Ramp Rate Constrained Unit Commitment by Improved Adaptive Lagrangian Relaxation

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ABSTRACT

This paper proposes an improved adaptive Lagrangian relaxation (ILR) for ramp rate constrained unit commitment problem. The proposed ILR minimizes the total supply cost subject to the power balance, 15 minute spinning reserve response time constraint, generation ramp limit constraints, minimum up and new down constraints. ILR is improved by new minimum down time to account for startup and shut down ramp constraints, new initialization to obtain a high quality initial solution, dynamic economic dispatch to include the operating ramping limits, and adaptive Lagrangian multipliers to speedup the convergence. If the 24 hour schedule is feasible, dynamic economic dispatch by quadratic programming is used to minimize the production cost subject to the power balance and new generation ramp operating frame limit. For hours with insufficient 15 minute response time spinning reserves, repairing strategy by quadratic programming is used to minimize the generator fuel cost subject to power balance, generator operating ramp rate limit, and 15 minute spinning reserve response time constraints. The proposed ILR algorithm is tested on the 26 unit IEEE reliability test system. It is shown that ILR could obtain a higher quality solution than dynamic economic dispatch based on artificial neural network.

1. INTRODUCTION

Unit commitment (UC) is used to schedule the generators such that the total system production cost over the scheduled time horizon is minimized under the spinning reserve and operational constraints of generator units. UC problem is a nonlinear, mixed integer combinatorial optimization problem. The global optimal solution can be obtained by complete enumeration, which is not applicable to large power systems due to its excessive computational time requirements [1].

Ramp rate constrained unit commitment (RUC) was solved by mixed integer linear programming (MILP) [2], enhanced dynamic programming [3], artificial neural network (ANN) [4], Lagrangian relaxation (LR) [5-12], augmented Lagrangian relaxation (ALR) [13-15], and dynamic priority list [16]. In [4], a unit commitment solution was first obtained without ramping constraints. Thereafter, a dynamic adjusting process is adopted to obtain unit commitment schedule considering the ramping constraints. To satisfy the startup process, the constrained units were adjusted by changing their status from ‘0’ to ‘1’ for a few hours earlier, leading to overcommitment. In [7], the objective function was augmented with an additional ramping cost, which was related to the depreciation in shaft life.

LR is the most efficient method to solve UC problem [17]. LR decomposes the main problem into unit subproblems solved by dynamic programming. This simplifies the problem significantly. However, ramping constraints in UC problem requires enlarging state spaces dramatically [6], [8], [9], [14], for dynamic programming to solve each unit subproblem. The total number of states is the sum of number of down states, number of ramp up states, number of up states, and number of ramp down states [6].

The additional Lagrangian multipliers corresponds to ramping constraints for each hour and each unit were proposed in [8], [9], and [11]. These extra Lagrangian multipliers cause additional computational burden, leading to a slower convergence rate. Furthermore, backward economic dispatch is required to reduce the generation output from its constrained maximum to zero under the ramp down limit within one hour [4].

This paper proposes an improved adaptive Lagrangian relaxation (ILR) for ramp rate constrained unit commitment problem. The proposed ILR minimizes the total supply cost subject to the power balance, 15 minute spinning reserve response time constraints, generation ramp limit constraints, minimum up and new down constraints without enlarging state spaces for dynamic programming to solve unit subproblems. ILR is improved by new minimum down time to account for startup and shut down ramp constraints, new initialization to obtain a high quality initial solution, dynamic economic dispatch to include the operating ramping limits, and adaptive Lagrangian multipliers to speedup the convergence. If the 24 hour schedule is feasible, dynamic economic dispatch by quadratic programming is used to minimize the production cost subject to the power balance and new generation ramp operating frame limit. For hours with insufficient 15 minute response time spinning reserves, repairing strategy by quadratic programming is used to minimize the generator fuel cost subject to power balance, generator operating ramp rate limit, and 15 minute spinning reserve response time constraints. The proposed ILR algorithm is tested on the 26 unit IEEE reliability test system.

The organization of this paper is as follows. Section 2 addresses the RUC problem formulation. The ILR algorithm is described in Section 3. Numerical results are presented in Section 4. Lastly, the conclusion is given.

2. RAMP RATE CONSTRAINED UNIT COMMITMENT PROBLEM FORMULATION

In this paper, three types of ramping constraints for each unit are considered.

- Startup ramp constraints: when an off-line unit is turned on, it takes $T_{i,SR}$ minutes to increase its generation output from zero to its minimum level as shown in Fig. 1a. During this period, this unit is off-line but consuming fuel.
- Shut down ramp constraints: when an online unit is turned off, it takes some times to decrease its generation output from the current generation output to zero. However, decommitting the online unit with generation output higher than minimum level is not allowed unless it is forced shut down (generation shedding). This is because before the unit status is changed from '1' at hour t to '0' at hour t+1, this unit is operated at the equilibrium point where mechanical input power equals to electrical output power. At hour t+1 when this unit is disconnected from the system, the electrical power of this unit is changed from its current generation to zero. This results in accelerating power on the rotor and causes transient instability, which may further affect the generator's life time and its performance. Hence, the generation output of this unit should be at the minimum level at the last committed hour [9]. It will take $T_{i,SDR}$ minutes to decrease its generation output from its minimum level to zero as shown in Fig. 1b.

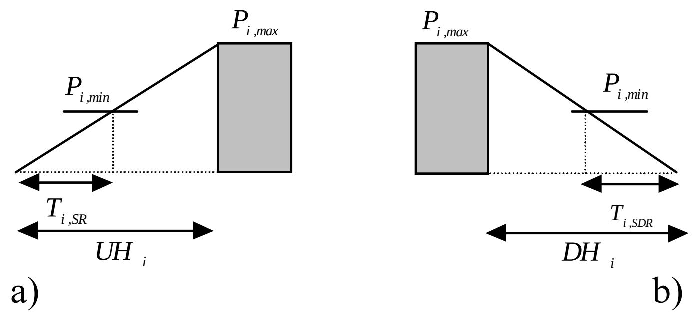

Fig. 1 Startup and shut down ramp constraints

- Operating ramp constraints: The generation output of current hour must be limited by the up ramp limit and down ramp limit,

$$P_i^t - P_i^{t-1} \le UR_i \cdot 60 \text{, as generation increases} \quad (1)$$

$$P_i^{t-1} - P_i^t \le DR_i \cdot 60 \text{, as generation decreases} \quad (2)$$

Unit ramp rate constraints require modification of operating constraints (unit basis) and spinning reserve constraints (system basis). On the unit basis, the change of the generation level of each unit on any two successive periods must be within ramp rate limitation. The sum of ramp limits of committed units must be at least sufficient to meet the change in the system load from one hour to the next hour. On the system basis, the spinning reserve amount contributed by each unit must be calculated by considering ramp rate constraints. The spinning reserve calculated from the difference between maximum committed power and actual load can be large enough to meet the reserve requirement, but ramping limitations may cause the actual available spinning insufficient. In this paper, generation ramp limit, new minimum down time, and unit reserve contribution are used.

