1	2	3	4	5	26
$P_{g\ max}$	700	200	300	150	200	120
$P_{g\ min}$	100	50	80	50	50	50
$Q_{g\ max}$	-	250	150	80	160	50
$Q_{g\ min}$	-	-40	-40	-40	-40	-15
a	240	200	220	200	220	190
b	7.0	10.	8.5	11.0	10.5	12.0
c	0.007	0.0095	0.009	0.009	0.008	0.0075