Generation ramp limit: To satisfy the generation operating limit constraint in (1) and (2), generation ramp limit, $P_{i,high}^t$ and $P_{i,low}^t$ are used as shown in Fig. 2.

New Minimum down time: To satisfy the shut down and startup ramping constraints, the generation output of the shut down hour and the startup hour is limited to its minimum level. The new minimum down time is extended by the time for shut down process (from minimum generation level to zero) and time for startup process (from zero to minimum generation level) as shown in Fig. 3.

$$NT_{i,down} = T_{i,SDR} + T_{i,down} + T_{i,SR} \text{ .} \quad (3)$$

Unit reserve contribution: Due to the unit ramp up limits, the spinning reserve contributed by each unit within τ minutes is

$$r_i^t = min \ [P_{i,max} - P_i^t , \ \tau \cdot UR_i] \text{ .} \quad (4)$$

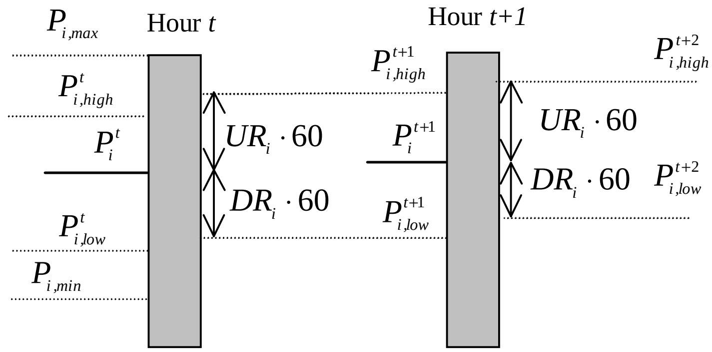

Fig. 2 Generation ramp limit for unit i

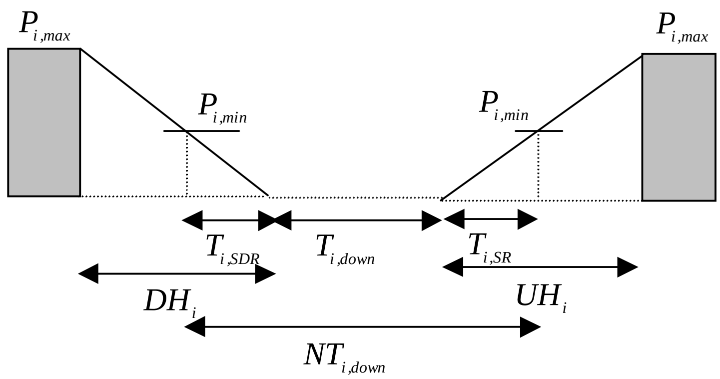

Fig. 3 New minimum down time

The objective of UC problem is to minimize the production cost over the scheduled time horizon (e.g. 24 hours) under the generator operational, power balance, and spinning reserve constraints. The objective function to be minimized is:

$$F(P_i^t, U_{i,t}) = \sum_{t=1}^{T} \sum_{i=1}^{N} [F_i(P_i^t) + ST_{i,t}(1 - U_{i,t-1})] U_{i,t} \tag{5}$$

Subject to:

(a) Power balance constraint

$$P_{load}^t - \sum_{i=1}^{N} P_i^t U_{i,t} = 0, \tag{6}$$

(b) 15 minute spinning reserve response time constraint

$$R^t - \sum_{i=1}^{N} r_i^t U_{i,t} \le 0, \tag{7}$$

(c) Generation ramp limit constraints

$$P_{i,low}^t U_{i,t} \le P_i^t \le P_{i,high}^t U_{i,t}\,,\ i = 1,\ldots,N, \tag{8}$$

where,

$$P_{i,high}^t = \begin{cases} \min[P_{i,max}, P_i^{t-1} + UR_i \cdot 60]\,, \text{ if } U_{i,t} = U_{i,t-1} = 1\,, \\ P_{i,min}\,, \text{ if } U_{i,t-1} = 0,\ U_{i,t} = 1\,, \end{cases} \tag{9}$$

$$P_{i,low}^t = \begin{cases} \max[P_{i,min}, P_i^{t-1} - DR_i \cdot 60]\,, \text{ if } U_{i,t} = U_{i,t-1} = 1\,, \\ P_{i,min}\,, \text{ if } U_{i,t-1} = 0,\ U_{i,t} = 1\,. \end{cases} \tag{10}$$

(d) Minimum up and new down time constraints

$$U_{i,t} = \begin{cases} 1, \text{ if } T_{i,on}^{t-1} < T_{i,up}\,, \\ 0, \text{ if } T_{i,off}^{t-1} < NT_{i,down}, \\ 0 \text{ or } 1, \quad \text{otherwise,} \end{cases} \tag{11}$$

(e) Startup cost

$$ST_i = [\chi_i + \delta_i (1 - exp(\frac{-T_{i,off}^{t-1}}{\gamma_i})]\,, \tag{12}$$

where, χ_i, δ_i, and γ_i are startup cost parameters.

3. AN IMPROVED ADAPTIVE LAGRANGIAN RELAXATION FOR RUC

The LR procedure solves the UC problem by decomposing the main problem into subproblems which are coupled by the Lagrangian multipliers.

The LR procedure solves the UC problem by relaxing or temporarily ignoring the coupling constraints and solving the problem as if they did not exist. This is done through the dual optimization procedure attempting to reach the constrained optimum by maximizing the Lagrangian,

$$L(P,U,\lambda,\mu) = F(P_i^t, U_{i,t}) + \sum_{t=1}^{T} \lambda^t (P_{load}^t - \sum_{i=1}^{N} P_i^t U_{i,t}) + \sum_{t=1}^{T} \mu^t (R^t - \sum_{i=1}^{N} r_i^t U_{i,t}), \tag{13}$$

With respect to nonnegative λ^t and μ^t whereas minimizing it with respect to the other control variables in the problem, that is:

$$q^*(\lambda,\mu) = \underset{\lambda^t,\mu^t}{\text{Max}}\, q(\lambda,\mu)\,, \tag{14}$$

where,

$$q(\lambda,\mu) = \underset{P_i^t, U_{i,t}}{\text{Min}} \; L(P,U,\lambda,\mu) \,. \tag{15}$$

Eqs. (6) and (7) are the coupling constraints across the units. In particular, what is done to one unit affects the other units. The Lagrangian function is rewritten as:

$$L = \sum_{i=1}^{N} \sum_{t=1}^{T} \{[F_i(P_i^t) + ST_{i,t}(1-U_{i,t-1})]U_{i,t} - \lambda^t P_i^t U_{i,t} - \mu^t r_i^t U_{i,t}\} + \sum_{t=1}^{T} (\lambda^t P_{load}^t + \mu^t R^t). \tag{16}$$

The term $\sum_{t=1}^{T} \left\{ [F_i(P_i^t) + ST_{i,t}(1-U_{i,t-1})]U_{i,t} - \lambda^t P_i^t U_{i,t} - \mu^t r_i^t U_{i,t} \right\}$ can be minimized separately for each generating unit, when the coupling constraints are temporarily ignored. Then the minimum of the Lagrangian function is solved for each generating unit over the time horizon, that is

$$\underset{P_i^t, U_i^t}{\text{Min}} \; L(P,U,\lambda,\mu) = \sum_{i=1}^{N} \min \sum_{t=1}^{T} \{[F_i(P_i^t) + ST_{i,t}(1-U_{i,t-1})]U_{i,t} - \lambda^t P_i^t U_{i,t} - \mu^t r_i^t U_{i,t}\},$$

Subject to $U_{i,t} P_{i,low}^t \le P_i^t \le U_{i,t} P_{i,high}^t$ for $t = 1,\ldots,T$, and the constraints in (11).

3.1 Dynamic Programming

In the conventional Lagrangian relaxation method, the dual solution is obtained by using dynamic programming for each unit separately. Since generation ramp limit and new minimum down time are used to account for the ramping constraints, the dynamic programming does not require enlarging state spaces. This can be visualized in Fig. 4 showing the only two possible states for unit i (i.e., $U_{i,t} = 0$ or 1):

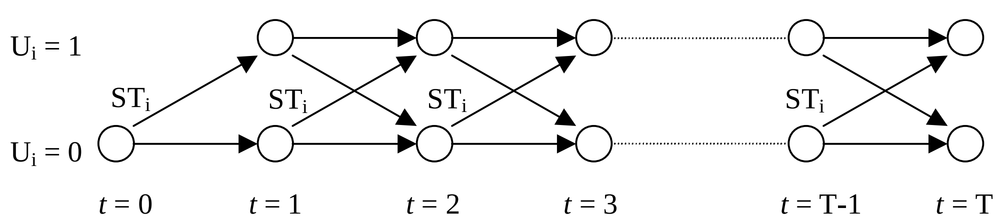

Fig. 4 Search paths of each unit in dynamic programming

At the $U_{i,t} = 0$ state, the value of the function to be minimized is trivial (i.e., it equals zero), at the state where $U_{i,t} = 1$, the function to be minimized is (the start-up cost is dropped here since the minimization is with respect to P_i^t) $\min[F_i(P_i^t) - \lambda^t P_i^t - \mu^t r_i^t]$.

To find the dual power, the term $\min[F_i(P_i^t) - \lambda^t P_i^t - \mu^t r_i^t]$ will be minimized by the optimality condition,

$$\frac{d}{dP_i^t}[F_i(P_i^t) - \lambda^t P_i^t - \mu^t r_i^t] = 0. \tag{17}$$

The solution to this equation is

$$\frac{dF_i(P_i^{t,opt})}{dP_i^t} = \lambda^t - \mu^t. \tag{18}$$

The dual power is obtained,

$$P_i^{t,opt} = \frac{\lambda^t - \mu^t - b_i}{2c_i}. \tag{19}$$

There are three cases to check $P_i^{t,opt}$ against its limits:

1. If $P_i^{t,opt} < P_{i,low}^t$, $P_i^t = P_{i,low}^t$.
2. If $P_{i,low}^t \le P_i^{t,opt} \le P_{i,high}^t$, $P_i^t = P_i^{t,opt}$.
3. If $P_i^{t,opt} > P_{i,high}^t$, $P_i^t = P_{i,high}^t$.

Dynamic programming is used to determine the optimal schedule of each unit over the scheduled time period. More specifically, for each state in each hour, the on/off decision making is needed to select the lower cost by comparing the combination of the start up cost and the accumulated costs from two historical routes. At hour *t*, the dual power calculated by (19) and within the limit in (8), will be substituted in

$$[F_i(P_i^t) + ST_{i,t}(1 - U_{i,t-1})]U_{i,t} - \lambda^t P_i^t U_{i,t} - \mu^t r_i^t U_{i,t} \tag{20}$$

Then, dynamic programming searches for the optimal scheduling for each unit to obtain the lowest value of the term

$$\sum_{t=1}^{T} \{[F_i(P_i^t) + ST_{i,t}(1 - U_{i,t-1})]U_{i,t} - \lambda^t P_i^t U_{i,t} - \mu^t r_i^t U_{i,t}\} \tag{21}$$

Subject to minimum up and down time constraints and generation ramp limit in (8).

3.2 Initialization

The initial values of Lagragian multipliers are very critical to the LR solution since they may prevent LR from reaching the optimal solution or require a longer computational time to reach one [18]. Different initial values may also lead LR to different solutions. In [14], the initial multiplier λ^t was set to the hourly system marginal cost of the schedule to satisfy the power balance constraint and the initial multiplier μ^t was set to zero, leading to an infeasible initial solution. On the other hand, the initial multiplier λ^t was set to the hourly system marginal cost of the schedule to satisfy both the power balance and spinning reserve constraint, whereas the initial multiplier, μ^t was set to zero which was generally lower than the optimal value μ^t [19].

Our initialization procedure intends to create a high quality feasible schedule in the first iteration. The generating units are sorted in the ascending order of full load average production cost, $FL_{avg}(P_{i,max})$. For each of the 24 hours, the unit with the least $FL_{avg}(P_{i,max})$ will be committed one by one until the power balance constraint is satisfied. Subsequently, economic dispatch in each hour is carried out to obtain the hourly equal lambda which is initially set to Lagrangian multipliers $\lambda^{t(0)}$. For the hours with insufficient spinning reserve requirement considering the maximum capacity constraint

$$P_{load}^t + R^t - \sum_{i=1}^{N} P_{i,max} U_{i,t} \leq 0, \tag{22}$$

More unit(s) are needed to be committed to give the initial feasible solution. This is obtained by committing a unit with the least $FL_{avg}(P_{i,max})$ one by one until the spinning reserve is satisfied.

For each of the 24 hours, each nonnegative $\mu_i^{t(0)}$ is determined by the upper bound of zero on/off decision criteria of the committed unit *i* as follows:

$$\mu_i^{t(0)} = \max[\frac{1}{P_{i,max}}[F_i(P_i^t) + \frac{\chi_i + \delta_i}{T_{i,up}} - \lambda^{t(0)} P_i^t],0]. \tag{23}$$

The initial $\mu^{t(0)}$ is determined by the highest $\mu_i^{t(0)}$ among the committed units as:

$$\mu^{t(0)} = \max[\mu_1^{t(0)},...,\mu_m^{t(0)}] \tag{24}$$

where, *m* is the marginal unit with the highest $FL_{avg}(P_{i,max})$ giving the sufficient spinning reserve at hour *t*.

3.3 Adaptive Updating of the Lagrangian multiplier

In general, adjusting Lagrangian multiplier by subgradient method is not efficient in the presence of the spinning reserve constraint [20]. The LR performance is heavily dependent on the method used to update the multipliers. In this paper, the Lagrangian multiplier update rule is to design the large step size at the beginning of iterations and smaller as the iteration grows as proposed in [18]. The values of α and β are determined heuristically [21]. Each nonnegative λ^t and μ^t are adaptively updated by:

$$\lambda^{t(k)} = \max[\lambda^{t(k-1)} + \frac{pdif^t}{(\alpha + \beta \times k) \times norm(pdif)}, 0], \tag{25}$$

where,

$$pdif^{\,t} = P_{load}^{t} - \sum_{i=1}^{N} P_i^t U_{i,t}\,, \tag{26}$$

$$norm(pdif) = \sqrt{(pdif^{\,1})^2 + (pdif^{\,2})^2 + \ldots\ldots\ldots + (pdif^{\,T})^2}\,, \tag{27}$$

$$\mu^{t(k)} = \max[\mu^{t(k\text{-}1)} + \frac{rdif^{\,t}}{(\alpha + \beta \times k) \times norm(rdif)}, 0], \tag{28}$$

where,

$$rdif^{\,t} = max(P_{load}^{t} + R^t - \sum_{i=1}^{N} P_{i,max} U_{i,t}\,,\;\; R^t - \sum_{i=1}^{N} \tau \cdot UR_i \cdot U_{i,t}), \tag{29}$$

$$norm(rdif) = \sqrt{(rdif^{\,1})^2 + (rdif^{\,2})^2 + \ldots\ldots\ldots + (rdif^{\,T})^2} \tag{30}$$

α and β are divided into three cases depending on the signs of $pdif^t$ and $rdif^t$ as follows:

Case 1: $pdif^t \geq 0$ and $rdif^t$ 0 : updating both λ^t and μ^t by using $\alpha = 0.02$ and $\beta = 0.05$

Case 2: $pdif^t < 0$ and $rdif^t$ 0 : updating both λ^t and μ^t by using $\alpha = 0.6$, $\beta = 0.3$

Case 3: $pdif^t < 0$ and $rdif^t > 0$: updating only μ^t by using $\alpha = 0.02$, $\beta = 0.05$

In fact, updating these two multipliers λ^t and μ^t in hour *t* must move them in the same direction. In hour *t*, if $pdif^t$ and $rdif^t$ have the same signs, either positive or negative, λ^t and μ^t will be updated (increase or decrease) by (25) and (28) respectively. When the total dual generation output is larger than the load in that hour ($pdif^t < 0$) but the spinning reserve is insufficient ($rdif^t > 0$), more committed unit(s) are required to satisfy the spinning reserve constraints. However, updating λ^t by (25), will decrease its value, resulting in committing less units. Therefore, when $pdif^t < 0$ and $rdif^t > 0$, only μ^t will be updated.

3.4 Dynamic Economic Dispatch

The unit ramping constraint links the generation output of the previous hour to that of the present hour, thus introducing a dynamic characteristic in the economic dispatch procedure known as dynamic economic dispatch (DED).

Since forward economic dispatch starts from the first hour to the last hour, the resulting generation level at the last committed hour may be higher than the minimum level before decommitting this unit (shut down generation level constraint). Similarly, backward economic dispatch, starting from the last hour to the first hour may result in the generation level at the first committed hour higher than the minimum level (startup generation level constraint). If the economic dispatch is initially performed at the maximum demand hour as in [15], performing forward economic dispatch for the subsequent hours and backward economic dispatch for the previous hours does not guarantee that the startup and shut down generation level constraints will be satisfied. Therefore, in this paper, the new limit frame consisting of down and up frame is proposed for dynamic economic dispatch. The down frame limit is used to guarantee that the generation output at the last committed hour will be at its minimum power output before decommitting. On the other hand, the up frame limit is used to guarantee that the generation output at the first committed hour will be at its minimum power output.

At the last committed hour, the generation output is set at the minimum level. Thus, at the hour prior to this hour, the upper operating limit is set by a down frame limit, $DF_{i,high}^{t_l - t_b}$, as shown in Fig. 5(a).

$$DF_{i,high}^{t_l-t_b} = \begin{cases} \min[P_{i,max},\ P_{i,min} + DR_i \cdot 60 \cdot t_b], \text{ if } t_l = T-1,\ldots,2, t_b \geq 0 \text{ ,and } U_{i,t_l-t_b} = 1, \\ P_{i,max}, \text{ if } U_{i,t} = 1, t = 1,\ldots,T, \\ 0, \text{ if } U_{i,t_l-t_b} = 0, \end{cases} \tag{31}$$

Afterward, a limit frame is constructed starting from the first scheduled hour to the last scheduled hour. At the first committed hour, the generation output is set at the minimum level. Thus, at the subsequent hours, the upper operating limit is set by an up frame limit, $F_{i,high}^{t_f - t_a}$, as shown in Fig. 5(b).

$$F_{i,high}^{t_f-t_a} = \begin{cases} \min[DF_{i,high}^{t_f-t_a},\ P_{i,min} + UR_i \cdot 60 \cdot t_a], \text{ if } U_{i,t_f-t_a} = 1, t_f = 1,\ldots,T, \text{ and } t_a \geq 0, \\ P_{i,max}, \text{ if } U_{i,t} = 1, t = 1,\ldots,T, \end{cases} \tag{32}$$

A whole frame for unit *i* is shown in Fig. 6, the dotted block representing the unconstrained capacity, regardless ramp rate limit, whereas the bold line representing ramp rate limit frame.

After the complete frame for each unit is obtained, the economic dispatch is performed initially at the hour that corresponds to the maximum system demand. Then, the dispatching process proceeds forward and backward in the other time intervals.

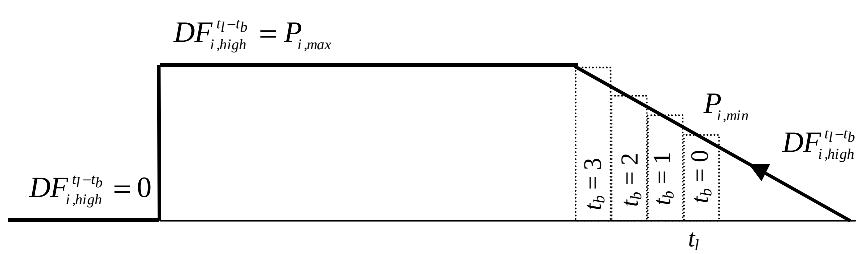

Fig. 5(a) An down frame for unit *i*

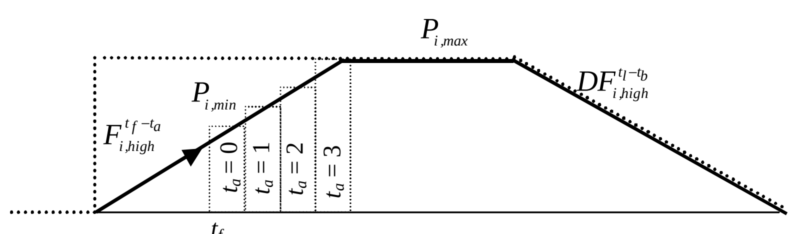

Fig. 5(b) An up frame for unit *i*

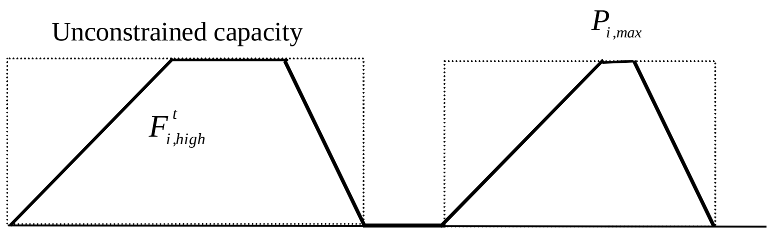

Fig. 6 A whole frame for unit *i*

In this paper, the unit commitment schedule obtained from dynamic programming described in Section 4.1 is checked for the feasibility for each hour by considering the maximum capacity constraint (22), reserve ramping capacity constraint

$$R^t - \sum_{i=1}^{N} \tau \cdot UR_i \cdot U_{i,t} \leq 0, \tag{33}$$

and committed unit constraint

$$\sum_{i=1}^{N} P^t_{i,low} U_{i,t} \leq P^t_{load}. \tag{34}$$

The 15 minute spinning reserve response time constraint (7) is needed to guarantee that the sum of the reserve contribution by each unit determined by both the difference between its capacity and current generation, and ramping capability is at least sufficient to meet the spinning reserve requirement within 15 minutes. The maximum capacity constraint (22) forces commitment of sufficient generation to meet the demand and reserve requirement. Whereas the reserve ramping capacity constraint (33) is necessary to assure that the committed generators have sufficient reserve ramping capacity to satisfy the reserve requirements. To satisfy the 15 minute spinning reserve response time constraint (7), both maximum capacity constraint (22) and reserve ramping capacity constraint (33) must be satisfied. Because satisfying only maximum capacity constraint (22), the spinning reserve may be limited by the ramping capability whereas satisfying only reserve ramping capacity constraint (33), the spinning reserve may be limited by the generation capacity. The committed unit constraint (34) assures that the demand can be met with all dispatch generators loaded above their respective lower limits.

If the 24 hour schedule does not violate (22) and (33), hourly dynamic economic dispatch by quadratic programming is used to minimize the total production cost subject to power balance equation and operating limit. Otherwise, current Lagragian multipliers are not suitable to obtain a feasible schedule.

$$\underset{P^t_{i,ed}}{Minimize} \sum_{i=1}^{N} F_i(P^t_{i,ed}), \tag{35}$$

Subject to:

(a) Power balance constraint

$$P^t_{load} - \sum_{i=1}^{N} P^t_{i,ed} U_{i,t} = 0, \tag{36}$$

(b) New generation ramp operating frame limit

$$P_{i,ed-low}^{t} U_{i,t} \le P_{i,ed}^{t} \le P_{i,ed-high}^{t} U_{i,t}\ ,\ i = 1,\ldots,N, \tag{37}$$

where,

$$P_{i,ed-high}^{t} = \begin{cases} \min[F_{i,high}^{t}, P_{i,ed}^{t-1} + UR_i \cdot 60],\ \text{if}\ U_{i,t} = U_{i,t-1} = 1, \\ P_{i,min},\ \text{if}\ U_{i,t-1} = 0,\ U_{i,t} = 1\ \text{or}\ U_{i,t} = 1, U_{i,t+1} = 0, \end{cases} \tag{38}$$

$$P_{i,ed-low}^{t} = \begin{cases} \max[P_{i,min}, P_{i,ed}^{t-1} - DR_i \cdot 60],\ \text{if}\ U_{i,t} = U_{i,t-1} = 1, \\ P_{i,min},\ \text{if}\ U_{i,t-1} = 0,\ U_{i,t} = 1\ \text{or}\ U_{i,t} = 1, U_{i,t+1} = 0. \end{cases} \tag{39}$$

$P_{i,ed-high}^{t}$ in (38) and $P_{i,ed-low}^{t}$ in (39) are different from $P_{i,high}^{t}$ in (9) and $P_{i,low}^{t}$ in (10). $P_{i,ed-high}^{t}$ uses the limit frame as the upper limit instead of the maximum real power generation, $P_{i,max}$ which is used for $P_{i,high}^{t}$. In addition, the new generation ramp operating frame limit is set to the minimum real power generation, $P_{i,min}$ whenever the unit status is changed either from ‘0’ to ‘1’ or from ‘1’ to ‘0’.

3.5 Repairing Strategy

To meet the system load without violating ramp up rate, backward economic dispatch was used in [4]. However, backward economic dispatch generally may end up with an infeasible solution, violating 15 minute response time spinning reserve constraint. Thus, forward and backward economic dispatch strategies were used to resolve the problem in [11]. Initially, the forward economic dispatch is performed. If a feasible solution can not be found from the forward path, a backward economic dispatch is needed to satisfy the 10 minute response time spinning reserve constraint. Constructing both forward and backward paths require excessive computing requirement. Starting at the maximum system demand hour as in [15], performing forward economic dispatch for the subsequent hours and backward economic dispatch for the previous hours does not guarantee that the 10 minute spinning reserve constraint will be satisfied, especially for the sharp increase or decrease loading level. In this paper, repairing strategy is proposed to relieve the 15 minute response time spinning reserve constraint. For each hour, after the dynamic economic dispatch in Section 4.4 is performed and the generation power outputs, $P_{i,ed}^{t}$ are obtained, the 15 minute spinning reserve response time constraint in (7) is checked. If it is violated, the generation power outputs will be redispatched. The committed units are classified into two categories, shown in Fig. 7.

Category 1: Units whose generations can increase without reducing their reserve contribution and their generations satisfy condition,

$$P_{i,max} - P_{i,ed}^{t} \ge \tau \cdot UR_i\ . \tag{40}$$

Their redispatched output power must not be less than their generation dispatched in Section 4.4. Therefore, their new lower limit is set by their dispatched power output,

$$P^t_{i,ed-low} = P^t_{i,ed}\ ,\ \ i \in \Omega_1\ . \tag{41}$$

Whereas the new upper limit is set by,

$$\text{If } P^t_{i,ed-high} > \left(P_{i,max} - \tau \cdot UR_i\right),\ \ P^t_{i,ed-high} = \left(P_{i,max} - \tau \cdot UR_i\right),\ \ i \in \Omega_1\ . \tag{42}$$

Category 2: Units whose reserve contribution can be increased if their generations are decreased and their generations satisfy condition,

$$P_{i,max} - P^t_{i,ed} < \tau \cdot UR_i\ . \tag{43}$$

Their redispatched output power must not be more than their generation dispatched in Section 4.4. Therefore, their new upper limit is set by their dispatched power output,

$$P^t_{i,ed-high} = P^t_{i,ed}\ ,\ \ i \in \Omega_2\ . \tag{44}$$

Whereas the new lower limit is set by,

$$\text{If } P^t_{i,ed-low} < \left(P_{i,max} - \tau \cdot UR\right)_i,\ \ P^t_{i,ed-low} = \left(P_{i,max} - \tau \cdot UR_i\right),\ \ i \in \Omega_2\ . \tag{45}$$

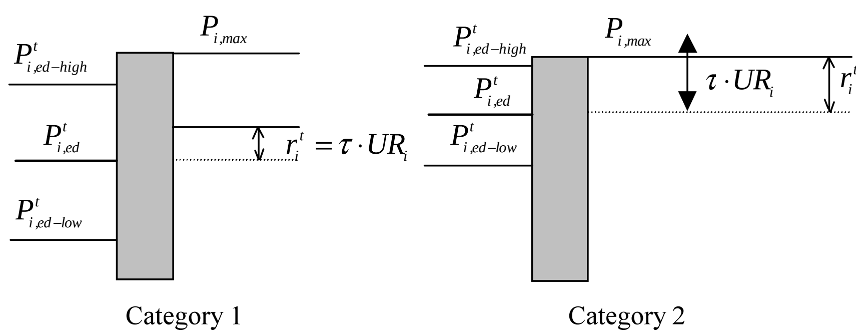

Fig. 7 Classification committed units into category 1 and 2

In this paper, the repairing strategy by quadratic programming is proposed to minimize the production cost in (35) subject to balance constraint (36), operating limit (37), 15 minute spinning reserve response time constraint (7) and additional constraints,

$$\sum_{i \in \Omega_1}^{N} P^t_{i,ed} = TP^t_{\Omega_1} + \Delta SPR^t\ , \tag{46}$$

$$\sum_{i \in \Omega_2}^{N} P^t_{i,ed} = TP^t_{\Omega_2} - \Delta SPR^t\ , \tag{47}$$

where,

$$\Delta SPR^t = R^t - \sum_{i=1}^{N} r_i^t U_{i,t} \; . \tag{48}$$

3.6 Stopping Criteria

The relative duality gap is

$$G^{(k)} = \frac{J([U_{i,t}^{(k)}]) - L(P^{(k)}, U^{(k)}, \lambda^{(k)}, \mu^{(k)})}{L(P^{(k)}, U^{(k)}, \lambda^{(k)}, \mu^{(k)})}\,, \tag{49}$$

where, $J([U_{i,t}^{(k)}]) = \sum_{t=1}^{T}\sum_{i=1}^{N}[F_i(P_{i,ed}^t) + ST_{i,t}(1 - U_{i,t-1})]U_{i,t}$ and $L(P^{(k)}, U^{(k)}, \lambda^{(k)}, \mu^{(k)})$ is calculated from (16).

The relative duality gap is used to measure the solution quality, by checking against the stopping criteria. The iteration process stops when either the relative duality gap is less than the specified tolerance or the iteration counter exceeds the maximum allowable number of iterations.

3.7 Overall Procedure

Step 1: Initialize λ^t and μ^t described in Section 4.2.
Step 2: Initialize the ILR iteration counter, $k = 1$, and $J_B = \$10^7$.
Step 3: Solve the unit subproblems by using dynamic programming described in Section 4.1.
Step 4: If the dual solution does not satisfy maximum capacity constraint (22), reserve ramping capacity constraint (33), and committed unit constraint (34), go to Step 10.
Step 5: Carry out the dynamic ED by quadratic programming described in Section 4.4. If 15 minute spinning reserve response time constraint (7) is satisfied, go to Step 7.
Step 6: Perform repairing strategy by quadratic programming described in Section 4.5.
Step 7: Calculate the primal cost $J([U_{i,t}^{(k)}])$, the dual cost $L(P^{(k)}, U^{(k)}, \lambda^{(k)}, \mu^{(k)})$ and the relative dual gap $G^{(k)}$ as described in Section 4.6.
Step 8: If $J([U_{i,t}^{(k)}]) < J_B$, $J_B = J([U_{i,t}^{(k)}])$ and $[U_{i,t}^{B}] = [U_{i,t}^{k}]$.
Step 9: If the relative dual gap $G^{(k)} < \varepsilon$, go to Step 11.
Step 10: If $k < K_{max}$, $k = k + 1$, update Lagrangian multiplier adaptively as described in Section 4.3 and return to Step 3.
Step 11: Terminate ILR.

4. NUMERICAL RESULTS

The ILR algorithm is tested on the IEEE reliability system consisting of 26 generating units [4], [22], and [23]. In the simulation, the daily load, unit data are shown in Appendix. There are two cases:

Case 1: To compare the results with those in [4], the spinning reserve is calculated from the difference between the maximum capacity and the current generation output of each unit. The spinning reserve requirement is set to the largest committed unit with respect to the constraint in (22), neglecting the 15 minute spinning reserve response time constraint in (7).

Case 2: The spinning reserve is calculated from unit reserve contribution within 15 minutes. The spinning reserve is set to 4% of the total load demand.

In both cases, there are two load levels, load A and B. Load B is smaller than load A, thus there are more medium size units to start up and shut down. ILR computational times are obtained from a pentium IV, 1.6 GHz personal computer.

As shown in Table 1, the total cost obtained from ILR is lower than ANN [4] in both load levels. The CPU times may not be directly comparable because the computer used in [4] was not specified. In addition, ILR could be used with 15 minute spinning reserve response time constraints as shown in Table 2.

Table 1 Production Cost in Case 1

Load	Method	CPU time (s)	Cost($)
Load A	ANN [4]	12.0	729,326.5
	ILR	76.9	725,996.9
Load B	ANN [4]	14.0	613,653.6
	ILR	119.6	594,116.5

Table 2 Production Cost in Case 2

Load	CPU time (S)	Cost($)
Load A	161.5	720,641.9
Load B	122.0	576,625.7

5. CONCLUSIONS

In this paper, the proposed ILR is efficiently and effectively implemented to solve the RUC problem. ILR total production costs over the scheduled time horizon are less expensive than dynamic economic dispatch based on artificial neural network. ILR is suitable for RUC due to the substantial production cost savings.

6. ACKNOWLEDGEMENTS

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7. NOMENCLATURE

DH_i = the shut down time of unit i to decrease its generation from $P_{i,max}$ to zero (h),

DR_i = the ramp down rate limit of unit i (MW/min),

$DF_{i,high}^{t_l - t_b}$	=	the down frame of unit i at t_b hours prior to the last committed hour t_l (MW),
$F_i(P_i^t)$	=	the generator fuel cost function in a quadratic form,
$F_i(P_i^t)$	=	$a_i + b_i P_i^t + c_i \left(P_i^t\right)^2$ (\$/h),
$F_{i,high}^{t_f - t_a}$	=	the limit frame of unit i at t_a hours after the first committed hour t_f (MW),
$FL_{avg}(P_{i,max})$	=	the full load average production cost of unit i, $F(P_{i,max}) / P_{i,max}$ (\$/MWh),
$G^{(k)}$	=	the relative duality gap at iteration k,
J_B	=	the best total economic dispatch production cost reached (\$),
$J([U_{i,t}^{(k)}])$	=	the total economic dispatch production cost at iteration k (\$),
k	=	the ILR iteration counter,
K_{max}	=	the maximum allowable number of iterations,
N	=	the total number of generator units,
$NT_{i,down}$	=	the new minimum down time of unit i (h),
$P_{i,high}^t$	=	the highest possible power output of unit i at hour t (MW),
$P_{i,low}^t$	=	the lowest possible power output of unit i at hour t (MW),
$P_{i,ed-high}^t$	=	the highest possible dispatched power output of unit i at hour t (MW),
$P_{i,ed-low}^t$	=	the lowest possible dispatched power output of unit i at hour t (MW),
$P_{i,min}$	=	the minimum real power generation of unit i (MW),
$P_{i,max}$	=	the maximum real power generation of unit i (MW)
P_i^t	=	the generation output power of unit i at hour t (MW),
$P_{i,ed}^t$	=	the economic dispatch generation output of unit i at hour t (MW),
P_{load}^t	=	the load demand at hour t (MW),
r_i^t	=	the unit reserve contribution at hour t, $\min[P_{i,max} - P_i^t, \tau \cdot UR_i]$ (MW),
R^t	=	the spinning reserve at hour t (MW),
$ST_{i,t}$	=	the startup cost of unit i at hour t (\$),
T	=	the total number of hours,
$T_{i,down}$	=	the minimum down time of unit i (h),
$T_{i,on}^{t-1}$	=	the continuously on time of unit i up to hour t-1,
$T_{i,off}^{t-1}$	=	the continuously off time of unit i up to hour t-1,
$T_{i,SR}$	=	the time for unit i to increase its generation from zero to $P_{i,min}$ (h),
$T_{i,SDR}$	=	the time for unit i to decrease its generation from $P_{i,min}$ to zero (h),
$T_{i,up}$	=	the minimum up time of unit i (h),
$TP_{\Omega_1}^t$	=	the total generation output of units in category 1 at hour t (MW),

$TP_{\Omega_2}^{t}$	=	the total generation output of units in category 2 at hour t (MW),
t_a	=	the number of hours after the first committed hour,
t_b	=	the number of hours prior to the last committed hour,
t_f	=	the first committed hour,
t_l	=	the last committed hour,
$U_{i,t}$	=	the status of unit i at hour t (on = 1, off = 0),
UH_i	=	the startup time of unit i to increase its generation from zero to $P_{i,max}$ (h),
UR_i	=	the ramp up rate limit of unit i (MW/min),
$[U_{i,t}^{B}]$	=	the best feasible solution reached,
ΔSPR^t	=	the deficit spinning reserve at hour t (MW),
ε	=	the relative duality gap tolerance,
Ω_1	=	the set of committed unit classified in category 1,
Ω_2	=	the set of committed unit classified in category 2,
τ	=	the reserve response time frame (i.e., 10-15 min),
$\lambda^{t(0)}, \mu^{t(0)}$	=	the initial Lagrange multipliers at hour t (\$/MWh, \$/MW),
$\lambda^{t(k)}, \mu^{t(k)}$	=	the Lagrange multipliers at hour t at iteration k (\$/MWh, \$/MW), and
$\mu_i^{t(0)}$	=	the initial Lagrangian multiplier of unit i at hour t (\$/MW),

9. REFERENCES

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10. APPENDIX

The data were obtained from [4], [22], and [23]. The load and unit data are summarized in Tables 3, 4 and 5.

Table 3 The Daily Load A

	Hour					
	1	2	3	4	5	6
Load (MW)	1700	1730	1690	1700	1750	1850
	Hour					
	7	8	9	10	11	12
Load (MW)	2000	2430	2540	2600	2670	2590
	Hour					
	13	14	15	16	17	18
Load (MW)	2590	2550	2620	2650	2550	2530
	Hour					
	19	20	21	22	23	24
Load (MW)	2500	2550	2600	2480	2200	1840

Table 4 The Daily Load B

	Hour					
	1	2	3	4	5	6
Load (MW)	1430	1450	1400	1350	1350	1470
	Hour					
	7	8	9	10	11	12
Load (MW)	1710	2060	2300	2380	2290	2370
	Hour					
	13	14	15	16	17	18
Load (MW)	2290	2260	2190	2130	2190	2200
	Hour					
	19	20	21	22	23	24
Load (MW)	2300	2340	2300	2180	1910	1650

Table 5 Unit Data

Unit	$P_{i,min}$ (MW)	$P_{i,max}$ (MW)	c_i (k$)	b_i (k$/MW)	a_i (k$/MW^2)	$T_{i,up}$ (h)	$T_{i,down}$ (h)	Init. Stat. (h)	UH_i (h)	DH_i (h)	UR_i (MW/min)	DR_i (MW/min)	$NT_{i,down}$ (h)	χ_i ($)	δ_i ($)	γ_i (h)
1	2.4	12.0	0.02533	25.5472	24.3891	0	0	-1	0	0	0.8	1.0	0	0	0	1
2	2.4	12.0	0.02649	25.6753	24.4110	0	0	-1	0	0	0.8	1.0	0	0	0	1
3	2.4	12.0	0.02801	25.8027	24.6382	0	0	-1	0	0	0.8	1.0	0	0	0	1
4	2.4	12.0	0.02842	25.9318	24.7605	0	0	-1	0	0	0.8	1.0	0	0	0	1
5	2.4	12.0	0.02855	26.0611	24.8882	0	0	-1	0	0	0.8	1.0	0	0	0	1
6	4.0	20.0	0.01199	37.5510	117.7511	0	0	-1	1	0	0.508	1.167	1	20	20	2
7	4.0	20.0	0.01261	37.6637	118.1083	0	0	-1	1	0	0.508	1.167	1	20	20	2
8	4.0	20.0	0.01359	37.7770	118.4576	0	0	-1	1	0	0.508	1.167	1	20	20	2
9	4.0	20.0	0.01433	37.8896	118.8206	0	0	-1	1	0	0.508	1.167	1	20	20	2
10	15.2	76.0	0.00876	13.3272	81.1364	3	2	3	2	1	0.642	1.333	3	50	50	3
11	15.2	76.0	0.00895	13.3538	81.2980	3	2	3	2	1	0.642	1.333	3	50	50	3
12	15.2	76.0	0.00910	13.3805	81.4641	3	2	3	2	1	0.642	1.333	3	50	50	3
13	15.2	76.0	0.00932	13.4073	81.6259	3	2	3	2	1	0.642	1.333	3	50	50	3
14	25.0	100.0	0.00623	18.0000	217.8952	4	2	-3	2	2	0.850	1.233	3	70	70	4
15	25.0	100.0	0.00612	18.1000	218.3350	4	2	-3	2	2	0.850	1.233	3	70	70	4
16	25.0	100.0	0.00598	18.2000	218.7752	4	2	-3	2	2	0.850	1.233	3	70	70	4
17	54.25	155.0	0.00463	10.6940	142.7348	5	3	5	3	2	0.917	1.300	5	150	150	6
18	54.25	155.0	0.00473	10.7154	143.0288	5	3	5	3	2	0.917	1.300	5	150	150	6
19	54.25	155.0	0.00481	10.7367	143.3179	5	3	5	3	2	0.917	1.300	5	150	150	6
20	54.25	155.0	0.00487	10.7583	142.5972	5	3	5	3	2	0.917	1.300	5	150	150	6
21	68.95	197.0	0.00259	23.0000	259.1310	5	4	-4	4	2	0.917	1.650	6	200	200	8
22	68.95	197.0	0.00260	23.1000	259.6490	5	4	-4	4	2	0.917	1.650	6	200	200	8
23	68.95	197.0	0.00263	23.2000	260.1760	5	4	-4	4	2	0.917	1.650	6	200	200	8
24	140.0	350.0	0.00153	10.8616	177.0575	8	5	10	5	3	1.167	2.000	6	300	200	8
25	100.0	400.0	0.00194	7.4921	310.0021	8	5	10	8	4	0.842	1.667	8	500	500	10
26	100.0	400.0	0.00195	7.5031	311.9102	8	5	10	8	4	0.842	1.667	8	500	500	10